

LEPTON MAGNETIC MOMENTS: WHAT THEY TELL US*

FRED JEGERLEHNER 

Humboldt-Universität zu Berlin, Institut für Physik
Newtonstrasse 15, 12489 Berlin, Germany

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Recently, the exciting new Fermilab (FNAL) Muon $g - 2$ measurement impressively confirmed the final Brookhaven (BNL) result from 2004 and, with a result four times more precise, has launched a new serious attack on the Standard Model (SM). On the theoretical side, *ab-initio* lattice QCD (LQCD) calculations of hadronic vacuum polarization have made remarkable progress. They are now the new standard for studying the leading non-perturbative contributions, which have previously hindered matching with the precision required for full exploitation of the experimental results. The lattice results affected both leading hadronic contributions — the hadronic vacuum polarization (HVP) and the hadronic light-by-light (HLbL) contributions — by increasing the previously generally accepted $e^+e^- \rightarrow$ hadrons based dispersion relation results. The shifts reduced the discrepancy between theory and experiment to “nothing missing.” One of the most prominent signs of “beyond the Standard Model” (BSM) physics has disappeared: the SM appears validated more than ever, in agreement with what also searches at the Large Hadron Collider (LHC) at CERN tell us! A triumph of the SM, in spite of the fact that the SM cannot explain known cosmological puzzles such as dark matter or baryogenesis and why neutrino masses are so tiny, the absence of strong CP violation, for example. I also argue that the discrepancy between the data-driven dispersive result and the lattice QCD results for the hadronic vacuum polarization can be largely explained by correcting the e^+e^- data for $\rho^0-\gamma$ mixing effects.

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1. Introduction

This text is an update of an earlier article [1] and the more detailed book [2], which were written when the 2004 BNL measurement of $a_\mu = (g_\mu - 2)/2$ at 540 ppb [3] had set the precision limit before the Fermilab Muon $g - 2$

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experiment [4, 5] provided new, more precise measurements of a_μ , which recently reached the milestone of 124 ppb [5]. With four times the precision and a statistical error roughly equal to the combined systematic error, the SM was subjected to an unprecedented precision test. While the Fermilab experiment confirmed the final BNL result, the theoretical prediction moved closer to the experimental result due to significant advances in *ab-initio* lattice QCD calculations of the non-perturbative QCD contributions that had been an obstacle to the precision of the SM prediction of a_μ .

The story of the lepton magnetic moments is the story of “The closer you look, the more there is to see” and highlights the SM to the deepest. The magnetic moments of leptons (g_ℓ factors) and the associated anomalies of the muon and electron play a decisive role in high-precision verification of the Standard Model. These quantities are not only among the most precisely measured in particle physics, but can also be predicted with high accuracy. At the same time, they offer promising insights into physics beyond the Standard Model. The key relationship for testing possible new heavy states is

$$\frac{\delta a_\ell}{a_\ell} \propto \frac{m_\ell^2}{M^2} \quad (M \gg m_\ell),$$

where m_ℓ is the mass of the lepton and M is the mass scale of New Physics. This equation shows that the fractional change in a_ℓ is proportional to the square of the lepton mass over the square of the New Physics scale, making the muon about $(m_\mu/m_e)^2 \sim 4 \times 10^4$ times more sensitive to BSM physics than the electron. For the muon, theory and experiment now agree to *9 decimal places*, so that $\delta a_\mu \sim \frac{\alpha}{\pi} \frac{m_\mu^2}{M_{\text{NP}}^2}$ can effectively probe intermediate mass scales beyond the heaviest SM ingredient, the top quark mass M_t up to about 1 TeV, a region where new heavy states are essentially ruled out directly by the LHC.

In any case, the “Muon $g - 2$ drama” will have an essential impact on which SM extensions remain viable. Paradigms such as naturalness and the supposed related hierarchy problem are definitely not constructive, and new guiding principles must take their place. The most fruitful approach here is to search for emergent structures, as outlined in [6, 7], for example.

2. A new milestone in the experimental determination of the muon anomaly

Any particle with spin \vec{s} has a magnetic moment $\vec{\mu}$ (internal current circulating)

$$\vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c} \vec{s}; \quad g_\mu = 2(1 + a_\mu).$$

The extreme precision one is able to reach in measuring and calculating the lepton anomalous magnetic moments is possible by taking the simplest object you can think of in the static limit which is provided with the electromagnetic vertex

$$= (-ie) \bar{u}(p') \left[\gamma^\mu F_1(q^2) + i \frac{\sigma^{\mu\nu} q_\nu}{2m_\mu} F_2(q^2) \right] u(p),$$

where

$$F_1(0) = 1; \quad F_2(0) = a_\mu. \quad (1)$$

The point is that a_μ is responsible for the Larmor (spin) precession frequency $\vec{\omega}_a$ which shows up when polarized muons are orbiting in a homogeneous magnetic field as illustrated in Fig. 1. This¹ “magic γ ” muon storage ring principle developed at CERN has been applied in later experiments at Brookhaven (BNL) and Fermilab (FNAL)². The new Fermilab Muon $g-2$ measurement (see Fig. 1), applying sophisticated improved technology,

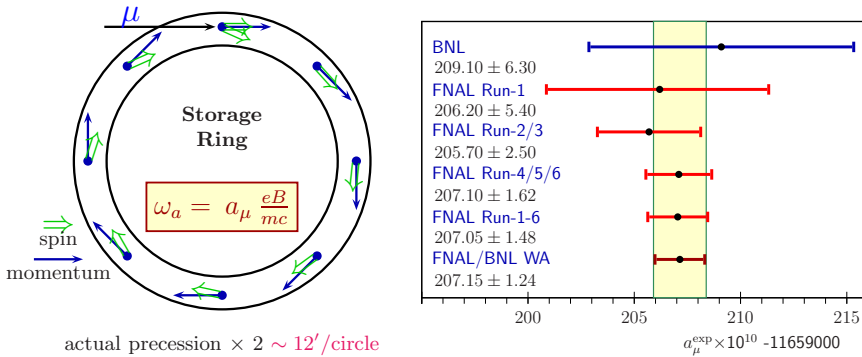


Fig. 1. Left: At the Magic Energy (muon beam energy ~ 3.1 GeV), the angular frequency $\vec{\omega}_a$ is directly proportional to the magnetic field \vec{B} , so that the a_μ measurement is a frequency counting experiment. Right: The measurements of a_μ by the Muon $g-2$ Collaboration at the Fermi National Accelerator Laboratory (FNAL) have significantly improved on the results obtained by Brookhaven National Laboratory (BNL) in 2004.

¹ γ is the Lorentz factor of relativistic time dilation. It is adjusted so that $a_\mu - 1/(\gamma^2 - 1) = 0$ to cancel out the dependence on the electric field \vec{E} needed to focus the beam.

² There are good reasons why a very different experimental technique for measuring a_μ is much-needed. To come — novel complementary experiment at J-PARC in Japan [8–10], where a strict $\vec{E} = 0$ cavity is utilized. In place of the ultra relativistic (CERN, BNL, Fermilab) muons, one can then work with ultra cold (J-PARC) muons such that one obtains a measurement with very different systematics.

brightly confirmed the 2004 final BNL result

$$a_\mu[\text{BNL}] = 11659209.1(5.4)(3.3)[6.3] \times 10^{-10} \quad (2)$$

now improved to

$$a_\mu[\text{FNAL}] = 11659207.05(1.14)(0.95)[1.48] \times 10^{-10} \quad (3)$$

and potentially triggers a new serious attack on the SM. The combined result is

$$a_\mu^{\text{exp}} = (11\,659\,207.15 \pm 0.80 \pm 0.95)[1.24] \times 10^{-10} \quad (\text{FNAL/BNL}). \quad (4)$$

This is a remarkable achievement with the precision of 124 ppb today from 540 ppb at BNL, a factor 4 improvement with statistical error as small as the systematic one.

3. Theory today

The theoretical prediction of a_μ includes the entire SM, and the result presented in the White Paper in 2020 (WP20) reads [11]³

$$\begin{aligned} a_\mu^{\text{SM}} = F_2(0) &= a_\mu^{(\text{QED}+\text{EW}+\text{HVP}_{\text{LO}}+\text{HVP}_{\text{NLO}}+\text{HVP}_{\text{NNLO}}+\text{HLbL}_{\text{LO}}+\text{HLbL}_{\text{NLO}})} \\ &= 0.00116591810(43). \end{aligned} \quad (5)$$

This result, worked out by the Muon $g - 2$ Theory Initiative, relies on the values $a_\mu^{\text{HVP,LO}} = 693.1(4.0) \times 10^{-10}$ and $a_\mu^{\text{LbL,LO}} = 90(17) \times 10^{-10}$ for the problematic leading hadronic contributions⁴. An interesting point concerns the inclusion of τ decay-spectra⁵ in the calculation of $a_\mu^{\text{HVP,LO}}$, which leads to the increased value $a_\mu^{\text{HVP,LO}}[e^+e^- + \tau] = 705.3 \pm 4.5 \times 10^{-10}$ [28]. Similarly, calculations of $a_\mu^{\text{HVP,LO}}$ based solely on τ data⁶ [34, 35] yield $a_\mu^{\text{HVP,LO}}[\tau] = 703.1_{-4.0}^{+4.1} \times 10^{-10}$. We learn that by including the τ

³ Recently, in 2025, the WP20 result was updated [12], which we will refer to below as WP25.

⁴ My value from 2007 was $a_\mu^{\text{SM}} = 0.00116591793(68)$ in [13, 14], *i.e.*, the central value of WP20 changed only by $81.0 - 79.3 = 1.7$ in 10^{-10} , while the error could be reduced by 30% mainly due to more precise e^+e^- data in the calculation of the hadronic contributions. The WP20 result for $a_\mu^{\text{HVP,LO}}$ was obtained by the e^+e^- -data driven dispersion relation approach. The relevant e^+e^- data have been obtained from the energy scan data by CMD-2 [15], SND [16], SND20 [17], and CMD-3 [18], and the Initial State Radiation (ISR) data from KLOE [19–22], BaBar [23, 24], BESIII [25], and CLEO-c [26].

⁵ This was pioneered by Davier *et al.* in [27].

⁶ $\tau^- \rightarrow \nu_\tau \pi^0 \pi^-$ spectra were recorded by ALEPH [29, 30], OPAL [31], CLEO [32], and Belle [33].

data, both calculations yield values close to the known lattice QCD result from the Budapest–Marseille–Wuppertal (BMW) Collaboration [36]: $a_\mu^{\text{HVP,LO}}[\text{LQCD : BMW}] = 707.5 \pm 5.5 \times 10^{-10}$. These results compare to the value $a_\mu^{\text{HVP,LO}} = 717.0 \pm 1.5 \times 10^{-10}$ which would close the mismatch between theory and experiment. Also, if we compare the result determined from τ data by Belle 2008: $a_\mu^{\pi\pi}[2m_\pi, 1.8\text{GeV}] = (523.5 \pm 3.9) \times 10^{-10}$ (τ : Belle), with the result obtained from the e^+e^- data $a_\mu^{\pi\pi}[2m_\pi, 1.8\text{GeV}] = (504.6 \pm 10.1, 1) \times 10^{-10}$ (e^+e^- : CMD-2, SND), there is a difference of 18.9×10^{-10} . Adding this difference to the dispersive result of the e^+e^- data $a_\mu^{\text{HVP,LO}}[e^+e^-] = (694.79 \pm 4.18) \times 10^{-10}$, we obtain 713.7×10^{-10} . Recently, the energy-scan experiment CMD-3 reported an $e^+e^- \rightarrow \pi^+\pi^-$ cross-section measurement in the energy range from 0.327 to 1.2 GeV, which also yielded the significantly higher value $a_\mu^{\text{HVP,LO}}(2\pi, \text{CMD-3}) = (526.0 \pm 4.2) \times 10^{-10}$, which, compared to the corresponding WP20 estimate $(506.0 \pm 3.4) \times 10^{-10}|_{\text{WP20}}$, results in a difference of 20.0×10^{-10} . Adding this to $a_\mu^{\text{HVP,LO}}[e^+e^-] = (693.1 \pm 4.0) \times 10^{-10}|_{\text{WP}}$ yields $(713.1 \pm 4.7) \times 10^{-10}$, a surprising result since the second experiment with the SND detector at the same storage ring yielded a result of $a_\mu^{\text{HVP,LO}}(2\pi, \text{SND20}) = (508.3 \pm 4.2) \times 10^{-10}$, which roughly corresponds to the other e^+e^- -data-based analyses. The large difference between the CMD-3 value and the SND20 result (both e^+e^- scan results from the e^+e^- collider facility in Novosibirsk) is another puzzle⁷!

While the $e^+e^- \rightarrow \pi^+\pi^-$ $R(s)$ -ratio measurements show significant discrepancies between the ISR experiments KLOE and BaBar, as well as between the scan experiments CMD-3 and SND20, for which we still have no explanations, we learn that taking the τ data into account yields results that systematically increase $a_\mu^{\text{HVP,LO}}$, making them compatible with the results of lattice QCD, in particular with the BMW value.

The unexpected shift in theory has resulted from recent advances in lattice calculations of the leading non-perturbative hadronic contributions, the vacuum polarization $a_\mu^{\text{HVP,LO}}$, and the light-light scattering $a_\mu^{\text{LbL,LO}}$. In particular, the significantly higher $a_\mu^{\text{HVP,LO}}$ result from BMW 2019 [36, 38] and subsequent validations by the Mainz/CLS [39–41] and RBC/UKQCD 2024 [42, 43] collaborations have opened a new window for the $a_\mu^{\text{HVP,LO}}$ contributions. For the significant shifts resulting from the lattice QCD results, the Muon $g - 2$ Theory Initiative estimated in the WP25 update [12]

$$a_\mu^{\text{HVP,LO}} = 693.1(4.0) \rightarrow 713.2(6.1) \quad \text{and} \quad a_\mu^{\text{LbL,LO}} = 90(17) \rightarrow 112.6(9.6) \quad (6)$$

⁷ CMD-3 seems to apply different radiative corrections (sQED *vs* RLA?) from other experiments; see [18, 37].

as necessary corrections. The resulting discrepancy between theory and experiment has consequently decreased to

$$\delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 38 \pm 63 \times 10^{-11}, \quad (7)$$

and this deviation must be confronted with subdominant SM effects such as the contribution of the weak interactions 12.9σ , the HLbL effect 9.6σ , and the higher-order (HO) HVP effect -8.2σ . It is noteworthy that if one were to stick with the WP20 prediction, an apparently significant BSM effect $\Delta a_\mu^{\text{BSM}} \stackrel{?}{=} \delta a_\mu$ would be 1.6 times larger than the phenomenologically well-established weak contribution. Furthermore, what we know from the intensive LHC searches makes it very unlikely that a gap of 5.6σ could be real. We now have the unbelievably precise agreement to 9 decimal places in the muon g factor.

The updated QED prediction of a_μ is (see [44–47])

$$\begin{aligned} a_\mu^{\text{QED}} &= \frac{\alpha}{2\pi} + 0.765,857,423(16) \left(\frac{\alpha}{\pi}\right)^2 + 24.050,509,82(28) \left(\frac{\alpha}{\pi}\right)^3 \\ &+ 130.8734(60) \left(\frac{\alpha}{\pi}\right)^4 + 750.010(872) \left(\frac{\alpha}{\pi}\right)^5 = 116584718.8(2) \times 10^{-11}. \quad (8) \end{aligned}$$

For the weak contributions, we have [48–51]

$$a_\mu^{\text{EW}} = \underset{\text{LO}}{194.79(1)} \times 10^{-11} - \underset{\text{NLO}}{40.38(36)} \times 10^{-11} = 154.4(4) \times 10^{-11}. \quad (9)$$

The hadronic radiation corrections are the challenge for theory. Contributions from hadrons (quark loops) at low-energy scales are a general problem in electroweak precision physics. About the muon anomaly, three classes of hadronic contributions in which light quark loops appear as hadronic “blobs” are: (a) Hadronic vacuum polarization (HVP) of order $O(\alpha^2)$, $O(\alpha^3)$; (b) Hadronic light-light scattering (HLbL) of order $O(\alpha^3)$; (c) Hadronic effects in 2-loop electroweak (EW) radiative corrections of order $O(\alpha G_F m_\mu^2)$, with G_F the Fermi constant. The calculation of non-perturbative effects is possible using hadron production data in conjunction with the dispersion relations approach (DRA), effective low-energy modeling within a resonance Lagrangian approach (RLA), and *ab-initio* lattice QCD calculations. Table 1 provides a summary of the hadronic and weak contributions (based on dispersive approach), where the LO-HVP and LO-HLbL results (see Fig. 2) have to be updated according to Eq. (6) by LQCD results! The uncertainties of the subleading NLO and NNLO and weak contributions are below one σ , *i.e.*, they are “DRA save”.

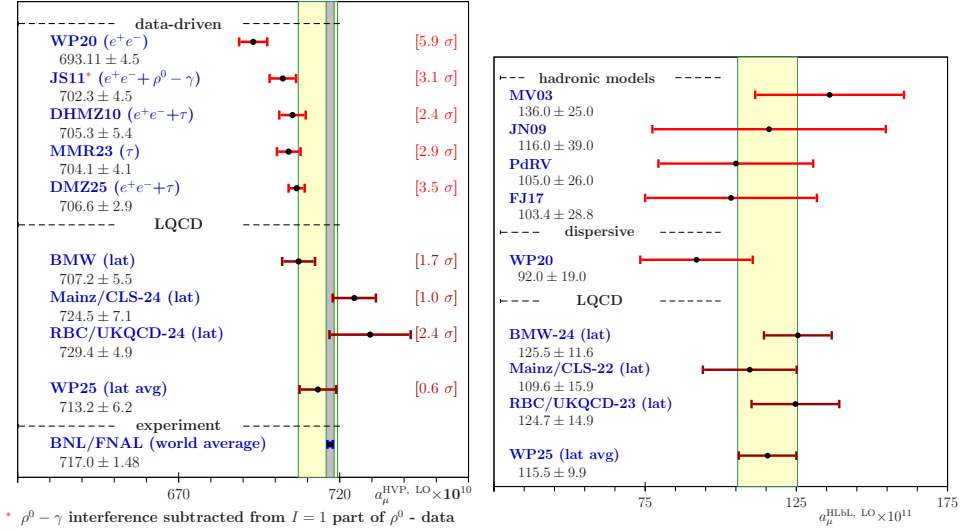


Fig. 2. Left: A selection of $a_\mu^{\text{HVP, LO}}$ results obtained using different approaches. WP20 [11] is the dispersive result based exclusively on $e^+e^- \rightarrow$ hadron data. JS11 refers to an e^+e^- -data-based result corrected for $\rho^0 - \gamma$ interference, which convincingly explains the puzzle surrounding the e^+e^- vs τ -spectra relation [52]. This correction is necessary to separate the irreducible QCD component from the normally ignored QED-QCD mixing inherent in the experimental $\pi^+\pi^-$ production data. DHMZ10 [28] and the update DMZ25 [53] have included the isospin breaking (IB)-corrected τ data in addition to the e^+e^- data. MMR23 is an analysis based exclusively on τ data [54, 76]. BMW, Mainz/CLS, and RBC/UKQCD are the latest lattice QCD results, which currently provide the most reliable hadronic contributions. The last point is the update of WP25 [12] (wheat-colored band), the result of the consensus theory compared to the experimental result from BNL/FNAL (gray band), represented by the fictitious $a_\mu^{\text{HVP, LO}}$ term required to close the gap between experiment and the theory prediction when dropping $a_\mu^{\text{HVP, LO}}$. Right: A selected history of $a_\mu^{\text{HLbL, LO}}$ results. Pioneered by Hayakawa, Kinoshita, and Sanda [55], Bijmens, Pallante, and Prades [56], and Knecht and Nyffeler [57, 58]. Data points shown are MV03 [59], JN09 [60], the consensus PdRV [61], my book FJ17 [2], and the result based on the dispersive approach by Colangelo *et al.* [62] of the White Paper WP20. The lattice QCD results are BMW-24 [63], Mainz/CLS-22 [64, 65], and RBC/UKQCD-23 [66, 67]. WP25 represents the current consensus of the Muon $g - 2$ theory initiative [12].

Table 1. HVP, HLbL, and weak contributions to a_μ , where the HVP contributions rely on e^+e^- data. The HLbL is estimated mainly using transition form factors like $\gamma\gamma^* \rightarrow \pi^0$ and $\gamma\gamma^* \rightarrow \pi^+\pi^-, \pi^-\pi^0$.

$a_\mu^{\text{had}(1)}$	=	$(697.17 \pm 4.18) \times 10^{-10}$	(LO)
$a_\mu^{\text{had}(2)}$	=	$(-10.89 \pm 0.067) \times 10^{-10}$	(NLO)
$a_\mu^{\text{had}(3)}$	=	$(1.242 \pm 0.010) \times 10^{-10}$	(NNLO)
$a_\mu^{\text{had,LbL}}$	=	$(10.34 \pm 2.88) \times 10^{-10}$	(HLbL)
a_μ^{weak}	=	$(15.4 \pm 0.4) \times 10^{-10}$	(LO+NLO)

4. What is the problem?

Here, we have to remind the role of τ decay spectra for the $a_\mu^{\text{HVP,LO}}$ evaluation. The lesson we learn from Fig. 2 is that including the charged channel $\tau \rightarrow \nu_\tau \pi \pi^0$ spectra brings the data-driven dispersive results close to the lattice QCD results from the BMW Collaboration. The problem with the DRA-HVP evaluation must therefore be related to the experimental e^+e^- data.

Despite determined efforts to calculate and model QED corrections for hadronic processes, open questions remain. What can explain the large difference between experimental e^+e^- -HVP data and lattice QCD-HVP data? In the process $e^+e^- \rightarrow$ hadrons, experiments measure the photon propagator, *i.e.*, a mixture of QCD and QED effects that is difficult to disentangle. This requires the removal of external QED corrections, where the radiation of hadrons is not well understood, even though scalar QED (sQED) is a good approximation for meson production at low energies. In contrast, lattice QCD measures the hadronic blob exclusively, as a correlator of two hadronic currents

$$\begin{array}{ccc}
 \begin{array}{c} \text{---} \otimes \text{---} \\ \langle A(x) A(0) \rangle \end{array} & \leftrightarrow & \begin{array}{c} \text{---} \otimes \text{---} \\ \langle j(x) j(0) \rangle \end{array} \\
 \text{photon propagator, } e^+e^- \text{ data} & & \text{current correlator, LQCD data}
 \end{array}$$

It is interesting to compare the dipion spectra of $e^+e^- \rightarrow \pi^+\pi^-$ and $\tau^- \rightarrow \nu_\tau \pi^- \pi^0$ taking into account the Isospin Breaking (IB) effects. In many respects, the τ spectrum is much simpler than that of e^+e^- , since the charged ρ^\pm does not mix with other states in the dominant low-energy range below about 1.05 GeV (above ϕ , below ρ'). The τ decays are mediated by the very heavy W boson, in stark contrast to e^+e^- annihilation, which is mediated far

below the Z boson by the massless photon, which mixes with ρ^0 , ω , and ϕ . τ spectra provide isovector $I = 1$ dipion spectra which should agree with the $I = 1$ part of the e^+e^- annihilation upon appropriate isospin breaking corrections. In contrast to the charged current (CC) data, understanding the neutral current (NC) data requires more complex QCD-driven hadron phenomenology modeling: Chiral perturbation theory (CHPT) has been extended to include spin-1 resonances, which are well described by the vector meson dominance model (VMD) or, in perfect form, by the Resonance Lagrangian Approach (RLA). These models predict the dynamic widths and dynamic mixing of γ , ρ^0 , ω , ϕ rather accurately.

A key effect that is often overlooked is the mixing of ρ^0 , ω , and ϕ with the photon, especially the ρ^0 - γ mixing, which directly impacts the relationship between the photon propagator and the One-Particle-Irreducible (1PI) HVP blob. This mixing appears in the NC measurements $e^+e^- \rightarrow \pi^+\pi^-$, but not in the CC data $\tau^- \rightarrow \nu_\tau\pi^-\pi^0$. This issue was examined in [52] to clarify the known discrepancy between the dipion spectra from τ decays $v_\pm(s)$ and the e^+e^- data $v_0(s)$, which persisted despite applying the commonly accepted isospin-breaking correction (IB) $R_{\text{IB}}(s)$ to the τ data.

In fact, taking into account the ρ - γ interference solves the mystery of τ (charged channel) *vs* e^+e^- (neutral channel). The ρ - γ interference (which does not occur in the charged channel) is often mimicked by large shifts in the mass M_ρ and width Γ_ρ , but such large shifts are not consistent with known calculations of $M_{\rho^\pm} - M_{\rho^0}$, for example according to the Cottingham–Sakurai formula (based on the VMD-I model, which breaks electromagnetic gauge invariance) is the omission of the γ - ρ^0 mixing propagator, which we calculated using the gauge-invariant VMD-II approach together with sQED, which should be valid below 1 GeV (as can be learned from the $\gamma\gamma \rightarrow \pi^+\pi^-$ *vs* $\pi^0\pi^0$ data). We thus are confronted with a non-diagonal (γ, ρ^0) 2×2 matrix propagator, which we must diagonalize. We do not obtain the correct diagonalization by omitting the non-diagonal mixing term. The γ - ρ^0 mixing effect, which is proportional to eg in terms of couplings, where e is the electric charge and g is the effective coupling $\rho\pi\pi$, undergoes a resonance amplification, resulting in unexpectedly large distortions of the $I = 1$ resonance peak⁸, as shown in Fig. 3.

In [52], we *wrongly* corrected the τ spectral function⁹ as follows:

$$v_0(s) = r_{\rho\gamma}(s) R_{\text{IB}}(s) v_-(s).$$

⁸ This correction is robust, as it involves no new parameters that do not already appear in the GS formula. The effect only depends on the well-known leptonic ρ width $\Gamma_{\rho \rightarrow ee}$.

⁹ That the correction $r_{\rho\gamma}(s)$ has to be applied to the e^+e^- data was already stated and discussed in Section 21.3 of [68].

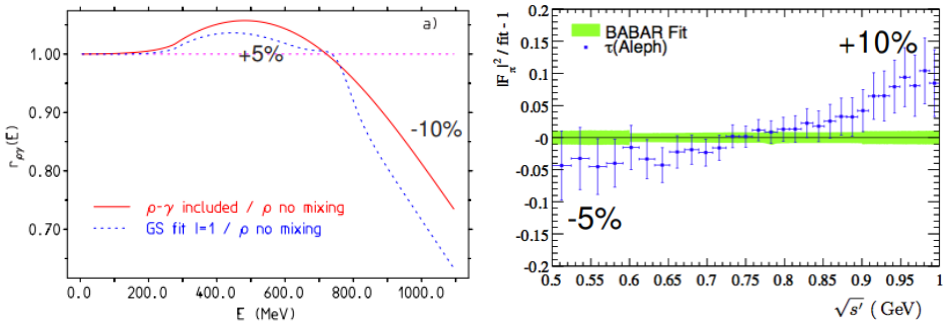


Fig. 3. Left: The ratio of the isospin $I = 1$ pion form factor $|F_\pi(E)|^2$ taking mixing into account, normalized to the case without mixing. Also shown is the same ratio of the $I = 1$ part of the e^+e^- data to the τ data GS fits, mimicked by fictitious parameter shifts in mass and width. Right: Best “proof” for our $\gamma-\rho^0$ mixing profile is the ratio of the ALEPH τ decay spectrum *vs* the BaBar e^+e^- spectrum [reproduced as part of Fig. 55 in [24] (also see [28]).

But as we just argued, it is not the τ spectra that require correction for the absence of $\rho-\gamma$ mixing. Instead, this QED–QCD interference effect must be removed from the e^+e^- data in order to get the purely hadronic part of the 1PI self-energy. Thus, the proper correction must read

$$v_0(s)|_{\text{corr}} = v_0(s)/r_{\rho\gamma}(s) = R_{\text{IB}}(s) v_-(s). \quad (10)$$

In fact, as in the τ -channel, the $\gamma-\rho^0$ mixing is missing in the LQCD data, since this data are obtained by simulating the “QCD-only” path integral¹⁰.

For the ρ^0 resonance range [0.63,0.96] GeV, the $\rho-\gamma$ interference leads to a shift [52]¹¹

$$\delta a_\mu^{\text{HVP, LO}}[\rho\gamma] \simeq (5.1 \pm 0.5) \times 10^{-10}. \quad (11)$$

This correction must be added to the standard e^+e^- -based $a_\mu^{\text{HVP, LO}}$

$$a_\mu^{\text{HVP, LO}}|_{\text{corr}} = a_\mu^{\text{HVP, LO}} + \delta a_\mu^{\text{HVP, LO}}[\rho\gamma] \simeq (702.3 \pm 4.2) \times 10^{-10}. \quad (12)$$

This result fits well with $a_\mu^{\text{HVP, LO}}[ee+\tau] = (705.3 \pm 4.5) \times 10^{-10}$ from Davier *et al.* [28], where τ decay spectra are taken into account.

In summary, removing the $\rho^0-\gamma$ mixing from the e^+e^- data is essential, as it brings the dispersive result into better agreement with the BMW lattice result, $a_\mu^{\text{HVP, LO}} = (707.5 \pm 5.5) \times 10^{-10}$.

¹⁰ Intrinsic QED effects relevant for HVP, such as final-state radiation (FSR), must be included separately.

¹¹ It is important to apply the correction only to the $I = 1$ contribution. Due to the narrow widths of ω and φ , the mixing for these states is much smaller.

With respect to the value $a_\mu^{\text{HVP,LO}} = 713 \times 10^{-10}$ accepted by WP25, we conclude that at least part of the missing shift is explained. If the BMW results were to prevail over the WP25 value, the $\rho^0\text{-}\gamma$ mixing would largely explain the discrepancy between the dispersive result based on e^+e^- data and the lattice and τ -based results.

5. Electron $g - 2$ status and its future

Here is a brief note on the status of the electron magnetic moment a_e , which is a test of QED that is highly sensitive to α electromagnetic as determined by atomic interferometry. In 2018, with $\alpha^{-1}(\text{Cs18}) = 137.035999046(27)$, one had $a_e^{\text{exp}} - a_e^{\text{the}} = (-84 \pm 36) \times 10^{-14}$, a deviation of -2.3σ between theory and experiment. This presented a difficult situation for developers of BSM models, as the sign of δa_e differed from that of δa_μ , which was difficult to explain. In 2020, however, a more precise value of $\alpha^{-1}(\text{Rb20}) = 137.035999206(11)$ was obtained [69], which meant that $a_e^{\text{exp}} - a_e^{\text{the}} = (51 \pm 30) \times 10^{-14}$ changed sign and now predicted a gap of $+1.7\sigma$. Note that α , the most fundamental parameter in physics, changed by 5.4σ due to the switch from cesium (Cs) to rubidium (Rb) atoms! Soon after, in 2022, a more accurate measurement improved the value of $a_e^{\text{exp}} = 0.00115965218073(28)$ to $a_e^{\text{exp}} = 0.00115965218059(13)$ [70]. But the QED prediction was also improved. In 2024, the universal 5-loop coefficient changed to $A_1^{(10)} = 5.873(128)$ [46] (crosschecked in [47]), which then leads to the prediction¹²

$$a_e^{\text{the}} = 0.00115965218023(9) \quad \text{and} \quad a_e^{\text{exp}} - a_e^{\text{the}} = (36 \pm 16) \times 10^{-14}. \quad (13)$$

Only a deviation of $+2.3\sigma$ still agrees quite well with the SM prediction. In future atomic interferometry experiments (as part of the AION project [71]) based on strontium and yttrium, are expected to reduce the uncertainty of the electromagnetic fine structure constant such that an a_e prediction with an accuracy of 2×10^{-14} would be possible.

6. Conclusions

One of the most famous oracles that promised New Physics beyond the SM has dissolved, thereby consolidating the SM up to the TEV scale, which is consistent with the LHC searches! However, this does not mean that the search for BSM physics as the main goal of particle physics is losing importance. What do a_μ and a_e tell us? High-precision physics may prove

¹² In this result, also $a_e^{\text{LO-HVP}}$ had to be slightly modified to take into account the change in $a_\mu^{\text{LO-HVP}}$ (lattice *vs* dispersive result), *i.e.*, $a_e^{\text{LO-HVP}} = 1.871(11) \times 10^{-12}$ changes to $a_e^{\text{LO-HVP}} = 1.923(09) \times 10^{-12}$.

more difficult than expected to reach the limits of the SM, and this applies to both a_μ^{exp} and a_μ^{the} . For the electron, the limitation arises from the need for extremely precise atomic interferometry to determine the electromagnetic fine structure constant in the Thomson limit. As far as a_μ^{exp} is concerned, all experiments to date (CERN, BNL, FNAL) have used the magic γ trick, and an alternative type of experiment is urgently needed. Hopefully, the J-PARC project [8–10], which is promoting an experiment with ultra-cold muons in an $\vec{E} = 0$ field cavity, will be realized in the not-too-distant future.

Given the failure to provide a clear answer based on data-driven DRA, the MUonE determination (elastic μe scattering) of the HVP at CERN (directly measuring what LQCD calculates) would be a mandatory experiment that would need to be carried out [72–75]. Lattice QCD has proven to be an indispensable tool for predicting non-perturbative hadronic effects in electroweak precision observables. However, since a significant fraction of the results still have to be estimated by extrapolation to small lattice spacings and large volumes, further progress is required to reduce the extrapolation uncertainties.

As for an improved α determination as input for predicting a_e , the AION project (using strontium and yttrium ions) [71] promises significant progress.

Although lattice QCD already provides satisfactory results for the hadronic vacuum polarization, it remains an urgent task to understand the inconsistencies in the experimental data required in the dispersive approach: KLOE *vs* BaBar, SND20 *vs* CMD-3, $e^+e^- \rightarrow \pi^+\pi^-$ *vs* $\tau^- \rightarrow \pi^0\pi^-\nu_\tau$ data (NC *vs* CC). A better determination of the HLbL contribution requires more $\gamma\gamma \rightarrow$ hadrons data to improve the precision of known channels and explore new channels that have not been experimentally accessible so far.

Beyond BSM science fiction: Scrutinizing SM predictions and making progress in determining SM parameters such as M_t and M_H (which requires a high-precision Higgs/top quark pair factory) is a major challenge that is also important for a better understanding of early cosmology. But that would be another topic [6]!

Outlook: We note that we have remarkable agreement between the following results:

$$\begin{aligned}
 a_\mu^{\text{HVP-LO}}[ee + \gamma\rho] &= 702.3 \pm 4.2 \times 10^{-10} && \text{Eq. (12),} \\
 a_\mu^{\text{HVP-LO}}[ee + \tau] &= 705.3 \pm 4.5 \times 10^{-10} && [28], \\
 a_\mu^{\text{HVP-LO}}[\tau] &= 704.1 \pm 4.1 \times 10^{-10} && [76], \\
 a_\mu^{\text{HVP-LO}}[\text{LQCD : BMW}] &= 707.5 \pm 5.5 \times 10^{-10} && [36].
 \end{aligned}$$

Since Mainz/CLS and RB/UKQCD achieved slightly larger lattice results (compared to BMW), the 2025 White Paper combined the lattice results to give the larger value $a_\mu^{\text{HVP-LO}} = 713.2 \pm 6.2 \times 10^{-10}$. If the progress

in the lattice results confirmed the slightly smaller BMW value, the mystery surrounding the discrepancy between the dispersive approach and the lattice results would be largely solved if the e^+e^- data were corrected for the $\rho^0\text{-}\gamma$ interference. Then, the difference between the theoretical prediction and the experimental measurement at a level of 0.9σ would still provide strong confirmation of the SM. In fact, the hybrid evaluation, which combines the “best” of the lattice and dispersive approaches: [77, 78] [BMW/DMZ-24], highlighted the result as “confirmation of the SM up to a level of 0.37 ppm.”

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