TWO-NEUTRINO $0^+ \to 0^+$ DOUBLE-BETA DECAY OF ^{48}Ca IN THE DFT-NCCI FRAMEWORK*

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We present the results of our calculation of the nuclear matrix element for the $2\nu\beta\beta$ decay $^{48}\mathrm{Ca} \to {}^{48}\mathrm{Ti}$, performed using a post-Hartree–Fock (HF) Density Functional Theory-based No-Core Configuration-Interaction (DFT-NCCI) framework developed by our group. The preliminary value we have obtained for the nuclear matrix element describing this process, $|\mathcal{M}^{2\nu}| = 0.056(6)$ MeV^{-1}, is in excellent agreement with the results of the shell-model study by Horoi *et al.*, which yielded 0.054 (0.064) MeV^{-1} for the GXPF1A (GXPF1) interactions, respectively. It is also in reasonable agreement with the most recent experimental estimate from the review by Barabash, which is 0.068(6) MeV^{-1}, assuming a quenching factor $qg_\mathrm{A} \approx 1$. The consistency of our prediction with the shell-model results strengthens our confidence in the nuclear modeling of this second-order, extremely rare process, which is of paramount importance for the further modeling of the $0\nu\beta\beta$ decay.

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1. Introduction

Two-neutrino double-beta decay ($2\nu\beta\beta$) is the rarest radioactive process known in physics. It has been observed only within 11 even–even nuclei that are close to the valley of stability [1]. For these nuclei, a single-beta transition of an even–even nucleus into a neighboring odd–odd nucleus is forbidden. However, if it is followed immediately with a secondary decay, a daughter even–even nucleus is lighter than the mother, and thus, the whole process turns out as energetically allowed. This reaction is a weak-interaction process of the second order, half-life $T^{2\nu}_{1/2}$ of which is given by

$$\left[T^{2\nu}_{1/2}\right]^{-1} = G^{2\nu} \left|\mathcal{M}^{2\nu}\right|^2 . \tag{1}$$

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In the above equation, $G^{2\nu}$ stands for the leptonic phase-space factor and $\mathcal{M}^{2\nu}$ — the so-called nuclear matrix element, which contains all information about the nuclear quantum states active in the process. The latter is of primary theoretical interest, as the accuracy of its evaluation directly reflects the capability of a nuclear model to reproduce the strong-interaction-driven eigenstates involved in the process.

The nuclear matrix element for $2\nu\beta\beta$ can be estimated using Fermi's golden rule. However, since single-β transitions are forbidden, the process must be treated as a second-order perturbation. The resulting formula for $\mathcal{M}^{2\nu}$ (in the case of $0^+ \to 0^+$ transition) is given by [2]

$$\mathcal{M}^{2\nu} = \sum_m \frac{\langle 0_f^+ | \sum_i \hat{\sigma}_i \hat{\tau}_i^- | 1_m^+ \rangle \langle 1_m^+ | \sum_i \hat{\sigma}_i \hat{\tau}_i^- | 0_i^+ \rangle}{\Delta E_m + \frac{1}{2} Q_{\beta\beta} + \Delta M}, \tag{2}$$

where the sum runs over the m-intermediate states. The transition operators are GT operators, $Q_{\beta\beta}$ is the $2\nu\beta\beta$ Q-value, ΔE_m is the excitation energy of the m^{th} intermediate state relative to the nucleus's ground state, and $\Delta M = M_{\text{int}} - M_i$ is the mass difference between the intermediate (M_{int}) and mother (M_i) nuclei. Due to the strong suppression of the Fermi transition between neighboring nuclei, only the Gamow–Teller mode will be discussed in this $2\nu\beta\beta$ analysis.

The nuclear level structure of ^{48}Ca and its neighboring nuclei is distinctive. Similarly to ^{96}Zr, a single-β transition to the ground state of the intermediate nucleus is, in principle, allowed [3]. However, because this would require the emission of a large angular momentum ($|^{48}\text{Ca}; 0^+_{\text{gs}}\rangle \to |^{48}\text{Sc}; 6^+_{\text{gs}}\rangle$), such a process is strongly suppressed. Consequently, the two-neutrino double-beta decay directly to ^{48}Ti is much more probable.

The goal of our work was to evaluate nuclear matrix element $\mathcal{M}^{2\nu}$ for the $^{48}\text{Ca} \to {}^{48}\text{Ti}$ transition. Applying the Fermi golden rule, we obtain the master formula in terms of the intermediate ^{48}Sc states

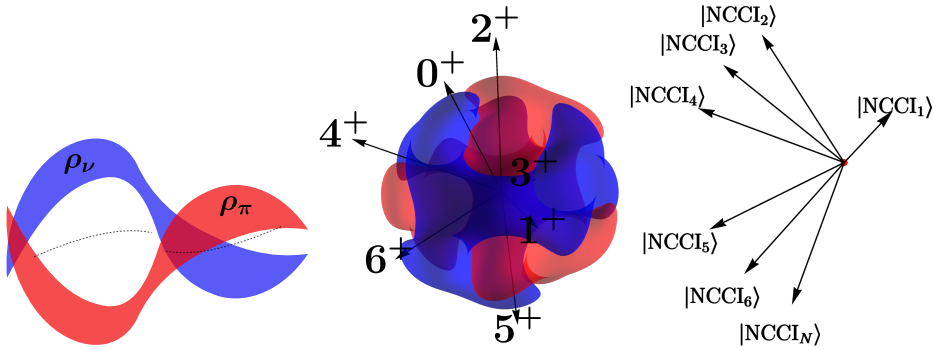
$$\mathcal{M}^{2\nu}\left(^{48}\text{Ca}\right) = \sum_m \frac{\langle {}^{48}\text{Ti}; 0_f^+ | \sum_i \hat{\sigma}_i \hat{\tau}_i^{\pm} | {}^{48}\text{Sc}; 1_m^+ \rangle \langle {}^{48}\text{Sc}; 1_m^+ | \sum_i \hat{\sigma}_i \hat{\tau}_i^{\pm} | {}^{48}\text{Ca}; 0_i^+ \rangle}{\Delta E_m + \frac{1}{2} Q_{\beta\beta} - \Delta M}. \tag{3}$$

2. DFT-NCCI model

2.1. Overview of the framework

For almost a decade, the research group at the Faculty of Physics, University of Warsaw [4] has been developing the so-called DFT-rooted No-Core Configuration-Interaction (DFT-NCCI) model, which combines projection techniques with configuration mixing. In this approach, depicted in Fig. 1,

the configuration space is first constructed as a set of mean-field Slater determinants $|\varphi_i\rangle$ corresponding to the nuclear ground state and a selected number of low-lying excitations, such as $1p$–$1h$, $2p$–$2h$, *etc.* This step is carried out within the single-reference density functional theory (SR-DFT) framework using the SV-parametrized Skyrme functional.



(a) Nuclear Skyrme SV energy functional

(b) Mean-field Slater $|\Psi_i\rangle$ determinant

(c) NCCI basis

Fig. 1. Depiction of the DFT-NCCI procedure. The nuclear Skyrme SV energy functional is variated over nuclear densities (a) → (b) resulting in a mean-field Slater determinant. It is subsequently projected onto a good angular momentum I. Performing this operation for a set of mean-field I-configuration space and rediagonalizing in the Hill–Wheeler procedure (b) → (c) results in an NCCI basis with good quantum numbers restored.

In order to determine the necessary transition rates in the regime of DFT, one needs to extend the formalism to be able to restore the relevant broken symmetries, so a state may again be labelled with a specific angular momentum. Each configuration is being projected first on the well-defined angular-momentum I, using the three-dimensional $\hat{P}^I_{MK}$ projection operator

$$|\varphi; IMK; T_z\rangle = \hat{P}^I_{MK}|\varphi\rangle\,, \tag{4}$$

where T_z denotes the z-component of total isospin T. The angular-momentum projector can explicitly be expressed as

$$\hat{P}^I_{MK} = \frac{2I+1}{8\pi^2}\int D^{I*}_{MK}(\Omega)\hat{R}(\Omega)\mathrm{d}\Omega\,, \tag{5}$$

where D^I_{MK} is Wigner function with M, K as the angular-momentum components along the laboratory (M) and intrinsic z-axis (K), and $\hat{R}(\Omega)$ is rotation in space (3D rotation). The proper symmetry restoration requires

additional mixing of the projected state (5) within the K quantum number

$$|\varphi; IM; T_z\rangle = \frac{1}{\sqrt{N_{\varphi;IM;T_z}}} \sum_K a_K^I |\varphi; IMK; T_z\rangle \,. \tag{6}$$

Finally, the eigenstates and eigenenergies in the DFT-NCCI model may be determined by solving the Hill–Wheeler equation in the specific subspace of the linearly-independent vectors $|\varphi; IM; T_z\rangle$. As a result of its diagonalization, one obtains normalized DFT-NCCI states

$$|IM; T_z\rangle^{(n)} = \frac{1}{\sqrt{N^{(n)}_{IM;T_z}}} \sum_{ij} \eta_{ij}^{(n)} |\varphi_i; IM; T_z\rangle^{(j)} \,, \tag{7}$$

where the sum runs over all i-configurations and over the index j, which denotes consecutive K-mixed states of the same I within the fixed i-configuration.

2.2. Gamow–Teller matrix elements in DFT-NCCI

Calculation of the matrix elements is based on the computation of the expected values of Fermi and Gamow–Teller operators between two I-projected mean-field Slater determinants and their further reevaluation in the basis of DFT-NCCI states. In the case of the neutrino-abundant β-decays, representing both operators as rank-1 spherical tensors

$$\left(\hat{\mathcal{O}}_{\mathrm{F}}\right)_\mu = \tau^{\pm}_{1\mu} \,, \tag{8}$$

$$\left(\hat{\mathcal{O}}_{\mathrm{GT}}\right)_{\mu\nu} = \tau^{\pm}_{1\mu}\sigma_{1\nu} \tag{9}$$

allows to write matrix elements between I-projected mean-field Slater determinants $|\varphi; I', M', K'\rangle, |\psi; I, M, K\rangle$ in the DFT-NCCI framework as [5]

$$\begin{aligned}
(\mathcal{M}_{\mathrm{F}})_\mu &= \langle \varphi; I'M'K'; T'_z | \left(\hat{\mathcal{O}}_{\mathrm{F}}\right)_\mu |\psi; IMK; T_z\rangle \\
&= \frac{2I+1}{8\pi^2}\delta_{II'}\delta_{M'K} \sum_{K'} \int \mathrm{d}\Omega\, a^I_{K'} D^{I\,*}_{M,K'}(\Omega) \left\langle \varphi | \tau_{1\mu} | \tilde{\psi} \right\rangle \,, \qquad (10) \\
(\mathcal{M}_{\mathrm{GT}})_{\mu\nu} &= \langle \varphi; I'M'K'; T'_z | \left(\hat{\mathcal{O}}_{\mathrm{GT}}\right)_{\mu\nu} |\psi; IMK; T_z\rangle \\
&= C^{I'M'}_{IM,1\nu} \sum_\xi C^{I'K'}_{IK'-\xi,1\xi} \frac{2I+1}{8\pi^2} \int \mathrm{d}\Omega\, D^{I\,*}_{K'-\xi\,K}(\Omega) \left\langle \varphi | \tau_{1\mu} \hat{\sigma}_{1\xi} | \tilde{\psi} \right\rangle \,, \qquad (11)
\end{aligned}$$

where $|\tilde{\psi}\rangle \equiv \hat{R}(\Omega)|\psi\rangle$ denotes the spatially rotated Slater determinant of the mother nucleus. As a result, the evaluation of $\mathcal{M}_{\rm F}$ and $\mathcal{M}_{\rm GT}$ boils down to the determination of the kernels $\langle\varphi|\tau_{1\mu}|\tilde{\psi}\rangle$, $\langle\varphi|\tau_{1\mu}\hat{\sigma}_{1\xi}|\tilde{\psi}\rangle$ via the Generalized Wick Theorem. In the case of $2\nu\beta\beta$ decay, our interest concerns only the Gamow–Teller transitions.

3. Nuclear matrix element

3.1. Configuration space

We constructed a configuration space in the DFT-NCCI framework referring to ^{48}Ca, ^{48}Sc, and ^{48}Ti nuclear states. The ^{48}Ca nucleus is doubly magic; hence, the only configurations that can mix with the ground-state configuration are seniority-zero neutron–neutron (nn) pairing excitations carrying no alignment, *i.e.*, coupled to $\Omega = 0$. In the case of ^{48}Sc, we assume that the dominant contributions to the Gamow–Teller (GT) matrix element arise from virtual $|1^+\rangle$ states built upon the lowest neutron $1p$–$1h$ and proton $1p$–$1h$ seniority-two configurations with $|\Omega| = 0, 1$. Finally, the configuration space of ^{48}Ti consists of 49 non-aligned ($\Omega = \Omega_\nu + \Omega_\pi = 0$) configurations. For clarity, it is convenient to divide these configurations into classes, which are specified in Fig. 2.

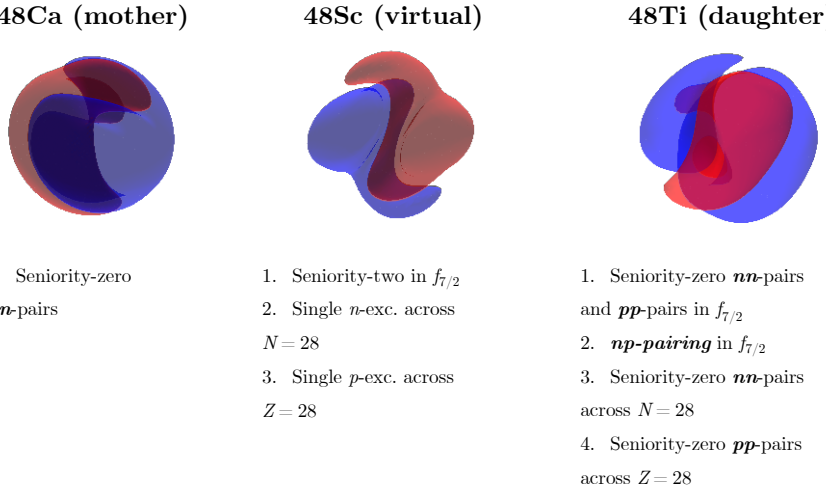
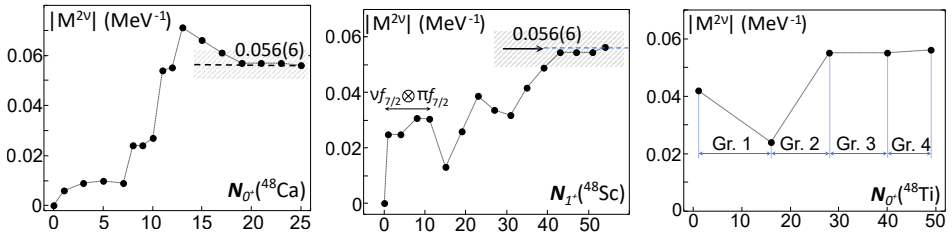


Fig. 2. Summary of the nuclear mean-field configuration groups for mother ^{48}Ca, virtual ^{48}Sc, and daughter ^{48}Ti nuclei.

3.2. Numerical results

The impact of configuration mixing in the three nuclei participating in the $2\nu\beta\beta$ decay of ^{48}Ca is illustrated in Fig. 3.



(a) ^{48}Ca (b) ^{48}Sc (c) ^{48}Ti

Fig. 3. Stability of the matrix element $\mathcal{M}^{2\nu}$.

Figure 3 (b) shows the convergence of the matrix element $\mathcal{M}^{2\nu}$ with respect to the number of configurations in the intermediate nucleus ^{48}Sc. The first 11 configurations (Group 1), formed from in-shell $\nu f_{7/2} \otimes \pi f_{7/2}$ excitations, contribute over 50% to the final value $\mathcal{M}^{2\nu} \approx 0.056$ MeV^{-1}. The remaining ones come from cross-shell excitations involving orbitals above $N = Z = 28$, although their contribution is modest due to energy suppression in the denominator of Eq. (3).

Figure 3 (a) demonstrates that the ground-state configuration of ^{48}Ca plays a surprisingly small role. Instead, the matrix element is driven by $2p$–$2h$ seniority-zero neutron excitations across the $N = 28$ shell gap. These configurations are crucial for capturing the collectivity needed to describe the transition. Proton excitations, in contrast, are found to be negligible.

In ^{48}Ti (Fig. 3 (c)), Group 1 (in-shell nn/pp pairing) configurations reduce $\mathcal{M}^{2\nu}$ relative to the ground state. Group 2 (np-pairing) configurations increase $\mathcal{M}^{2\nu}$, largely determining its final value. Cross-shell pairing excitations (Groups 3 and 4) have a marginal influence.

The interplay of neutron and proton pairing, particularly the role of np correlations in ^{48}Ti, highlights the interpretive clarity of the DFT-NCCI approach. Despite lacking the fine-tuned local calibration of the Nuclear Shell Model, our prediction, $\mathcal{M}^{2\nu} = 0.056(6)$ MeV^{-1}, aligns well with the NSM results [6–8] and approaches the empirical estimate of 0.068(6) MeV^{-1} [1] assuming $g_{\mathrm{A}}^{(\mathrm{eff})} = 1$.

We conservatively estimate a theoretical uncertainty of 10%. While the model appears well converged, missing configurations or energy normalization choices (*e.g.*, shifting the 1^+ reference state in ^{48}Sc from 2.2 to 3.0 MeV changes $\mathcal{M}^{2\nu}$ from 0.056 to 0.054 MeV^{-1}) could introduce small shifts.

4. Summary

We have performed comprehensive numerical calculations, which estimated the nuclear matrix element for $2\nu\beta\beta$ decay of ^{48}Ca as $\mathcal{M}^{2\nu} = 0.056(6)$ MeV^{-1}. Table 1 presents an overview of NME for the same decay across different nuclear models. All values have been renormalized from dimensionless $\mathcal{M}^{2\nu}_{\text{eff}}$ to the effective axial-vector coupling constant $g_{\text{A}}^{\text{eff}} = 1$.

Table 1. $\mathcal{M}^{2\nu}$ estimation for ^{48}Ca decay within various nuclear models, renormalized to $g_{\text{A}}^{\text{eff}} = 1$. See the references for dimensionless $\mathcal{M}^{2\nu}_{\text{eff}}$ values.

Reference	Method	$\lvert\mathcal{M}^{2\nu}\rvert$ [MeV^{-1}]
Barabash [1]	Experiment	0.068 ± 0.006
Horoi *et al.* [6]	Shell model[a]	0.054
	Shell model[b]	0.064
Iwata *et al.* [7]	Shell model	0.0539
Šimkovic *et al.* [9]	ChER	0.0832
Kostensalo, Suhonen [10]	Shell model	0.100
Novario *et al.* [11]	Coupled cluster	0.082
Terasaki, Iwata [8]	Shell model	0.0515
	QRPA	0.0745
Veselý *et al.* [12]	STDA	0.1668
Miśkiewicz, Konieczka, Satuła [13]	**DFT-NCCI**	**0.056**

[a]GXPF1A interaction, [b]GXPF1 interaction.

Although our result is noticeably lower than the experimental estimate by Barabash [1], it remains consistent within the uncertainty limits of the cited works. This agreement supports the validity of our calculation and motivates the planned extension of the DFT-NCCI framework to the neutrinoless double-beta-decay case.

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