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# Bhabha SCATTERING AT FUTURE COLLIDERS WITH BHLUMI/BHWIDE[*]

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In this paper, we briefly present the Monte Carlo event generators BHLUMI and BHWIDE for small- and large-angle Bhabha scattering, respectively, and discuss possible ways of their improvements in order to satisfy precision needs of future electron–positron colliders.

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## 1. Introduction

Several future collider projects are being considered by the high-energy physics (HEP) community for the post-LHC era of particle physics research. Since no signals of physics beyond the Standard Model (BSM) have been observed so far at the LHC, in the 2020 update of the European Strategy for Particle Physics (ESPP), an electron–positron collider was indicated as the preferred choice to study the Higgs boson in greater detail as well as to perform precision measurements that would possibly reveal deviations from

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the Standard Model (SM) predictions, to be further studied in a next generation higher-energy (electron–positron, hadron or muon) collider [1]. The possible options for the $e^+e^-$ collider discussed then included the International Linear Collider (ILC) in Japan [2], Compact Linear Collider (CLIC) [3] or Future Circular Collider (FCC-ee) [4] at CERN, and Circular Electron Positron Collider (CEPC) in China [5]. The aim of the ongoing process of the 2026 update of ESPP is to choose the preferred option for CERN. In addition to the aforementioned colliders, three other projects have been taken into consideration: LEP3 [6], Linear Collider Factory (LCF) [7], and LHeC [8]. At the moment, FCC-ee seems to be the preferred option as the next HEP collider for CERN [9].

FCC-ee is planned to run in a few stages: at the $Z$-boson peak, at the $W^+W^-$ production threshold, at the maximum of the $ZH$ production cross section, at the $t\bar{t}$ production threshold, and maybe at some more collision energy points. The number of events to be collected at the $Z$ peak and at the $WW$ threshold are expected to exceed that of LEP experiments by several orders of magnitude. This would allow for the improvement the LEP measurements of key electroweak observables by factors from 10 to 500 [10–12]. In order to correctly interpret these measurements within SM or BSM, the experimental precision needs to be matched by theoretical predictions at the same level or better.

Among the processes of importance for experimental studies at electron–positron colliders is Bhabha scattering, *i.e.* the $e^+e^- \to e^+e^-$ process. It is usually divided into two classes depending on the range of an electron/positron scattering angle: (1) small-angle Bhabha scattering (SABS), typically with $\theta_e \lesssim 100\,\mathrm{mrad}$, and (2) large-angle Bhabha scattering (LABS), typically with $\theta_e \gtrsim 100\,\mathrm{mrad}$. Theoretical predictions for these processes that could be useful for experimental analyses should be provided in the form of Monte Carlo event generators (MCEGs).

In the following, we discuss two MCEGs — developed originally for LEP — in the context of the future $e^+e^-$ collider needs for theory predictions of the Bhabha scattering processes. Section 2 is devoted to BHLUMI [13–15] for SABS and Section 3 — to BHWIDE [16, 17] for LABS. In Section 4, we conclude our paper.

## 2. SABS with BHLUMI

Integrated luminosity is a very important parameter of particle colliders. It is used to translate numbers of experimentally observed events into cross sections of physical processes

$$\sigma = \frac{N}{\mathcal{L}}\,, \tag{1}$$

where $\sigma$ denotes a cross section of some physical process, $N$ is the number of events observed in a particle detector, and $\mathcal{L}$ is the luminosity. Then, the resulting cross section can be compared with predictions of theoretical models. Thus, precise knowledge of the collider luminosity is important for confronting experimental measurements with theory — in precision tests of the Standard Model (SM), on the one hand, and in searches for beyond the Standard Model (BSM) phenomena, on the other hand.

One of the methods of the luminosity measurement is to identify a reference process for which, on the experimental side, high event statistics $N_{\rm ref}$ can be collected, with low background and good control of systematics, and, on the theory side, high-precision calculations of the corresponding cross section $\sigma_{\rm ref}$ are possible, with negligible New Physics contributions, *i.e.*

$$\mathcal{L} = \frac{N_{\rm ref}}{\sigma_{\rm ref}}\,, \qquad \frac{\delta\mathcal{L}}{\mathcal{L}} = \frac{\delta N_{\rm ref}}{N_{\rm ref}} \oplus \frac{\delta\sigma_{\rm ref}}{\sigma_{\rm ref}}\,. \tag{2}$$

As can be seen from the second formula in Eq. (2), the luminosity error consists of two contributions: (1) an experimental error related to the measurement of $N_{\rm ref}$ and (2) a theoretical error resulting from accuracy of calculating $\sigma_{\rm ref}$. Generally, it is required that $\delta\sigma_{\rm ref}/\sigma_{\rm ref} \lesssim \delta N_{\rm ref}/N_{\rm ref}$, *i.e.* the luminosity measurement should not be limited by theory predictions.

At LEP, small-angle Bhabha scattering (SABS) was chosen as the main reference process for the luminosity measurement [18]. It is dominated by the $t$-channel $\gamma$ exchange which, in principle, is a pure QED process, *i.e.* high-precision theoretical calculations of $\sigma_{\rm ref}$ are possible. Since $\mathrm{d}\sigma/\mathrm{d}\theta_e \propto 1/\theta_e^3$, high event statistics can be collected by experiments at low angles, $\theta_e \lesssim 100$ mrad. Thanks to both experimental and theoretical efforts, the relative precision of the luminosity measurement at LEP reached the level of $\sim 5\times 10^{-4}$ at the $Z$-boson peak [19]. For the calculations of $\sigma_{\rm ref}$, all four LEP experiments used the MCEG BHLUMI 4.04 [14, 15].

At the future $e^+e^-$ colliders, the respective experimental precision can reach $\lesssim 10^{-4}$ at the $Z$ pole and $\mathcal{O}(10^{-3})$ at higher energies. This poses a big challenge for theoretical calculations. The main effects contributing to the error budget of luminometry at these colliders on the theory side and the ways to achieve the required precision were discussed in Refs. [20, 21]. There, the theoretical error for LEP at the $Z$ pole was reassessed down to 0.037%, and forecasts were made for achieving the precision of $10^{-4}$ at the $Z$ pole at FCC-ee and the sub-permil precision level at higher energies at this as well as other planned $e^+e^-$ colliders. These estimates were more recently updated in Refs. [22, 23]. In particular, the expected error for FCC-ee at the $Z$ pole was reduced to $0.7\times 10^{-4}$, and precision predictions for some higher collision energies were also improved.

It was argued [18] that when aiming at high numerical precision of the SABS cross section, it is better to reorder a perturbative series in calculations of radiative corrections from the usual expansion in the QED coupling constant $\alpha$ to the expansion in this coupling multiplied by the so-called big log, *i.e.* $\alpha^n L^m (m, n = 1, 2, \ldots, m \leq n)$, where $L \equiv \ln(|t|/m_e^2) - 1$. The latter was called the "pragmatic" QED expansion. In particular, it was shown that at LEP energies, the $\mathcal{O}(\alpha^2 L^2)$ coefficient, being formally of the second order in the traditional expansion, is by a factor up to 6 larger than the sub-leading first-order $\mathcal{O}(\alpha)$ factor. Following this pragmatic expansion, it turned out that for LEP, it was sufficient to include terms with the coefficients $\alpha L$, $\alpha$, and $\alpha^2 L^2$. This was done in BHLUMI 4.04 and was called $\mathcal{O}(\alpha^2)_{\rm prag}$. In order to reach the precision required for the future $e^+e^-$ colliders, in particular FCC-ee, this has to be upgraded to $\mathcal{O}(\alpha^3)_{\rm prag}$, *i.e.* additional terms proportional to the coefficients $\alpha^2 L$ and $\alpha^3 L^3$ need to be included.

The important feature of BHLUMI is that it is based on the Yennie–Frautschi–Suura (YFS) exclusive exponentiation (EEX) method [24] in which all the infrared (IR) singularities are summed up to the infinite order and canceled properly in the so-called YFS form factor. Then, calculated perturbatively IR-finite residuals feature faster convergence than radiative corrections evaluated in the standard order-by-order approach. An important merit of BHLUMI is also a dedicated efficient Monte Carlo (MC) algorithm which features, among others, the exact multiphoton phase space.

We have a three-stage plan of improvements in BHLUMI that would allow us to reach the luminosity precision goals set up by the future $e^+e^-$ colliders.

**Stage 1:** The mentioned above photonic $\mathcal{O}(\alpha^2 L)$ and $\mathcal{O}(\alpha^3 L^3)$ corrections needed for $\mathcal{O}(\alpha^3)_{\rm prag}$ predictions are already available and can be implemented in BHLUMI. The $\mathcal{O}(\alpha^3 L^3)$ corrections are included in the leading-log Monte Carlo program LUMLOG which is part of the BHLUMI 4.04 package. The $\mathcal{O}(\alpha^2 L)$ corrections were calculated and tested numerically in Refs. [25, 26]. The next large contribution to the theoretical error of $\sigma_{\rm ref}$ comes from the vacuum polarisation, more precisely from uncertainty of its hadronic part [20–23]. This, however, should be reduced considerably in the near future due to lattice QCD computations as well as new low-energy $e^+e^-$ and $e\mu$ experiments, see *e.g.* Refs. [27, 28]. The other significant uncertainty of the BHLUMI predictions comes from the light-fermion pair corrections. They, however, can be computed with our Monte Carlo program KoralW for all $e^+e^- \to 4f$ processes [29]. Although for SABS the $t$-channel $\gamma$-exchange contribution dominates, for the precision goals of the future $e^+e^-$ colliders, the $s$-channel $\gamma$-exchange as well as $s$- and $t$-channel $Z$-exchange contributions (with all their interferences) need to be included. All these contributions are implemented in the BHWIDE MC generator, including $\mathcal{O}(\alpha)$ electroweak corrections in the YFS EEX scheme. Therefore, at this stage, our theoretical predictions for SABS will be provided with the use of three MC generators BHLUMI $\oplus$ BHWIDE $\oplus$ KoralW.

**Stage 2:** Then, the matrix elements from BHWIDE for all $\gamma$- and $Z$-exchange contributions can be implemented in BHLUMI, and similarly for the light-fermion pairs, as described in Refs. [30, 31]. At this stage, all the necessary effects will be included in one MCEG — the $\mathcal{O}(\alpha^3)_{\rm prag}$ YFS EEX version of BHLUMI.

**Stage 3:** Our ultimate goal is to apply the coherent exclusive exponentiation (CEEX) formalism [32] to SABS, in a similar way as in the MCEG for $e^+\mathrm{e}^- \to 2f, f \neq e$, KKMC [33, 34]. This will result in the $\mathcal{O}(\alpha^3)_{\rm prag}$ YFS CEEX BHLUMI MCEG that should satisfy the theoretical precision needs of the luminosity measurements in the future $e^+e^-$ colliders.

In addition to the above main MCEG for SABS, testing tools of BHWIDE 4.04 will need to be upgraded. They are necessary for assessing both the theoretical and physical precision of the main MCEG. They include two MC generators: the pure leading-log $\mathcal{O}(\alpha^3 L^3)$ LUMLOG and the fixed-order $\mathcal{O}(\alpha)$ OLDBIS, as well as the semi-analytical calculations of Ref. [35]. For the $\mathcal{O}(\alpha^3)_{\rm prag}$ precision of BHLUMI, the terms $\sim \alpha^4 L^4$ would need be included in LUMLOG, while OLDBIS would need to be upgraded to the fixed-order $\mathcal{O}(\alpha^2)$ precision, which should be possible due to the recent progress of two-loop QED calculations with massive leptons [36]. In the above scheme, the MCEG developed at the earlier stage will also play a role of a testing tool for the MCEG of the next stage.

At LEP, important for establishing the final theoretical precision of the luminosity measurements were cross-checks of BHLUMI with external MCEGs of a similar physical precision, mainly SABSPV [37]. Something similar will be needed for the future $e^+e^-$ colliders, with BabaYaga [38] as a possible candidate.

## 3. LABS with BHWIDE

Large(or wide)-angle Bhabha scattering (LABS), with $\theta_e \gtrsim 100$ mrad, is used at the $Z$ peak for a direct measurement of the electron partial decay width $\mathit{\Gamma}_e$ of the $Z$ boson [18]. Since the total cross section for a process of fermion-pair production at the $Z$ peak is

$$\sigma_{e^+e^-\to 2f}\left(s = M_Z^2\right) \;\propto\; \mathit{\Gamma}_f\,\mathit{\Gamma}_e\,, \tag{3}$$

LABS is also indirectly used for the determination of other fermions partial widths $\Gamma_f$ in a model-independent way. Measurements of the $Z$-boson partial widths (or, equivalently, its decay branching ratios) are important for precision tests of SM as well as for BSM searches.

At higher electron–positron collision energies, $\sqrt{s} > M_Z$, LABS plays mainly a role of a significant background for other processes, in particular di-photon production ($e^+e^- \to \gamma\gamma$) which is also considered as a good candidate for the reference process of the luminometry at the future $e^+e^-$ colliders [39, 40].

At lower energies, $\sqrt{s} \lesssim 10$ GeV, in the so-called flavour factories, LABS is used mainly for the luminosity measurements [41], with the precision at the $\sim 0.1\%$ level.

All of the above requires precise theoretical predictions for LABS in the form of an MCEG. Most experiments at LEP used BHWIDE [16, 17] as the main MCEG for LABS. Its precision was estimated at 0.3% near the $Z$ peak (LEP1) and at 1.5% for higher energies (LEP2) [18], which was sufficient for these experiments. For flavour factories (BaBar, Belle, VEPP, BES, KLOE, *etc.*), its precision was assessed at $\mathcal{O}(0.1\%)$ [41]. Precision requirements of theoretical predictions for LABS at the future $e^+e^-$ colliders will be much higher than at LEP, particularly at FCC-ee, where a relative experimental precision can reach $\mathcal{O}(10^{-4})$ at the $Z$ peak and $\mathcal{O}(10^{-3})$ at higher energies [40].

LABS is much more complicated than SABS or other fermion-pair production processes because it involves both the $\gamma$ and $Z$ exchanges in both the $s$ and $t$ channels, with complex interference patterns [18]. Therefore, calculations of radiative corrections are more involved for LABS than for the other processes.

The current version of BHWIDE [17] features YFS EEX with $\mathcal{O}(\alpha)$ electroweak (EW) radiative corrections for LABS, which means that in addition to the resummation of the QED IR singularities to the infinite order, the non-IR residuals are calculated up to $\mathcal{O}(\alpha)$ within SM EW theory. For one-loop virtual corrections, BHWIDE is interfaced with two EW software libraries: BABAMC [42] and ALIBABA [43]. The latter goes beyond the strict $\mathcal{O}(\alpha)$ calculations by using dressed $\gamma$ and $Z$ propagators through the Dyson resummation of light-fermion contributions to intermediate bosons self-energy corrections. This is important because at high energies, these contributions include big logs $\sim \ln(q^2/m_f^2)$, where $q^2 = s, |t|$ and $m_f$ is the light-fermion mass. Thus, for the precision $\lesssim 1\%$ at $\sqrt{s} \gtrsim M_Z$, such a resummation is necessary. For LABS, these self-energy corrections cannot be factorised in terms of the overall running QED coupling $\alpha(q^2)$, like in SABS, but need to be resummed at the propagator-level due to the $\gamma$- and $Z$-exchange contributions in both the $s$ and $t$ channels, with all their interferences.

To satisfy the precision needs of the future high-energy $e^+e^-$ colliders, the $\mathcal{O}(\alpha^2)$ (NNLO) QED corrections will be necessary in BHWIDE, and at the $Z$ peak also the NNLO EW corrections (and perhaps $\mathcal{O}(\alpha^3L^3)$ QED corrections). The NNLO QED corrections consist of: (1) the two-loop corrections with massive leptons, which have recently been calculated [36], (2) the one-loop corrections to single bremsstrahlung with massive leptons, which can be computed with the use of automated software packages, such as OpenLoops [44] or Recola [45, 46], and (3) the tree-level double-bremsstrahlung matrix element, which can be calculated 'by hand' using spin amplitudes (*e.g.* like for single-bremsstrahlung in the current version of BHWIDE [16]) and/or with the help of some automated software packages. The NNLO EW corrections at the $Z$ peak can be computed with the GRIFFIN software library [47]. Therefore, reaching the precision levels of the future $e^+e^-$ colliders for LABS by BHWIDE looks feasible, although it will require considerable effort.

## 4. Conclusion

In this paper, we have discussed the role of the small- and large-angle Bhabha scattering processes and the question of precision needs for their theoretical predictions at the past, current, and future electron–positron colliders. We have briefly presented the Monte Carlo event generators BHLUMI and BHWIDE for these processes, discussed their current theoretical precision, and proposed strategies of their improvements in order to reach the precision goals of the future $e^+e^-$ colliders. The theoretical precision of both these generators can be further improved by using the recently developed collinearly-enhanced YFS resummation [48].

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