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STABILIZING DARK MATTER WITH QUANTUM SCALE SYMMETRY: EXECUTIVE SUMMARY* **

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In these proceedings, we report on a secluding mechanism for dark matter from quantum scale symmetry: in the presence of an asymptotically safe ultraviolet fixed point at trans-Planckian scales, the renormalization group flow of the gauge–Yukawa couplings can control dark matter decays, without the need for discrete or global symmetries. We show explicitly how the mechanism works for an SU(6) model and indirectly constrain physical mass scales of the theory.

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1. Quantum scale symmetry in gravity–matter systems

In [1], we explored the concept of quantum scale symmetry as a fundamental symmetry of nature, manifested by the existence of a non-trivial, asymptotically safe fixed point [2, 3], present at ultraviolet (UV) scales, beyond which there is scale invariance. In asymptotically safe quantum gravity (ASQG), such a fixed point at Planckian or trans-Planckian scales, followed by a finite number of free parameters, renders a non-perturbative predictive UV completion to the gravity–matter system. Robust indications for asymptotic safety (AS) in gravity and gravity–matter systems have been found using functional renormalization group (FRG) techniques [3–8] (see also complementary searches for ASQG with lattice-based dynamical triangulations [9] and tensor field theory [10]). Ultimately, one would like to obtain measurable predictions and, despite being typically very difficult to make in a theory of quantum gravity, such predictions are possible in certain systems in ASQG, thanks to its predictive power. In this regard, the

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"landscape of ASQG" (low-energy matter theories that are UV completed by ASQG) has been explored for the Standard Model (SM) and beyond the Standard Model (BSM) physics, including the asymptotically safe SM [11, 12], flavor anomalies [13, 14], the muon anomalous magnetic moment [15], baryon number [16, 17], neutrinos [18–24], dark matter (DM) [25–29], cosmology [30–32], proton decay [33], and grand unification theory (GUT) [34, 35]. For discussion of uncertainties, see [36, 37].

Then, in [1], we proposed a new secluding mechanism for DM: Yukawa couplings that would be allowed by the underlying gauge symmetry of the theory are actually forbidden by their renormalization group (RG) flow, solely predicted by the fixed-point structure from ASQG. This is in contrast with the recurrent practice in the DM literature, in which *ad-hoc* global or discrete symmetries are introduced in order to eliminate couplings that lead to undesired decays. We used an SU(6) GUT model as a proof-of-concept to show how the secluding mechanism works.

2. The gravitational contribution to gauge–Yukawa systems

We can parametrize quantum gravity effects to the running of gauge and Yukawa couplings with universal trans-Planckian contributions f_g and f_y, at the leading order, by

$$\frac{\mathrm{d}g_i}{\mathrm{d}t} = \beta_i^{(\mathrm{matter})} - f_g\, g_i\,, \tag{1}$$

$$\frac{\mathrm{d}y_j}{\mathrm{d}t} = \beta_j^{(\mathrm{matter})} - f_y\, y_j\,, \tag{2}$$

where $t = \ln\mu$ is the RG scale, g_i and y_j (with $i,j = 1,2,3\ldots$) are the set of gauge and Yukawa couplings, and $\beta_{i,j}^{(\mathrm{matter})}$ are the matter beta functions without gravity. Notice that since gravity is universal, it does not distinguish among the SU(N) symmetries of the gauge sector, thus, only one f_g for all gauge groups is sufficient. We also assume this holds for the Yukawa sector. Then, one can derive scheme-dependent expressions for f_g and f_y as a function of the gravitational couplings. In particular, FRG computations support $f_g \geq 0$ [38, 39], which preserves asymptotic freedom for non-Abelian gauge couplings, while it anti-screens the Abelian gauge coupling, solving the U(1) triviality problem [40–42]. Unlike f_g, FRG computations for f_y are subject to greater uncertainties and at leading order, it is not clear whether $f_y > 0$, which is the necessary condition for a non-trivial fixed point solution. In our work, we assume such solution exists, which is consistent with recent "next-to-leading order" findings in [43]. We work with $\beta_{i,j}^{(\mathrm{matter})}$ at the one-loop level and we adopt a heuristic approach for f_g and f_y, treating them as free, positive, small parameters.

We neglect the quantum gravity contributions to the scalar potential, assuming there exists some truncation of the matter–gravity system in which the couplings of the scalar potential are free parameters.

3. Model, without quantum scale symmetry

Guided by the SU(5) group, which can be used to get SM content, in order to describe DM and neutrino masses, we consider SU(N) representations, straightforwardly broken as $\mathrm{SU}(N) \to \mathrm{SU}(5)_{\mathrm{SM}} \times \mathrm{SU}(N-5)_X$. For simplicity, we selected a minimal SU(6) anomaly-free model, whose fermion content, for each fermion generation, is $\bar{\mathbf{6}}_{\mathbf{1}}^{(F)}$, $\bar{\mathbf{6}}_{\mathbf{2}}^{(F)}$, and $\mathbf{15}^{(F)}$. We also consider four scalar multiplets, $\mathbf{6_1}^{(S)}$, $\mathbf{6_2}^{(S)}$, $\mathbf{15}^{(S)}$, and $\mathbf{21}^{(S)}$, apart from adjoints of $\mathbf{35}^{(S)}$, following the breaking chain in [1]. We use doublet–triplet splitting techniques to eliminate heavy degrees of freedom, so that only two SU(2) doublet fields and two SM-singlet scalars remain light, and all fermions receive mass through spontaneous symmetry breaking. The gauge–Yukawa sector, for each fermion generation, is

$$\begin{aligned}
\mathcal{L} \supset\ & y_{11}\mathbf{15}^{(F)}\bar{\mathbf{6}}_{\mathbf{1}}^{(F)}\bar{\mathbf{6}}_{\mathbf{1}}^{(S)} + y_{12}\mathbf{15}^{(F)}\bar{\mathbf{6}}_{\mathbf{1}}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(S)} \\
& + y_{21}\mathbf{15}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(F)}\bar{\mathbf{6}}_{\mathbf{1}}^{(S)} + y_{22}\mathbf{15}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(S)} \\
& + \tilde{y}_{11}\,\bar{\mathbf{6}}_{\mathbf{1}}^{(F)}\bar{\mathbf{6}}_{\mathbf{1}}^{(F)}\mathbf{15}^{(S)} + \tilde{y}_{12}\,\bar{\mathbf{6}}_{\mathbf{1}}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(F)}\mathbf{15}^{(S)} + \tilde{y}_{22}\,\bar{\mathbf{6}}_{\mathbf{2}}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(F)}\mathbf{15}^{(S)} \\
& + \hat{y}_{11}\,\bar{\mathbf{6}}_{\mathbf{1}}^{(F)}\bar{\mathbf{6}}_{\mathbf{1}}^{(F)}\mathbf{21}^{(S)} + \hat{y}_{12}\,\bar{\mathbf{6}}_{\mathbf{1}}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(F)}\mathbf{21}^{(S)} + \hat{y}_{22}\,\bar{\mathbf{6}}_{\mathbf{2}}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(F)}\mathbf{21}^{(S)} \\
& + y_u\,\mathbf{15}^{(F)}\mathbf{15}^{(F)}\mathbf{15}^{(S)} + \mathrm{H.c.}
\end{aligned} \tag{3}$$

For simplicity, we assume there is no mixing among the three SM fermion generations. After breaking the symmetries to $\mathrm{SU}(3)_{\mathrm{c}} \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_X$, in the low scale, we have the light scalar fields

$$\begin{aligned}
H_d &= \left(\mathbf{1}, \bar{\mathbf{2}}, -\frac{1}{2}; -1\right)\,, \qquad & H_u &= \left(\mathbf{1}, \mathbf{2}, \frac{1}{2}; -4\right)\,, \\
s_6 &= (\mathbf{1}, \mathbf{1}, 0; 5)\,, & s_{21} &= (\mathbf{1}, \mathbf{1}, 0; -10)\,,
\end{aligned} \tag{4}$$

and, for each generation, left-chiral Weyl fermion multiplets

$$\begin{aligned}
&Q : \left(\mathbf{3}, \mathbf{2}, \frac{1}{6}; 2\right)\,, \qquad u : \left(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3}; 2\right)\,, \qquad d_1, d_2 : \left(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3}; -1\right)\,, \\
&d' : \left(\mathbf{3}, \mathbf{1}, -\frac{1}{3}; -4\right)\,, \qquad L_1, L_2 : \left(\mathbf{1}, \mathbf{2}, -\frac{1}{2}; -1\right)\,, \qquad L' : \left(\mathbf{1}, \bar{\mathbf{2}}, \frac{1}{2}; -4\right)\,, \\
&e : (\mathbf{1}, \mathbf{1}, 1; 2)\,, \qquad \nu_1, \nu_2 : (\mathbf{1}, \mathbf{1}, 0; 5)\,.
\end{aligned} \tag{5}$$

The low-scale Yukawa Lagrangian is $\mathcal{L}_{\text{IR}} \supset \mathcal{L}_{\text{IR1}} + \mathcal{L}_{\text{IR2}}$, where

$$\begin{aligned}
\mathcal{L}_{\text{IR1}} &= 2y_u\, uH_u^{c\dagger}Q + y_d\, d_1 H_d Q + y_e\, eH_d L_1 + y_\nu\, L' H_d^{c\dagger}\nu_1 \\
&\quad + y_D\, d_2 d' s_6 + y_L\, L' L_2 s_6 + y_{\nu_1}\, \nu_1\nu_1 s_{21} + y_{\nu_2}\, \nu_2\nu_2 s_{21} + \text{H.c.}\,, \qquad (6)\\
\mathcal{L}_{\text{IR2}} &= \Big[y'_d\, d_2 H_d Q + y'_e\, eH_d L_2 + y'_\nu\, L' H_d^{c\dagger}\nu_2 \\
&\quad + y'_D\, d_1 d' s_6 + y'_L\, L' L_1 s_6 + 2\tilde{y}_{11}\, \nu_1 H_u^{c\dagger} L_1 + 2\tilde{y}_{22}\, \nu_2 H_u^{c\dagger} L_2 \\
&\quad + \tilde{y}_{12} \left(\nu_1 H_u^{c\dagger} L_2 + \nu_2 H_u^{c\dagger} L_1 \right) + \hat{y}_{12}\, \nu_1\nu_2 s_{21} + \text{H.c.} \Big]\,, \qquad (7)
\end{aligned}$$

where $H_{u,d}^c \equiv i\sigma_2 H_{u,d}^*$, couplings y_d, y_e, and y_ν originate from the coupling y_{11}, couplings y_D and y_L originate from y_{22}, couplings y'_d, y'_e, y'_ν from y_{21}, and couplings y'_D, y'_L from y_{12}. Such a complicated system gives the mass matrix for the Majorana fermions,

$$\frac{1}{2}M_\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & y'_L v_{s_6} & 2\tilde{y}_{11}v_u & \tilde{y}_{12}v_u \\ 0 & 0 & y_L v_{s_6} & \tilde{y}_{12}v_u & 2\tilde{y}_{22}v_u \\ y'_L v_{s_6} & y_L v_{s_6} & 0 & y_\nu v_d & y'_\nu v_d \\ 2\tilde{y}_{11}v_u & \tilde{y}_{12}v_u & y_\nu v_d & y_{\nu_1} v_{s_{21}} & \hat{y}_{12} v_{s_{21}} \\ \tilde{y}_{12}v_u & 2\tilde{y}_{22}v_u & y'_\nu v_d & \hat{y}_{12}v_{s_{21}} & y_{\nu_2} v_{s_{21}} \end{pmatrix}, \qquad (8)$$

in the basis $\langle \nu_{L_1}, \nu_{L_2}, \nu_{L'}, \nu_1, \nu_2|$, $|\nu_{L_1}, \nu_{L_2}, \nu_{L'}, \nu_1, \nu_2\rangle$. The mass eigenstates are $(\nu, N_1, \ldots, N_4)$, where ν is the SM neutrino and the lightest of the neutral states N_i is a DM candidate. While it is simple to assign correct neutrino masses for ν, the couplings in Eq. (7) lead to decays of the DM candidate, so that the model cannot properly describe DM.

4. Model, with quantum scale symmetry

In order to eliminate the decays of the DM candidate N_i, we restrict the analysis to the third SM generation and impose quantum scale symmetry at trans-Planckian scales. Then, we search for fixed-point solutions of the gauge–Yukawa system in Eq. (3) with gravity, treating f_g and f_y as free parameters, Section 2. Then, we select a non-trivial UV fixed-point solution in which the eigendirections defined by $y_{12,*} = y_{21,*} = \tilde{y}_{ij,*} = \hat{y}_{12,*} = 0$ are irrelevant (IR attractive). Then, the RG flow predicts that these couplings remain at zero all along the flow. Therefore, the fixed-point structure induced by quantum scale symmetry itself ensures that all the couplings in Eq. (7) are zero. The only non-vanishing low-energy couplings are those from Eq. (6), and therefore the lightest states among N_i are stable and can be DM candidates. This is the secluding mechanism.

Moreover, the fixed-point structure for the couplings in Eq. (6) is *maximally predictive*, in the sense that almost all couplings are predictions (the coupling y_{11} is free). Then, we can play with different values of f_y and $\tan\beta = v_u/v_d$ to find the SM top mass (from y_u). The mass of the SM bottom (from y_{11}) is a free parameter, while the masses of the heavy leptons (from y_{22}) and the DM candidates (from $\hat{y}_{11}$ and $\hat{y}_{22}$) are predictions. The mechanism persists under different choices of f_g, which in fact can even be zero, as the underlying UV theory SU(6) is asymptotically free.

In this maximally predictive scenario, the masses of the heavy Majorana fermions, in the limit $v_d \ll v_{s_6}, v_{s_{21}}$, are

$$
\begin{aligned}
m_{N_1} &\simeq \sqrt{2}\, y_{\nu_2} v_{s_{21}}\,, & m_{N_4} &\simeq \sqrt{2}\, y_{\nu_1} v_{s_{21}}\,,\\
m_{N_2} &\simeq \sqrt{2}\left(y_L v_{s_6} - \frac{1}{2}\frac{y_\nu^2 v_d^2}{y_{\nu_1} v_{s_{21}}}\right), & m_{N_3} &\simeq \sqrt{2}\left(y_L v_{s_6} + \frac{1}{2}\frac{y_\nu^2 v_d^2}{y_{\nu_1} v_{s_{21}}}\right).
\end{aligned}
$$

Different DM scenarios are possible depending on the hierarchy between v_{s_6} and $v_{s_{21}}$

— $v_{s_{21}} < v_{s_6}$: $m_{N_1} < m_{N_4} < m_{N_{2,3}}$. Since N_2, N_3, and N_4 do not mix with N_1 at the tree level, the lightest of these three heavy neutrinos, N_4, is also stable. We have a *two-component* DM sector with two SM singlets N_1 and N_4.

— $v_{s_6} < v_{s_{21}}$: $m_{N_{2,3}} < m_{N_1} < m_{N_4}$. It results in a *two-component* DM sector with an $SU(2)_L$ doublet (from N_2 and N_3) and a singlet N_1.

The lightest neutrino of the third generation is massless (possible in the inverted ordering, phenomenologically allowed [44]).

Additionally, we can also search for a second fixed-point solution in which all but one coupling are relevant directions (free parameters) at zero. Together with the first fixed point, we can then find RG trajectories yielding very small, yet non-zero couplings $\tilde{y}_{11}$, $\tilde{y}_{22}$ that are dynamically generated at the low-energy scale [21, 22]. The presence of these extra, naturally small Yukawa couplings gives mass to the neutrino and allows the decay of the heaviest of the two-component DM particles into either the SM neutrino (if $\tilde{y}_{11} \neq 0$), or the lightest N_i (if $\tilde{y}_{22} \neq 0$), so that DM is one-component: SM singlet or weak doublet, depending on the hierarchy between v_{s_6} and $v_{s_{21}}$.

5. Relic abundance

In [1], we studied the DM relic abundance for the two candidates in the less predictive scenario in which we have only one DM component, assuming the heavy scalars are sufficiently heavy and decoupled. For the singlet case, we focus on the DM annihilation into SM particles via a resonant channel with the Z', following Refs. [45–47]. We obtain an upper bound on the Z' mass, $m_{Z'} \lesssim 9$ TeV since the annihilation cross section starts to become kinematically suppressed by the mass of the mediator. Notice that the Yukawa couplings and the gauge coupling $g_X(1 \text{ TeV}) \approx 0.07$ are entirely predicted by the ASQG UV completion, while the relic-abundance condition indirectly constrains two of the relevant, free parameters of the scalar potential, related to the v.e.v.s v_{s_6} and $v_{s_{21}}$, to a very narrow slice of the parameter space. See the left panel in Fig. 1.

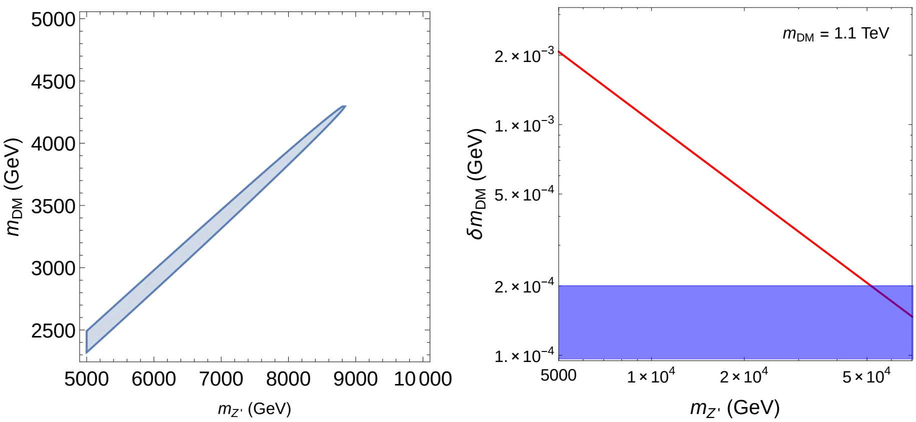

Fig. 1. Left: the region of under-abundant relic density (blue) in the SM-singlet case. Right: the upper bound on $m_{Z'}$ from inelastic scattering limits on the spin-independent DM-nucleon cross section in the case of doublet DM. From [1].

The doublet case is similar to a supersymmetric Higgsino-like neutralino, for which the effective thermally averaged cross section takes into account their co-annihilation into final states involving the gauge bosons of the SM. It is well known that condition $\langle \sigma v \rangle \approx 2.15 \times 10^{-26}$ cm^3/s requires $m_{\text{DM}} = m_{N_2} \approx 1.1$ TeV [48]. An upper bound on the Z' mass, $m_{Z'} \lesssim 50$ TeV, is extracted from the inelastic scattering limit of the spin-independent DM-nucleon cross section [49]. It indirectly constrains the heaviest v.e.v. of the DM sector, $v_{s_{21}}$. See the right panel in Fig. 1.

6. Conclusions

In these proceedings, we report on a secluding mechanism for DM from quantum scale symmetry introduced in [1]: couplings that would be allowed by the underlying gauge symmetry of the theory can be switched off by a particular fixed-point structure at trans-Planckian scales. In Sections 3 and 4, we illustrated the mechanism with one concrete example with a GUT

SU(6) model. We speculate that it is possible to generalize the mechanism for similar SU(N) models, even though phenomenology must be checked case-by-case. Then, in Section 5, we showed different ways in which UV boundary conditions based on quantum scale symmetry can constrain the DM sector. In particular, we demonstrated how one can constrain the physical mass scales of the theory by measuring observables at low energy. Such predictive power is one of the most attractive features of UV completions based on AS. The phenomenology of the most predictive scenario, together with the low-scale scalar potential and mixings across generations, is work in progress [50].

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