BabaYaga@NLO AT PRESENT AND FUTURE e^+e^- COLLIDERS. CELEBRATING 25 YEARS OF BabaYaga*

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Precise QED radiative corrections for low- and high-energy electron–positron colliders are essential for accurate simulations of luminosity processes and precision tests of the Standard Model. We review the historical formulation and the recent developments of the BabaYaga@NLO event generator, which implements a QED Parton Shower (PS) matched with fixed-order calculations. We discuss the theoretical formulation of the code, as well as the assessment of its theoretical accuracy. Applications at low- and high-energy e^+e^- colliders are presented, including the latest result, together with the perspectives for future improvements, in view of the demanding precision requirements of future machines at the intensity frontier.

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1. Introduction

Precision measurements at electron–positron colliders rely on the accurate theoretical description of Quantum Electrodynamics (QED) radiative corrections. Photon radiation from the beams and charged final states can affect cross section and event shapes [1], therefore, its precise modelling is mandatory to reach the sub-percent accuracy required by the Standard Model (SM) precision measurements. An example of the importance of QED radiative corrections and PS algorithms is given by luminosity measurements [2], for which a precision below 0.1% (0.01%) level is required at low(high)-energy machines. A comparable level of precision is also required for the measurement of $e^+e^- \to$ hadrons cross section at centre-of-mass

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(c.m.)
energies below 2 GeV [3]. This quantity enters the dispersive calculation of the anomalous magnetic moment of the muon a_μ in the Standard Model, which exhibits a several-σ discrepancy with data [4].

For instance, on the theory side, the relevant logarithm appearing in Bhabha scattering luminosity — $L \equiv \log(Q^2/m_e^2)$, where Q^2 is the relevant energy scale squared and m_e the electron mass — is of the order of 15–20 at the energy scale of 1–10 GeV, considering a large angular acceptance at flavour factories, or a small one at high-energy machines. For these reasons, an accurate description of additional photon radiation is of utmost importance, making Monte Carlo generators for QED radiative corrections an essential tool for e^+e^- colliders phenomenology. A description and comparison between the available codes can be found in [5].

In this context, Parton Shower (PS) algorithms play a central role, by providing an event-by-event description of photon emissions in an exclusive way, therefore allowing for the resummation of large logarithms appearing in perturbation theory — see Table 1 — while preserving the differential structure of the final states. This latter feature is particularly important for realistic MC event generation, where all relevant experimental event selection criteria and acceptance cuts can be implemented.

Table 1. Perturbative structure of QED radiative corrections. From top to bottom, the power counting in α increases the fixed-order accuracy. From right to left, the logarithmic expansion $L \equiv \log Q^2/m_e^2$ becomes more relevant.

	LL	NLL	NNLL
LO	α^0		
NLO	αL	α	
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
H.O.	$\sum_{n=3}^{\infty} \frac{1}{n!}\alpha^n L^n$	$\sum_{n=3}^{\infty} \frac{1}{n!}\alpha^n L^{n-1}$	$\ldots$

In this work, we will review the theoretical description of the BabaYaga Parton Shower algorithm, retracing the main steps that brought us to the actual formulation of BabaYaga@NLO, as well as some applications of the code in relevant experimental setups, where it has been used during the 25 years of its development.

2. The QED Parton Shower algorithm

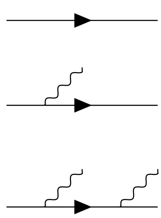
The original version of BabaYaga [6] was developed for luminosity measurements at flavour factories with Large Angle Bhabha Scattering (LABS), within the leading-logarithmic (LL) approximation; then, an updated version [7] introduced the Matching with next-to-leading (NLO) exact matrix elements, reducing the theoretical error to the 0.1% level. In this section, we review the main ideas underlying the theoretical formulation of the QED Parton Shower algorithm implemented in the most recent version BabaYaga@NLO (see [8] and references therein for a detailed review).

2.1. A Monte Carlo solution of DGLAP equations

The cross section for the generic process $e^+e^- \to X^+X^-$ at c.m. energy $\sqrt{s}$, including the effect of photon radiation emitted from charged legs, can be written as [9]

$$\sigma(s) = \prod_i \int \mathrm{d}x_i D\left(x_i, Q^2\right) \int \mathrm{d}\Omega \, \frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}(x_1 x_2 s)\Theta(\mathrm{cuts})\,, \tag{1}$$

where $i = 1, 2\,(3, 4)$ is the energy fraction of the initial(final)-state particles entering the hard-scattering process described by the Born-level differential cross section $\mathrm{d}\sigma_0$, while the Heaviside Θ implements the relevant cuts on the phase-space $\mathrm{d}\Omega$. The Structure Functions (SF) $D(x, Q^2)$ are an effective way to describe multiple emissions of hard and soft photons collinear to the emitting particles, and are solution of the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) equation [10]. The SF can be written as the sum of terms, each expressing the n^{th} photon emission [8]



$$\begin{aligned} D(x,s) = \;& \Pi\left(s, m^2\right)\, \delta(1-x) \\ & + \frac{\alpha}{2\pi} \int_{m^2}^{s} \frac{\mathrm{d}s'}{s'}\, \Pi\left(s, s'\right)\, \Pi\left(s', m^2\right) \int_0^{1-\varepsilon} \mathrm{d}y\, P(y)\, \delta(x-y) \\ & + 2 \text{ photon terms} \ldots \end{aligned} \tag{2}$$

by introducing the Sudakov form factor $\Pi(s, s')$, which represents the probability that the parton X evolves from the virtuality s' to s emitting a photon with energy fraction below ε. The MC implementation is based on updating the virtualities of each parton following Eq. (2), as detailed in [6], therefore generating the energy of each photon accordingly. The PS approach has the advantage of allowing for an exclusive generation of the photon angular spectrum beyond the collinear approximation. In BabaYaga, inspired by the YFS approach [11], the generation of a transverse momentum $k_{l,\mathrm{T}} \neq 0$ of the $l^{\rm th}$ photon is achieved by generating the $\cos\theta_l$ as the eikonal function $\mathcal{I}_{ij}(k_l)$, calculated with respect to the external charged legs i, j. By combining all these ingredients, we arrive at the expression of the cross section in PS approach at leading-logarithmic (LL) accuracy

$$\mathrm{d}\sigma^{\rm LL}_{\rm PS} = \Pi\left(\varepsilon\sqrt{s}/2, Q^2\right) \sum_{n=0}^{\infty} \frac{1}{n!} \left|\mathcal{M}^{\rm LL}_n\right|^2 \mathrm{d}\Phi_n\,, \tag{3}$$

where the Sudakov form factor is defined as

$$\Pi\left(\varepsilon, Q^2\right) = \exp\left\{-\frac{\alpha}{2\pi}\int_0^{1-\varepsilon} \mathrm{d}z\, P(z) \int \mathrm{d}\Omega_k I(k)\right\}\,, \tag{4}$$

and $P(z)$ is the Altarelli–Parisi vertex for the $X \to X + \gamma$ branching, which describes the probability density of emitting a photon with the energy fraction $1-z$ of the parent parton X and with $I(k) = \sum_{ij} \mathcal{I}_{ij}(k)$. The matrix element for the $n^{\rm th}$ photon emission $\mathcal{M}^{\rm LL}_n$ is an approximation of the exact $\mathcal{M}_n$ at LL accuracy, where each photon emission is approximated by a factor $\alpha/(2\pi)P(z)I(k)$ calculated on a kinematics mapped onto the $2 \to 2$ underlying Born process and factorised on $|\mathcal{M}_0|^2$, that can be iterated from the $n = 1$ case.

2.2. BabaYaga@NLO: matching and precision

In the LL approximation, all the tower of logarithms $\alpha^n L^n$, corresponding to the first column of Table 1, is correctly resummed, with the $\mathcal{O}(\alpha)$ being the first missing term. Introduced in [7], the matching procedure allows us to correctly include the exact NLO calculation while resumming all the $n \geq 2$ photon emissions. The procedure can be easily understood by comparing the $\mathcal{O}(\alpha)$ expansion of Eq. (3) with the exact calculation. Therefore, one can introduce the *soft-virtual* correcting factor in a multiplicative way $F_{\rm SV} = 1 - \left(C_\alpha - C^{\rm LL}_\alpha\right)$ — where $C^{\rm (LL)}_\alpha$ is the coefficient multiplying the zero-photon exact (LL) soft+virtual cross section — so that the exponentiated $\mathcal{O}(\alpha)$ correction is always exact. In the same spirit, the real matrix elements can be corrected by introducing a *hard* correction $F_{\rm H} = 1 + (|\mathcal{M}_1|^2 - |\mathcal{M}^{\rm LL}_1|^2)/|\mathcal{M}_1|^2$, arriving at the BabaYaga@NLO master formula, valid for any number of photonic emissions

$$\mathrm{d}\sigma_{\rm NLOPS} = F_{\rm SV}\, \Pi\left(\varepsilon\sqrt{s}/2, \{p\}\right) \sum_{n=0}^{\infty} \frac{1}{n!} \left(\prod_{i=1}^{n} F_{{\rm H},i}\right) \left|\mathcal{M}^{\rm LL}_n\right|^2 \mathrm{d}\,\Phi_n(\{p\},\{k\})\,. \tag{5}$$

The first term that we are missing in this approximation is the $\mathcal{O}(\alpha^2 L)$, which is $(\frac{\alpha}{\pi})^2 L \sim 0.1\%$ at $Q^2 \sim \mathcal{O}(\mathrm{GeV})$, giving a rough estimate of the theoretical precision of the approach. By a tuned comparison with other available codes, such as BHWIDE [7], as well as with analytical calculations [12], the accuracy of BabaYaga@NLO is assessed at the 0.1% level at flavour factories, as it will be discussed later.

2.3. Available final states

To conclude this section, we briefly show in Table 2 all the available processes in the current and upcoming version of BabaYaga@NLO, with the associated precision at which they are calculated.

Table 2. Processes ($e^+e^- \to XX$) implemented in BabaYaga@NLO and corresponding perturbative accuracy, where the NLOPS QED is intended for all final states. FF stands for Form Factor, included in the NLO calculation for $\pi\pi$ final states, with the approach indicated in parentheses.

	Process	Order (NLOPS QED $\oplus$)	Ref.
Published	e^+e^-	LO EW $\oplus$ LO SMEFT	[6, 7]
	$\mu^+\mu^-$	LO EW	[7]
	$\gamma\gamma$	NLO EW	[12]
	$\pi^+\pi^-$	FF (F$\times$sQED, GVMD, FsQED)	[13]
WIP	$e^+e^-\gamma$		
	$\mu^+\mu^-\gamma$		[14]
	$\pi^+\pi^-\gamma$	FF (F$\times$sQED)	

In the following sections, we show some phenomenological applications of radiative corrections, as computed with BabaYaga, in relevant experimental scenarios.

3. Low-energy colliders

Electron–positron colliders operating in the 1–10 GeV energy range continue to play a central role in precision tests of the Standard Model and in hadronic physics. Experiments such as KLOE, BESIII, BelleII, BaBar, CMD, and SND are either still taking data or analysing high-statistics datasets, which require a very good control of QED radiative corrections. One of the primary goals of these machines is the precise measurement of hadronic final states, which enter the dispersive calculation of the leading-order hadronic vacuum polarisation contribution to the muon $g-2$ anomaly,

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_{\pi^0}^2}^{\infty} \mathrm{d}s \frac{\hat{K}(s)}{s^2} R(s)\,, \tag{6}$$

where $R(s) = \sigma(e^+e^- \to \text{hadrons}\,(+\gamma))/\sigma_0(e^+e^- \to \mu^+\mu^-)$ is the ratio between the photon-inclusive hadronic cross section and the tree-level dimuon one. In this framework, Monte Carlo generators fulfil various tasks: they are employed for precision luminosity measurements via Large Angle Bhabha Scattering (LABS) or di-photon production, for QED tests through the measurement of the ratios like $N_{e^+e^-}/N_{\mu^+\mu^-}$, and for the modelling of exclusive hadronic channels such as $\pi^+\pi^-, K_L K_S, \ldots$ In this section, we present two examples to illustrate the crucial role played by Monte Carlo generators in low-energy precision measurements.

3.1. Luminosity measurements at flavour factory

In collider physics, the luminosity $\mathcal{L}$ is a fundamental machine parameter that allows us to convert the observed number of events for a certain process to its absolute cross section, via the relation $\sigma = N_{\mathrm{exp}}/\mathcal{L}$. At e^+e^- machines, it is convenient to measure the luminosity using a well-defined reference process, exploiting the relation $\mathcal{L} = N_0/\sigma_0^{\mathrm{th}}$, whose cross section σ_0 is calculable with very high accuracy in perturbation theory. At flavour factories, luminosity is typically measured with a relative error of some permille, making the resummation of multiple photon emissions a mandatory requirement for MC generators.

In experimental analyses, the theoretical precision is often estimated by comparing two independent generators. BabaYaga3.5 [15] was used as a benchmark in a number of experiments, with a theoretical precision of 0.5% for Large Angle Bhabha and of 1% for di-photon and $\mu^+\mu^+$ production, mainly due to the missing $\mathcal{O}(\alpha)$ constants [3]. The matched version BabaYaga@NLO has improved the theoretical precision to the level of 0.1%, as shown by tuned comparison [3] for the Bhabha channel at ϕ, τ, B factories, as well as for the $\mu^+\mu^-$ [5] in a CMD-like scenario.

3.2. The pion form factor

The cross section of the process $e^+e^- \to \pi^+\pi^-$, parametrised with the pion form factor $F_\pi(q^2)$, is the dominant contribution to $a_\mu^{\mathrm{HVP,LO}}$, due to the peaking behaviour of the dispersion integral (6) in the low-energy region. It can be alternatively extracted from energy-scan measurements or by the radiative return through the $e^+e^- \to \pi^+\pi^-\gamma$ process, with current experimental determinations exhibiting tensions up to 5σ level [4] between the two methods. Understanding the origin of this discrepancy needs a very good control of radiative corrections and, in particular, the availability of independent MC generators for the $\pi^+\pi^-(\gamma)$ final state. For example, in the latest measurement of $F_\pi(q^2)$ by the CMD-3 Collaboration [16] via energy scan, the uncertainty associated with radiative corrections is the 0.3% — more than 40% of the systematic error — mostly coming from the differences between BabaYaga@NLO and MCGPJ. In this context, the latest developments of BabaYaga@NLO [13] are in the direction of improving the theoretical description of the $\pi\pi$ channel, by implementing the PS algorithm, taking into account the internal structure of the pion at NLO accuracy. In particular, beyond the traditional scalar QED approach (F$\times$sQED), the form factor has been introduced in the Generalised Vector-Meson Dominance (GVMD) model [17] and in the dispersive approach (FsQED) [18]. These improved descriptions of $\pi^+\pi^-$ show a very good agreement with the charge asymmetry data, an observable extremely sensitive to the treatment of $F_\pi(q^2)$ in the loops. Ongoing efforts [14] aim to generalise the formulation of BabaYaga@NLO also for the $\mu^+\mu^-\gamma$, $\pi^+\pi^-\gamma$ final states, representing the first calculation at NLOPS accuracy for radiative processes.

4. Future high-energy colliders

Next-generation electron–positron colliders are being designed as Higgs, Top, and Electroweak (EW) factories [19], allowing for precision measurements at energies ranging from the Z-pole up to the $t\bar{t}$ production threshold. Luminosity calibration will play a crucial role in revisiting the Electroweak measurements, in order to fully exploit the very high statistics expected. We focus the discussion on the FCC-ee [20], which is expected to operate at c.m. energies of $\sqrt{s} = 91$, 160, 240, 365 GeV. Precision measurements at FCC-ee, such as the W-boson mass and width, will require a luminosity uncertainty at the level of $\delta\mathcal{L} \leq 10^{-4}$, while for the measurement of the HZ production cross section $\delta\mathcal{L}$ should be kept at least at the per-mille level. Building on the experience of LEP, luminosity at future machines is expected to be measured either with SABS or $e^+e^- \to \gamma\gamma$ at large angle. In this section, we present some recent developments in BabaYaga at high energy, including NLO electroweak corrections to the $\gamma\gamma$ and potential New Physics (NP) contamination in SABS.

4.1. High precision $\gamma\gamma$ luminosity measurements

The diphoton production, a pure QED process at leading order, is already used for luminosity calibration at few-GeV flavour factories with permille accuracy [21]. At high energy, $\gamma\gamma$ events are, in principle, statistically more limited w.r.t. the Large Angle Bhabha — $\sigma_{\rm LABS} \simeq 66\,\sigma_{\gamma\gamma}$ at $\sqrt{s} = 91$ GeV — which acts as a background. On the other hand, the $e^+e^- \to \gamma\gamma$ cross section has the advantage of a reduced sensitivity to the HVP, which enters only at the two-loop level, making it a theoretically sound alternative to the Bhabha. Moreover, experimentally, reaching 10 ppm ultimate relative precision on SABS events is technically challenging [19], requiring control of the position of calorimeters at the sub-μm level.

In order to meet the 10^{-4} precision goal, at least NLO electroweak corrections have to be included with higher orders QED on top. This has been done in [22, 23], where NLO EW corrections have been estimated to be $\mathcal{O}(1\text{–}2\%)$ in angular distribution, in a 20°–160° setup, of the same size of QED higher orders. Moreover, the effect of the fermionic and hadronic NNLO corrections, with their uncertainty, has been estimated by factorising the vacuum polarisation on the pure NLO corrections as

$$\sigma^{\Delta\alpha}_{\rm NNLO} \pm \delta\sigma_{\rm had} \simeq \left(\sigma^{\rm QED}_{\rm NLO} - \sigma_{\rm LO}\right) \times [\Delta\alpha(s) \pm \Delta\alpha_{\rm had}] \tag{7}$$

being of the order of 0.1% at FCC-ee. In order to lower the uncertainty to the 0.01% level, a first step would be the inclusion of the exact $\mathcal{O}(\alpha^2 N_f)$ fermionic virtual and real corrections, which is in progress in BabaYaga, as well as the full NNLO QED corrections, which have been implemented recently in McMule [24], possibly matched with a PS.

4.2. New Physics in SABS luminosity

On top of radiative corrections, one may ask if any unknown New Physics could contaminate luminosity processes at a level comparable with the FCC-ee precision target. The first step in this direction has been made in [25] for $e^+e^- \to \gamma\gamma$, while the latest update of BabaYaga concerns NP in SABS. In the Standard Model, the Bhabha at small angle is largely dominated by the t-channel photon exchange, being almost a pure QED process. At the 10^{-4} level, on top of NLOPS EW corrections, the process has an uncertainty due to the HVP. In the hypothesis of such error to be reduced by the time of future colliders, we investigated [26] whether any Light or Heavy New Physics could contaminate the process at the precision goal. Based on present bounds on beyond the SM interactions, it has been found that Light NP has negligible effects, while the Heavy NP, parameterised by means of the Standard Model Effective Field Theory (SMEFT) has an effect in the range of 10^{-5}–10^{-4}, representing a potential source of uncertainty for FCC-ee at the Z pole. A strategy to reduce such contaminations could be to measure the forward–backward asymmetry for the LABS, providing therefore model-independent constraints on Heavy NP by fitting $A_{\rm FB}(\sqrt{s})$.

5. Outlook and future developments

QED radiative corrections are a fundamental ingredient for low- and high-energy electron–positron colliders, where the resummation of multiple photon emission is mandatory for precision studies. The BabaYaga event generator has been developed for precision luminosity measurements at flavour factories more than 25 years ago, and has received continuous improvement. The latest version BabaYaga@NLO is able to generate many processes at NLOPS in QED, with an estimated theoretical accuracy of 0.1%. In this work, we have shown the theoretical formulation of the code, as well as some of the historical and more recent phenomenological applications. In the near future, an important update regarding radiative processes $X^+X^-\gamma$, for $X = \mu, \pi$ will be released, marking a milestone for the measurement of hadronic cross sections at low energy.

In the future, the BabaYaga team plans to further improve the code by introducing the next-to-next-to-leading order matching with the Parton Shower, proceeding on the path to 0.01% precision for future e^+e^- colliders.

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