ON SHAPE COEXISTENCE AND SHAPE ISOMERISM IN NEUTRON DEFICIENT PT–HG–PB NUCLEI*

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A Fourier parametrization of nuclear shapes and the macroscopic–microscopic method are used to evaluate the potential energy surfaces (PES) of nuclei in a 4-dimensional deformation space. The effect of different orientations in space of the nucleus is taken into account when deformations higher than quadrupole are considered. The effect on the PES of exact solving the pairing eigenproblem is also studied.

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1. Introduction

In recent years, neutron-deficient nuclei in the Pt–Hg–Pb region have been intensively studied experimentally (*cf.*, *e.g.*, [1–7]). Several excited 0^+ states corresponding to the shape isomers were found in these nuclei. These discoveries have motivated us to study the potential energy surfaces (PES) of even–even Pt, Hg, and Pb isotopes. The present research is a continuation of our previous studies [8], which were primarily focused on super-deformed (SD) shape isomers. In the present paper, the PESs are evaluated in the 4-dimensional Fourier-over-spheroid shape parametrization (η, c, a_3, a_4)

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[9, 10], where c stands for elongation, a_3 for the left–right asymmetry, a_4 for the neck degree of freedom, and η allows for non-axial deformations. One has shown that in this region of nuclei and for the considered deformation parameter range, the left–right asymmetry plays no role, *i.e.*, $a_3 \approx 0$ corresponds to an energy minimum in all (η, c, a_4) grid points.

2. Results

The set of deformation parameters (η, c, a_4) is projected, with simultaneous minimization with respect to a_4 [12], onto the traditional (β, γ) Bohr parameters [11] to study the effects of different nuclear orientations in space [12]. Potential energy surfaces (PES) are evaluated in the framework of the macroscopic–microscopic method based on the Lublin–Strasbourg-Drop (LSD) [13] macroscopic energy and the Strutinsky+BCS microscopic energy corrections obtained using the Yukawa-folded [14] mean-field potential. Examples of the PES for a few Pt, Hg, and Pb isotopes are shown in Fig. 1.

One can speak of a prolate–oblate shape coexistence when both prolate and oblate minima have similar energies and are separated by a barrier of the order of the zero-point energy (≈ 0.5 MeV) or smaller. Such a situation is found in some of the considered Pt and Hg isotopes. The ground-state deformation is on the prolate side at $\beta \approx 0.16$ in all the platinum isotopes considered here, with diminishing barrier height between the prolate and oblate minima with increasing mass number. In all three platinum isotopes, an oblate, well-seen minimum is found around $\beta \approx 0.18$.

In the neighboring mercury isotopes (see Figs. 1 and 2), the ground-state deformation is located on the oblate side and parallel exists a prolate minimum. As mass increases, the ground states of Hg isotopes become successively deeper, while the prolate shape isomer becomes shallower. The deeper oblate minima in $^{184-188}$Hg nuclei are located at $\beta \approx 0.15$, which is comparable with the deformation $\beta \approx 0.13$ extracted from the BE2($2_1^+ \to 0_1^+$) values measured in Ref. [3]. In all three mercury isotopes, triaxial shape-isomeric states are found around $\gamma \approx 20^\circ$ at an elongation of $\beta \approx 0.5$.

In lead isotopes, the ground state is spherical. An oblate shape-isomeric state at energy around 1 MeV and deformation around $\beta \approx 0.2$ is seen in ^{186,190}Pb, while on the prolate side, contrary to experimental findings reported in Refs. [4–6], we have got only a kind of plateau. This discrepancy can origin from our apprixmative treatment of the pairing correlations (see next section). Another prolate SD isomer develops with increasing mass number around the deformation $\beta \approx 0.45$ and $\gamma = 0^\circ$. A triaxial shape isomer at energy ~ 6 MeV and $\{\beta \approx 0.45, \gamma \approx 27^\circ\}$ is also found in all three lead isotopes.

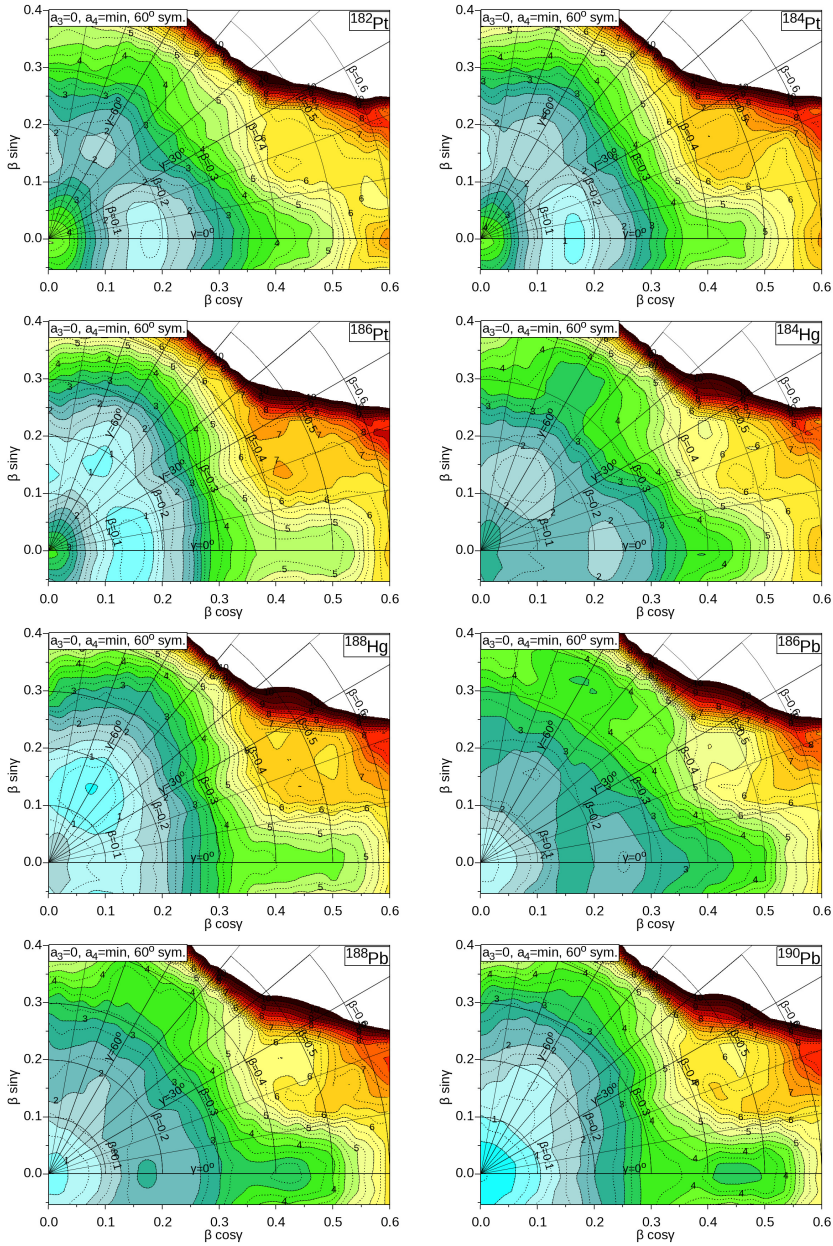

Fig. 1. (Color online) The ($\beta\,\cos\gamma$) and ($\beta\,\sin\gamma$) cross sections of the 4D PES evaluated relative to the spherical LSD energy of the corresponding $^{182-186}$Pt, ^{184,188}Hg, $^{186-190}$Pb even–even isotopes [12]. Solid lines correspond to layers separated by 0.5 MeV, with intermediate dashed lines at 0.25 MeV distance.

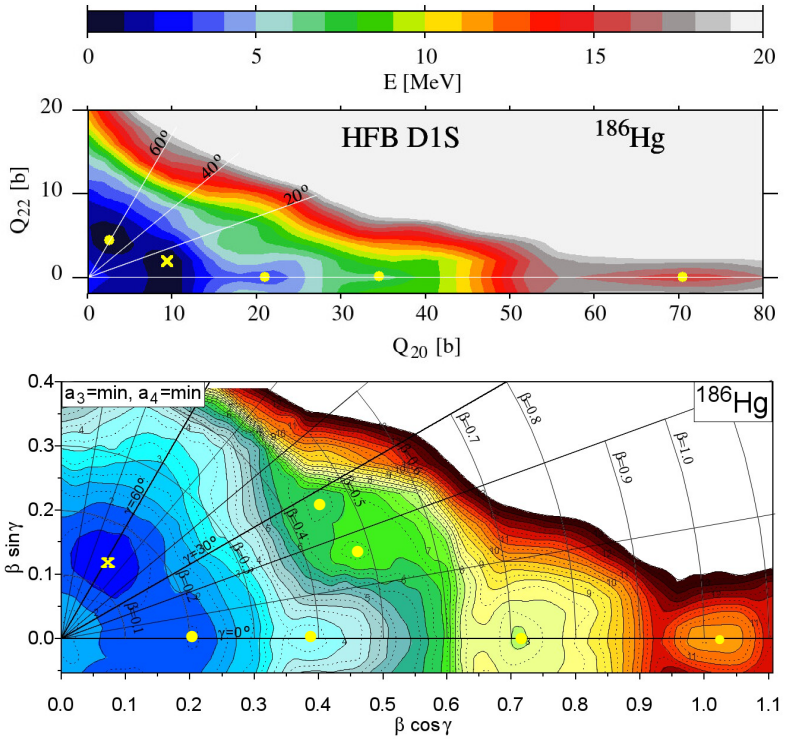

Fig. 2. (Color online) PES of ^{186}Hg evaluated within the HFB approach with the Gogny D1S force [12] (top), using constraints on the axial Q_{20} and non-axial Q_{22} mass quadrupole moments and, for comparison, a similar PES evaluated within the macroscopic–microscopic method (bottom). A cross denotes the ground state, while white/yellow dots indicate local minima.

In addition to our macroscopic–microscopic estimates, we have determined the structure of the ^{186}Hg isotope through a self-consistent HFB calculation with the Gogny D1S force using a constraint on the axial Q_{20} and non-axial Q_{22} mass quadrupole moments. The ground state was found with an almost prolate shape with a slight triaxiality of nearly $\gamma = 12°$, about 200 keV lower than the oblate minimum at $\beta \approx 0.12$. In the macroscopic–microscopic approach (bottom part of Fig. 2), the ground state shape, on the contrary, is oblate deformed and located about 300 keV lower than the prolate minimum. In both calculations, several prolate, axially symmetric, local minima are obtained, at $Q_{20} \approx 21.0$, 34.4, and 70.5 b, in the HFB approach corresponding, respectively, to values of $\beta = 0.56$, 0.79, and 1.13 obtained within the macroscopic–microscopic method. Despite minor quantitative differences, one clearly finds equivalence in the description of the structure of this nucleus across both methods, as shown in Fig. 2.

3. On exact and BCS solutions of the pairing eigenproblem

A question appears: how accurate are our predictions? In Ref. [8], we have shown that our macroscopic–microscopic model predicts well the positions and energies of the SD minima in the $^{190-194}$Hg and $^{192-196}$Pb isotopes. It means that the global (read: macroscopic) deformation dependence of the PES for these nuclei is well reproduced. On the other hand, at smaller deformations, a competition between the shell and pairing corrections is crucial for reproducing the shape coexistence phenomenon. The pairing eigenprob-

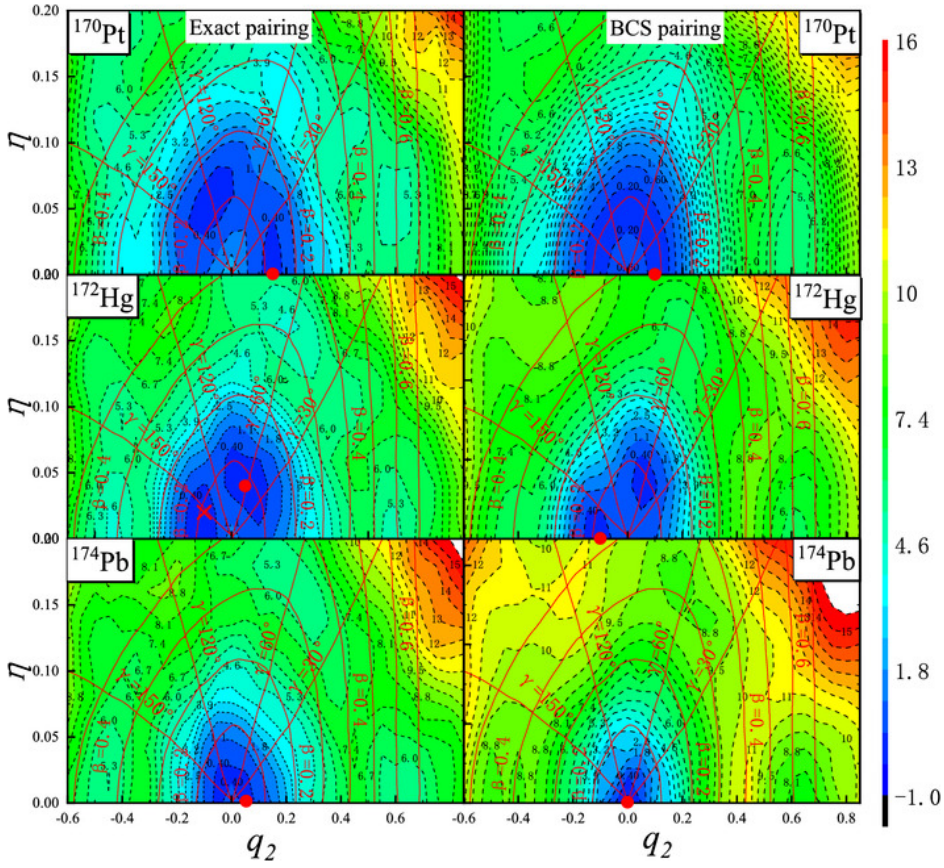

Fig. 3. (Color online) Comparison of the PESs of the nuclei ^{170}Pt, ^{172}Hg, and ^{174}Pb evaluated with an exact solution of the pairing eigenproblem (l.h.s.) and with the BCS pairing treatment (r.h.s.). The 4D PESs are projected here onto the Fourier parameter (q_2, η) 2D space [15, 16]. The ground state deformation of each isotope is denoted by a dark gray/red dot, while a coexistent minimum is indicated by a dark gray/red cross. The corresponding (β, γ) grid is marked by dark gray/red lines.

lem is usually solved in the BCS approximation, which by definition is not exact. To give the reader a sense of how accurate the BCS approximation is, we recall here the calculations by Guan and co-workers [16] performed in a similar macroscopic–microscopic model but for the lighter Pt–Hg–Pb isotopes. The maps in Fig. 3 are plotted using the Fourier deformation parameters (q_2, η) [17], but the (β, γ) grid is marked in addition for readers using this traditional parametrisation. Comparing the PES evaluated using the BCS approximation with that obtained from the exact solution of the pairing eigenproblem, one can judge the accuracy of this frequently used approximation. The pairing strengths in both exact and BCS pairing calculations are adjusted in Ref. [16] to the experimental odd–even mass differences for the platinum and lead isotopes. In Fig. 3, a comparison of the PESs of ^{170}Pt, ^{172}Hg, and ^{174}Pb obtained within the exact (l.h.s. column) and the BCS (r.h.s. column) pairing is shown. As one can see, the minima around the ground states are more pronounced in the exact PESs than the BCS ones. For small deformations, the difference between the two PESs is of the order of 0.5 MeV, which may influence our prediction of the shape coexistence. This is why we will use the exact pairing solution in our future calculations. Unfortunately, it takes much more computer time.

4. Conclusions

From our study of shape isomers in even–even nuclei of the isotopic chains of Pt, Hg, and Pb nuclei around ^{186}Hg, we conclude:

— The left–right asymmetry degree of freedom does not play an important role in any of the here-considered nuclei, and this is not only around the ground state but throughout the considered range of deformations, as long as the elongation does not become too large ($\beta < 0.6$).

— Minimizing the PESs with respect to the waist/neck parameter a_4 leads to a deeper ground-state minimum in the Pt and Hg isotopes and therefore it is important for the study of shape isomers,

— A self-consistent HFB calculation with the Gogny energy-density functional is shown to yield PESs that are close to the ones of the macroscopic–microscopic method.

— An exact pairing treatment confirms, on average, the BCS results, but tiny differences between the two estimates may be important for better reproduction of experimental data, especially the shape coexistence phenomenon.

— More extended calculations for neutron-deficient nuclei from the Pt–Hg–Pb region of nuclei, but using the exact solution of the pairing eigenproblem are planned.

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