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GRAVITATIONAL LENSING: A WINDOW INTO THE FUNDAMENTAL PROPERTIES OF NATURE*

Aleksandra Piórkowska-Kurpas

August Chełkowski Institute of Physics
Faculty of Science and Technology, University of Silesia
75 Pułku Piechoty 1, 41-500 Chorzów, Poland

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The discovery of gravitational lensing marked a significant milestone in observational astronomy. Massive galaxy surveys, combined with dedicated search strategies, have resulted in hundreds of known strong lensing systems. Consequently, this phenomenon is increasingly seen as a crucial tool in cosmology and fundamental physics. This article reviews selected developments in this field.

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1. Introduction

Historically, gravitational lensing has served to validate Einstein's theory of gravity (see, *e.g.* [1] and references therein); currently, it also provides a unique environment for probing the fundamental laws of nature in a broader context. In particular, the original idea of Refsdal from 1964 [2] links the measurement of time delays between images of a gravitationally lensed supernova with the possibility of estimating the Hubble constant H_0, which provides information concerning the present expansion rate of the Universe. With this technique, H_0 can be estimated independently and in a complementary way to other observations, emphasizing the importance of strong lensing in this area. For a long time, this concept could not be applied in its original form until the first detection of a gravitationally lensed supernova in 2014 [3], which allowed for the estimation of H_0 at 73.5 $\mathrm{km\,s^{-1}Mpc^{-1}}$ and 72.8 $\mathrm{km\,s^{-1}Mpc^{-1}}$ depending on the time delay measurement method (see [4] for more information). This ultimately confirmed the validity of the lensing phenomenon as a reliable alternative tool for cosmology.

Following the direction indicated by Refsdal, gravitational lenses can be successfully used as the so-called standard rulers in measuring distances to astrophysical objects, thus allowing for the estimation of key cosmological

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parameters, including not only H_0 (see [5] for a comprehensive overview) but also those present in the dark energy equation of state [6–9]. Accurate information concerning the latter may help to better understand the phenomenon of dark energy, the mysterious material content of the Universe responsible for its currently accelerating expansion [10]. The point is that any tool that allows us to study properties of this mysterious ingredient in a way complementary to other probes is priceless. In this light, strong gravitational lensing appears particularly promising.

In addition to their use in cosmology, gravitational lenses can also serve as a valuable tool for testing theories that extend beyond Einstein's theory of gravity. General relativity naturally loses its applicability at distances below the Planck length scale ($\sim 10^{-35}$ m), which is equivalent to an energy of the order $\sim 10^{19}$ GeV (Planck energy) [11]. Thus, it is expected that nonstandard effects related to such a new theory (widely accepted in the literature to be the quantum gravity) should manifest at energies of the order of the Planck energy, which is many times greater than the energy scales currently available to us. To overcome this problem, a strategy based on an effective phenomenology technique has been adapted. The idea is as follows: tiny corrections to the standard theory from physics at the Planck scale are considered to see how far current observations permit the presence of these effects. In this light, possible violations of fundamental symmetries such as Lorentz invariance [11, 12], the running nature of fundamental constants [13], or the nonzero graviton mass [14, 15] are of particular interest. This is because the smallness of quantum gravity effects at low energies enforces the careful selection of the testing area, such that tiny deviations from the well-established predictions of the standard theory could be more noticeable. In particular, time delays between lensed signals from distant astrophysical sources allow us to put constraints on quantum gravity effects. Here, we discuss this issue in more detail.

2. Strong lensing as standard rulers for cosmology

One idea of using individual lensing galaxies to estimate cosmological parameters is based on the fact that for a given strong lensing system, image separation depends only on the ratio of the angular diameter distances to the lens and to the source, provided one has a reliable model for the mass distribution within the intervening galaxy acting as a lens (see [9] and references therein). Early-type galaxies (ETGs or ellipticals) comprise the majority of stellar mass in the Universe, influencing the statistics of the gravitational lensing phenomenon and resulting in a sample of strong lenses dominated by this type of galaxies [16]. Consequently, the most common lens model fitted to the observed image configuration is based on a singular isothermal ellipsoid (SIE) model, featuring an elliptical projected mass distribution [17]. In our approach, the calculations were restricted, however, to a simpler but still reliable model of mass distribution in a lensing galaxy, that is, the singular isothermal sphere (SIS) model with its generalized form: $\rho \sim r^{\gamma}$ with the power-law index γ treated as an additional parameter. Within this model, one can calculate the total mass of the lens by solving the Jeans equation, assuming full isotropy and that both stellar and mass distributions follow the same power-law profile [18]. This mass (*i.e.* dynamical mass $M_{\rm dyn}$) should be understood as a galaxy mass assessed through the spectroscopy via stellar luminosity-averaged line-of-sight velocity dispersion $\sigma_{\rm ap}$ measured within an aperture (of angular size $\theta_{\rm ap}$) projected to the lens plane and scaled to the Einstein radius $\theta_{\rm E}$. On the other hand, knowledge of the image separation enables us to estimate the mass of the lens enclosed within the Einstein radius $\theta_{\rm E}$ (*i.e.* lensing mass $M_{\rm lens}$) [19]. Assuming equality between these two masses (*i.e.* $M_{\rm lens} = M_{\rm dyn}$), we obtain the following relation for the distance ratio — an actual observable, which can be determined through spectroscopic observations and astrometry:

$$\mathcal{D}^{\rm obs} = \frac{c^2 \theta_{\rm E}}{4\pi\sigma_{\rm ap}^2}\left(\frac{\theta_{\rm ap}}{\theta_{\rm E}}\right)^{2-\gamma} f^{-1}(\gamma)\,, \tag{1}$$

where $\mathcal{D}^{\rm obs} \equiv \frac{D_{\rm ls}}{D_{\rm s}}$, with $D_{\rm ls}$ and $D_{\rm s}$ being angular diameter distances, respectively, between the source and the lens and to the source. The function $f(\gamma)$ depends only on the slope γ through a combination of Euler's Gamma functions (see Eq. (6) in [9]); for the SIS model ($\gamma = 2$), the value of this function is 1. In this method, the cosmological model enters a distance ratio through the definition of angular diameter distance, which in the flat Friedmann–Robertson–Walker geometry is $D(z_1, z_2; \boldsymbol{p}) = \frac{1}{1+z_2}\frac{c}{H_0}\int_{z_1}^{z_2}\frac{{\rm d}z}{h(z;\boldsymbol{p})}$, such that the theoretical formula for the distance ratio is the following:

$$\mathcal{D}^{\rm th}(z_{\rm l}, z_{\rm s}; \boldsymbol{p}) = \frac{\int_{z_{\rm l}}^{z_{\rm s}}\frac{{\rm d}z}{h(z;\boldsymbol{p})}}{\int_0^{z_{\rm s}}\frac{{\rm d}z}{h(z;\boldsymbol{p})}}\,. \tag{2}$$

Here, c is the speed of light in vacuum and $h(z;\boldsymbol{p})$ is a dimensionless expansion rate depending on the redshift z and the cosmological model parameters $\boldsymbol{p}$. It is worth noting that the distance ratio (Eq. (2)) is independent of the H_0, which constitutes a strong point of this method, particularly in the context of the so-called Hubble tension, *i.e.* the discrepancy between local measurements of H_0 based on Cepheid variable stars and Type Ia supernovae, and those inferred from the early Universe through observations of the cosmic microwave background (see *e.g.* [20]). In two popular cosmological scenarios, the dimensionless Hubble function takes the following form: *(i)*

$h^2(z;w) = \Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^{3(1+w)}$ for wCDM cosmology, where dark energy is considered as a hypothetical homogeneous fluid phenomenologically described by the barotropic equation of state $p = w\rho$ (the cosmological constant Λ constitutes a special case in this model, with $w = -1$); *(ii)* $h^2(z;w_0,w_1) = \Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^{3(1+w_0+w_1)} \exp\left(-\frac{3w_1 z}{1+z}\right)$ for the CPL parameterization with the evolving equation of state for dark energy $w(z) = w_0 + w_1 \frac{z}{1+z}$ [21, 22]. Here, Ω_m is the mass density in the Universe, including baryonic and dark matter.

Fitting procedure (Monte Carlo simulations of the posterior likelihood) probing how well a statistical model (*i.e.* $\mathcal{D}^{\rm th}(z_{\rm l}, z_{\rm s}; \boldsymbol{p})$) explains observed data (*i.e.* $\mathcal{D}^{\rm obs}$) performed for a sample of strong lensing systems carefully chosen from known lensing catalogs allows for putting constraints on cosmological parameters, *e.g.* within the two mentioned above cosmologies. When γ is treated as a free parameter in the fitting procedure, information concerning the mass distribution within lensing galaxies can be obtained simultaneously with the cosmological parameters, thereby enhancing the reliability of all estimates. Moreover, the evolution of elliptical galaxies with redshift can be investigated by adopting the parametrization $\gamma(z_{\rm l}) = \gamma_0 + \gamma_1 z_{\rm l}$, where $z_{\rm l}$ denotes the redshift of the lensing galaxy (a new, dark energy-model-independent method for the investigation of the mass density slopes of lensing galaxies and their redshift evolution was recently proposed by [23]). Detailed information on the method and calculations can be found in [9]. The results displayed in Table 2 of [9] agree with the values of γ, w, w_0, and w_1 parameters obtained by other authors, demonstrating that strong gravitational lensing systems can probe cosmic equation-of-state parameters for dark energy and their potential time evolution. The analysis presented in [9] focused solely on the parameters in the dark energy equation of state, with the value of Ω_m treated as a fixed in accordance with Planck data [24]. More recently, the problem of constraining the Ω_m parameter using the distance ratio method was discussed in detail by [25] on an enriched sample of lenses.

3. Gravitational lensing and limits on the graviton mass

The first indirect evidence for gravitational waves (GWs) came from the binary pulsar system PSR 1913+16, discovered and monitored by Hulse and Taylor [26]. On the other hand, the first GW direct laboratory detection was achieved with GW150914 (a binary black hole merger) by the Advanced LIGO detector [27]. Both confirmed Einstein's prediction concerning GWs [28] and the existence of compact binary systems, opening a new window for testing fundamental physics. In particular, with GW detectors, it is possible to probe general relativity and beyond in a way that is unaccessible to other techniques. For example, within standard theory, GWs propagate through spacetime at the speed of light c. By contrast, violations of the Weak Equivalence Principle (WEP hereafter) or the presence of massive gravitons, as predicted within some alternative theories of gravity (see, *e.g.* [11, 15] and references therein), would imply a GW propagation speed different from c. GW observations therefore provide a unique opportunity to perform such tests, and indeed several of them have already been conducted (see, *e.g.* [29–31]).

Our method builds upon our previous proposal [32], which was originally formulated in the context of testing Lorentz Invariance Violation through gravitational time delays of electromagnetic (EM) signals from astrophysical sources monitored in different energy bands. In [33], we extended this idea to gravitational lensing of sources that emit both EM and GW signals simultaneously. In scenarios involving massive gravitons, the Einstein radius associated with GW signals would deviate slightly from that of the lensed EM waves. In this case, $\theta_{\rm E,GW} = \theta_{\rm E}(1 + \frac{m_{\rm GW}^2 c^4}{2E^2})$, where $\theta_{\rm E}$ is the Einstein radius for lensed EM signal, $m_{\rm GW}$ is the graviton mass, and E — the graviton energy (this formula was originally derived by [34] for massive photons but is also valid in the context of massive gravitons). This discrepancy leads to a measurable difference in lensing time delays between the GW and EM channels (*i.e.* $\delta T \equiv \Delta t_{\rm GW} - \Delta t_\gamma$), which encodes the information concerning the GW propagation speed $v_{\rm GW}$. The general form of the bound on $v_{\rm GW}$, valid for a broad class of analytical lens models, is as follows:

$$1 - \left(\frac{v_{\rm GW}}{c}\right)^2 \leq \frac{\delta T}{\Delta t_\gamma F_{\rm lens}(z_{\rm l}, z_{\rm s})}\,, \tag{3}$$

where $F_{\rm lens}(z_{\rm l}, z_{\rm s}) \sim O(1)$ is a factor specified by Eq. (18) in [33], weakly dependent on the lens model and the background cosmology. The calculations leading to Eq. (3) were based on a modified dispersion relation, and therefore our method is independent of any particular nonstandard model of gravity. We assumed, however, that: *(i)* the theory of gravity is metric and predicts gravitational lensing similar to that in general relativity; *(ii)* gravitons are massive and travel along timelike geodesics. Thus, our method may not be suitable for certain classes of alternative theories of gravity. Being differential in nature, our method is inherently free from assumptions regarding possible intrinsic delays between EM and GW signals at the source. Moreover, since the Shapiro delay is generated in the lens plane, our method is only weakly dependent on the underlying cosmological model.

It is clear from the formula given by Eq. (3) that the timing accuracy δT with which time delays are determined sets constraints on $v_{\rm GW}$. Since the timing of GW detectors is exact ($< 10^{-4}$ ms [35]), the accuracy of Δt_γ is the primary limitation of the method. In this case, the lensing of galaxies has a greater restrictive power in the light of our method than galaxy–galaxy lensing, as the cluster-scale images have significantly larger time delays. For example, considering Refsdal supernova value of time delay for SX image (the one which reappeared after one year, as predicted [3]), one would have $1-\left(\frac{v_{\rm GW}}{c}\right)^2 \leq 3.2\times10^{-11}$ with the assumption of $\delta T = 1$ ms timing accuracy [33]. It is worth mentioning here that the method based on the travel time technique is more restrictive in this context, due to the cumulative effect along the particle's path, provided the time delay between EM and GW signals generated at the source is known. In this case, if they are emitted from the same source at the same time, the GW signal would come later than its EM counterpart as a consequence of a nonzero graviton mass. This effect directly follows from the modified dispersion relation for a massive graviton. The difference between the EM and GW arrival times in lensed images is given by $\Delta t_{\gamma,{\rm GW}} = \frac{1}{2H_0}(1+z_{\rm s})^2\int_0^{z_{\rm s}}\frac{{\rm d}z}{(1+z)^2h(z)}$ [33]. For a source at the redshift $z_{\rm s} = 2$ and with the assumption of the standard (ΛCDM) cosmological scenario with parameters taken in accordance with the Planck results (*i.e.* $H_0 = 68$ km s^{-1}Mpc^{-1} and $\Omega_m = 0.3$ [36]), one can obtain the following bound: $1-\left(\frac{v_{\rm GW}}{c}\right)^2 \leq 9.92\times10^{-22}$ [33], which is a few orders of magnitude more restrictive than the strongest dynamical bounds obtained by probing the propagation of GW from compact binary mergers (see, *e.g.* [29]). In this context, strong gravitational lensing systems offer the possibility to compare the moments of arrival of the same image seen both in EM and GW windows, which may help to overcome the intrinsic time delay problem. This would be possible, however, only for transient EM sources, especially short gamma-ray bursts (SGRBs) associated with GW emission. The details of our methodology, together with a discussion of the associated analysis of its restrictive power, are provided in [33].

4. Discussion and perspectives

Modern physics continues to confront a range of fundamental questions that remain unsolved. Among the most significant is the unknown physical mechanism responsible for the accelerating expansion of the Universe. In observational cosmology, the most essential tools are those that rely on precise distance measurements to astrophysical objects [37], grounded in the concepts of standard candles, standard rulers, and, more recently, standard sirens [38, 39] (see also [40]). In this context, strong gravitational lensing systems can serve as the so-called (individual) standard rulers in constraining dark energy equation-of-state parameters through the distance ratio defined in Eq. (2) [7–9]. Until recently, the limited sample of known strong lensing systems was the primary obstacle to using this method. This is because strong lensing is an extremely rare phenomenon: even the most massive galaxies can distort flat spacetime such that the deflection angle of light would be only a few arcseconds. The rough estimates indicate that only one of 10000 massive galaxies would serve as a lens [41]. This explains why the first gravitational lensing system was identified decades after the formulation of the general theory of relativity (the twin quasars 0957+561 A and B, discovered in 1979 [42]) and why, by the late 1990s, only 11 such objects had been cataloged. A change in search strategy, now focused on identifying spectral lines at higher redshifts associated with background sources and embedded within the spectra of potential lensing galaxies, followed by imaging to confirm the presence of multiple images around these candidate galaxies, proved highly effective. As a result, massive spectroscopic surveys (SDSS, BOSS), supported by HST observations, have brought hundreds of known strong gravitational lensing systems. This, in turn, allowed us to examine and develop the method based on distance ratios initially proposed by [7]. Recent analyses [8, 9] show its potential in this field. Still, however, only a tiny section of the sky has been included in the search at the appropriate depth (redshift z) and with sufficient imaging resolution. At the dawn of the large-area sky surveys era, the prospects for strong lens detection in the near future appear optimistic. In particular, the forecasts for lens discoveries within the Euclid mission during its operational time and 10 years of data collection by the Viera C. Rubin (LSST) Observatory are such that we expect over two orders of magnitude more galaxy–galaxy lenses than those currently known [41, 43, 44]. The real problem lies, however, in the limited ability of the detection process: larger surveys cover an increasingly larger number of galaxies, which does not translate into an increasing number of images that astronomers can examine visually due to their limited processing power (see the discussion in [44]). In this light, new fully automated searching methods are necessary (see, *e.g.* [45] and references therein).

Another major problem in fundamental physics is the absence of clear theoretical and experimental guidance indicating the path toward a consistent theory of quantum gravity. The only robust way to test quantum gravity lies in searching for subtle, non-standard effects that may manifest in the low-energy regime, as predicted within some quantum gravity frameworks. Strong gravitational lensing may serve as a diagnostic tool in this context. In particular, following the idea proposed in [32] and in [33], a careful analysis of time delays between multiple images of a strongly lensed quasar observed in the EM window in different energy bands, or between lensed GW and the EM counterpart, seems promising for probing physics at the Planck scale (*e.g.* Lorentz invariance violation or massive graviton scenarios). The method is more robust compared to other quantum gravity tests based on time-of-flight measurements (see, *e.g.* [46] for a review), primarily because it is independent of the unknown intrinsic time lags originating at the source and is only weakly influenced by the background cosmology. The above idea is particularly interesting in the light of future ground-based GW interferometers such as the Einstein Telescope (ET; see [47]) or space-borne missions like Laser Interferometer Space Antenna (LISA; [48]) or DECihertz Interferometer Gravitational wave Observatory (DECIGO; [49] and its smaller-scale version B-DECIGO [50, 51]). These next-generation detectors are expected to elevate GW astronomy to an unprecedented level, primarily due to their vastly improved sensitivities, which will enable probing volumes many times larger than those currently accessible with the LIGO/Virgo observations [47, 52]. This should result in extensive catalogs of GW signals from coalescing double compact objects. The expected yearly detection rates in the GW window are of the order of 10^4–10^6 for ET [53], 10^2–10^6 for DECIGO, and 10^3–10^5 for B-DECIGO [54], depending on the type of system observed, the adopted population synthesis scenario, and galaxy metallicity evolution. Since our method discussed in Section 3 requires compact sources that produce both GW and electromagnetic (EM) signals, which must then be strongly lensed, the relevant ones from our perspective are binary neutron star mergers. These objects are expected to be observable as short gamma-ray bursts (GRBs) — transient EM signals at the highest energy range, followed by emission in other ranges of EM spectrum. After the first LIGO/Virgo evidence for a binary neutron star coalescence [55], accompanied by an EM counterpart observed in different wavelengths [56], our proposal seems realistic. Indeed, robust predictions suggest that ET will be able to observe a few such events per year [53, 57]. It is also worth noting that ET, and especially LISA or DECIGO/B-DECIGO, owing to their enhanced sensitivity to signals at frequencies below ~ 1 Hz, will be able to register double compact binaries during their inspiral phase for a long time (weeks to years; see [51]) prior to coalescence. This unique capability will enable continuous monitoring of such systems during their inspiral stage, thereby allowing for preparation for the eventual detection of their merger not only as chirping GW waveforms but also as GRBs in the EM window.

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