FACTORISATION SCHEMES FOR PROTON PDFs*

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Received 3 February 2026, accepted 5 February 2026,
published online 22 April 2026

Beyond leading order, perturbative QCD calculations require the choice of a factorisation scheme to define the coefficient functions and parton distribution functions. Through the years, a number of different schemes were proposed, with different motivations and purposes. We present the first-ever comparison of these factorisation schemes on a common basis. We compare their features, both at the analytical and numerical level, and assess the impact of this choice on phenomenology at the LHC.

DOI:10.5506/APhysPolBSupp.19.2-A6

1. Introduction

Beyond leading order QCD calculations, the factorisation between parton distribution functions (PDFs) and partonic cross sections is not uniquely defined anymore. Physical observables must be independent of unphysical choices, including the choice of factorisation scheme. For perturbative truncations, this is only guaranteed by cancellations up to the considered order in $\alpha_{\rm s}$, and residual scheme-dependence persists from higher-order terms, which may be numerically significant. Nowadays, the $\overline{\rm MS}$ scheme [1] is considered the default choice due to its simplicity, but alternative factorisation schemes exist, with various motivations for each of them, ranging from schemes for NLO matching to parton showers to schemes enforcing positivity of PDFs. We provide the first systematic comparison in a common notation of these schemes and analyse their differences both analytically and numerically. In this contribution, we highlight some of the results obtained from our paper [2].

* Presented at the XLVI International Conference of Theoretical Physics "Matter to the Deepest", Katowice, Poland, 15–19 September, 2025.

2. Factorisation schemes

We assume here collinear factorisation, which allows one to formulate a hadronic cross section as the convolution of a coefficient function (calculable perturbatively) with non-perturbative parton distribution functions. In order to obtain PDFs in a different scheme than $\overline{\mathrm{MS}}$, three routes are possible. The most natural one would be to directly fit them in the given scheme, but it is also the most complicated one, as one first needs to obtain the splitting functions in the considered scheme to perform the DGLAP evolution [3–5], but also needs to perform the actual fitting procedure. A second possibility is to transform the PDFs to the chosen scheme at an initial scale Q_0 and then evolve the transformed PDFs in the same scheme, which only requires the determination of splitting functions. Finally, the easiest way is to evolve the PDFs in the original $\overline{\mathrm{MS}}$ scheme, and transform them into the other scheme at each scale. This is the approach chosen throughout this work.

The transformed PDFs are obtained through their convolution with transformation kernels $\mathbb{K}_{ab}$

$$f_a^{\mathrm{FS}}(x,\mu_\mathrm{F}) = \sum_b \int_x^1 \frac{\mathrm{d}z}{z}\, \mathbb{K}_{ab}^{\overline{\mathrm{MS}}\to\mathrm{FS}}(z,\mu_\mathrm{F})\, f_b^{\overline{\mathrm{MS}}}\left(\frac{x}{z},\mu_\mathrm{F}\right), \tag{1}$$

with

$$\mathbb{K}_{ab}^{\overline{\mathrm{MS}}\to\mathrm{FS}}(z,\mu) = \delta_{ab}\,\delta(1-z) + \frac{\alpha_\mathrm{s}(\mu)}{2\pi}\, K_{ab}^{\overline{\mathrm{MS}}\to\mathrm{FS}}(z,\mu) + \mathcal{O}\left(\alpha_\mathrm{s}^2\right). \tag{2}$$

To allow for easier comparison between schemes, we decompose all kernels in a unified way

$$K(z) = \sum_{k=0}^{1} a_k\,\mathcal{D}_k(z) + b(z)\log(1-z) + c(z)\log z + P(z) - \Delta\,\delta(1-z), \tag{3}$$

where $\mathcal{D}_k(z) = \left[\frac{\log^k(1-z)}{1-z}\right]_+$.

We will focus here primarily on the Krk [6, 7], MPOS [8], and MPOSδ schemes, for which we briefly summarize their motivations:

- Krk: This scheme is tailored for general purpose Monte Carlo event generators and aims at reducing the complexity of matching NLO calculations to parton showers. It is the basis for the KrkNLO method [6, 9].
- MPOS/MPOSδ: These schemes enforce positivity of the PDFs, with MPOS enforcing momentum conservation via a soft function in diagonal kernels, while MPOSδ does it via a $\delta(1-z)$ term instead.

3. Scheme comparison

In this section, we present selected results from [2] where we performed a comprehensive comparison of the schemes. In what follows, we use CT18NLO [10] as the base $\overline{\text{MS}}$ PDF set which is then transformed to the other schemes.

3.1. PDFs

The transformed CT18NLO gluon PDFs in different schemes are shown in Fig. 1 at low scale (2 GeV) and high scale (100 GeV). The most striking feature is the drastic difference between schemes at low-x at 2 GeV, with the Krk and MPOS (MPOSδ) distributions being up to one order of magnitude larger than the $\overline{\text{MS}}$ one. At 100 GeV, the differences are reduced, but the overall trend persists. This effect originates primarily from the K_{gg} kernel which has a $\log(z)/z$ divergence at low-x[1]. Figure 2 shows the charm PDFs, for the same schemes and at the same scales as the gluon ones. At low scales, one observes a large spread between schemes, with notably clear negativity for the MPOS/MPOSδ schemes around $x = 10^{-2}$. It is the result of the breakdown of perturbation theory due to large α_s in the low-energy regime. This negativity is washed out at 100 GeV thanks to the DGLAP evolution. This issue will be touched upon further in Section 3.3.

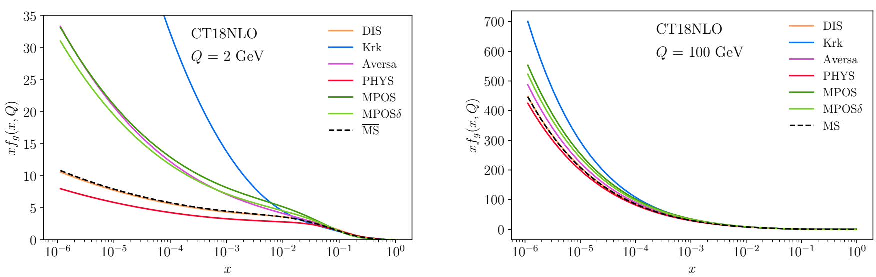

Fig. 1. Left: Comparison of transformed CT18NLO gluon PDFs in different schemes at $Q = 2$ GeV. Right: Same at $Q = 100$ GeV.

3.2. Sum rules

In the $\overline{\text{MS}}$ scheme, PDFs are constrained to satisfy both the momentum sum rule and the number sum rules. The former is defined as

$$\sum_i \int_0^1 \mathrm{d}x \; x f_i(x,Q) = 1 \,, \tag{4}$$

[1] For more details on the different contributions, one can refer to [2].

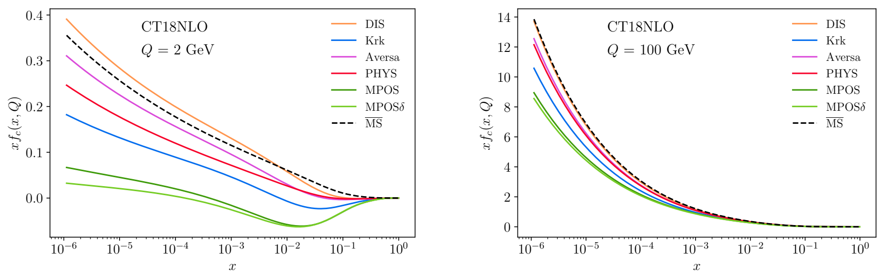

Fig. 2. Left: Comparison of transformed CT18NLO charm quark PDFs in different schemes at $Q = 2$ GeV. Right: The same at $Q = 100$ GeV.

while the latter are defined as

$$\int_0^1 \mathrm{d}x \ \left[f_u(x,Q) - f_{\bar{u}}(x,Q)\right] = 2\,, \qquad \int_0^1 \mathrm{d}x \ \left[f_d(x,Q) - f_{\bar{d}}(x,Q)\right] = 1\,,$$

$$\int_0^1 \mathrm{d}x \ \left[f_{s,c,b}(x,Q) - f_{\bar{s},\bar{c},\bar{b}}(x,Q)\right] = 0\,. \tag{5}$$

Figure 3 shows the numerical computation of the momentum sum rule in the case of transformed CT18NLO PDFs in different schemes and for the $\overline{\text{MS}}$ PDFs. The deviation for all schemes is found to be of the order of 10^{-5} which is negligible, especially since the original PDFs also deviate from the ideal value of 1. One can also note that while MPOS and MPOSδ impose momentum conservation in different ways, their results are very similar across the whole Q range[2].

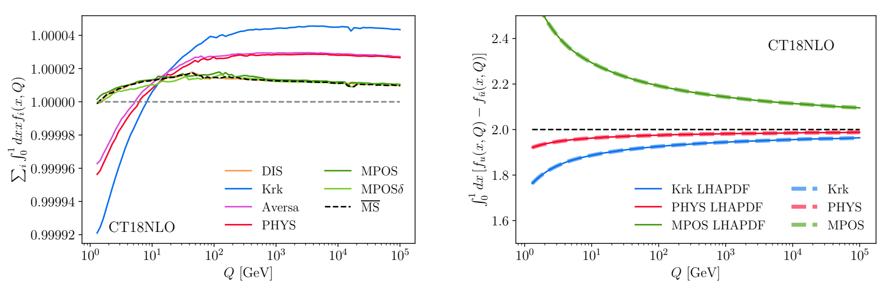

Fig. 3. Left: Momentum sum rule as a function of the factorisation scale Q for CT18NLO PDFs transformed into different schemes. Right: Number sum rule as a function of the factorisation scale for the up quark.

[2] The two schemes, however, differ in what fractions of momentum gluons, sea quarks, and valence quarks carry, as well as in the x dependence of the distributions.

When switching from the $\overline{\text{MS}}$ scheme to another one, the number sum rules are modified[3] and pick up corrections of the order of $\mathcal{O}(\alpha_s)$

$$N_q^{\text{FS}} = N_q^{\overline{\text{MS}}} \left\{ 1 + \frac{\alpha_s(\mu)}{2\pi} \int_0^1 \mathrm{d}z \, K_{qq}^{\overline{\text{MS}}\to\text{FS}}(z) \right\} , \tag{6}$$

where N_q^{FS} is defined as $N_q^{\text{FS}} = \int_0^1 \mathrm{d}x \, \left[f_q^{\text{FS}}(x,Q) - f_{\bar{q}}^{\text{FS}}(x,Q) \right]$ and similarly for the $\overline{\text{MS}}$ scheme.

On the right side of Fig. 3, we show the number sum rule for the up quark for transformed CT18NLO PDFs in the Krk, PHYS, and MPOS schemes, both computed directly from the LHAPDF [11] grids (plain lines) and from Eq. (6) (dashed lines). One can observe a perfect agreement between the dashed and plain lines, showing the deviation from the usual number sum rules, which depends on the scale considered. This provides a useful validation of our numerical transformation code. The $\overline{\text{MS}}$ sum rule is unmodified in the case of a $K_{qq}^{\overline{\text{MS}}\to\text{FS}}$ kernel integrating to 0[4]. One practical consequence of this is that since the number sum rules can be scale-dependent, it may be more natural to not impose the number sum rules during the fit of PDFs in these schemes, as it is done for $\overline{\text{MS}}$, but rather use them as a consistency check.

3.3. Positivity

As was shown in Fig. 2, heavy quark parton distribution functions can be negative close to the heavy quark mass thresholds. This, of course, is sensitive to the choice of PDF sets and how the chosen sets were constructed. In Fig. 4, we show heatmaps of the charm and bottom quark PDFs obtained from the transformation of the CT18NLO PDFs into the MPOS scheme as functions of the momentum fraction x and the factorisation scale Q. In both cases, we can see that the partonic functions are negative close to the mass thresholds, especially at high-x. This can be explained by a large negative contribution of the $\overline{\text{MS}}$ gluon PDF to the transformed heavy quark PDFs which is not balanced by the rather small $\overline{\text{MS}}$ heavy quark PDFs. Indeed, in the case of the CT18NLO sets, no intrinsic charm is present and the positivity of the distribution is not imposed during the fit, unlike other sets such as NNPDF40MC [12], where positivity is imposed globally or NNPDF40 [13] which includes intrinsic charm and imposes positivity for light flavours above $\sqrt{5}$ GeV.

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[3] One could also have corrections for the momentum sum rule, but these corrections were eliminated by fixing the $\delta(1-z)$ terms (or the soft function for the MPOS scheme) in the diagonal kernels.

[4] This is the case of the DIS scheme, which preserves the number sum rules by construction because K_{qq} is a plus-distribution.

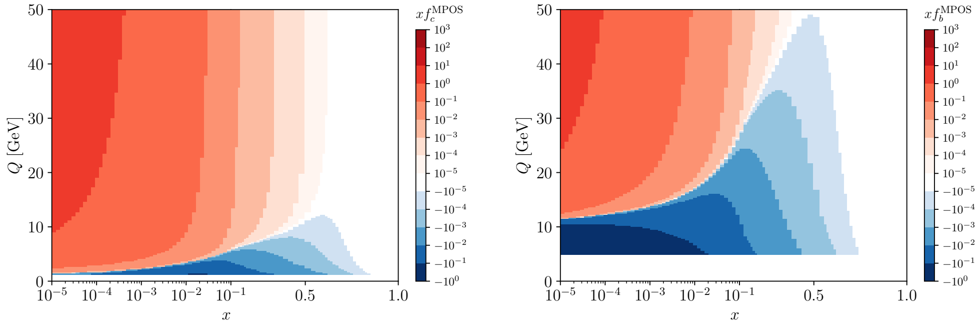

Fig. 4. Left: Heatmap of the charm quark CT18NLO PDFs in the MPOS scheme as functions of x and Q. Right: Same for the bottom quark.

In the case of the charm quark PDF, this negativity is effectively cured at higher scales by the generation of perturbative charm through the DGLAP evolution, at around 10 GeV[5]. Alternatively, the addition of intrinsic charm was also found to restrict considerably the region in (x,Q) where negativity is present. For the bottom quark PDF, the DGLAP evolution needs to be performed to a scale of at least 30 GeV in order to cure negativity, which is way above the bottom quark mass threshold. Unlike the charm PDFs, the addition of perturbative beauty would not be able to resolve the problem of negativity, which may point to the need for quark-mass effects in the transformation kernels for heavy quarks.

4. Impact on phenomenology

The choice of factorisation scheme for a calculation at a given order is compensated by a modification of the corresponding partonic cross section at the same order, inducing changes to the calculation at one order higher[6]. In some observables, only the real-emission kinematics result in a non-zero result, which at NLO effectively leads to a LO calculation in which the factorisation scheme choice is not compensated. In this section, we study the LO calculation of $pp \to Z + j$, for which the effect of the choice of scheme can be treated as a proxy for the effect on NLO $pp \to Z$. All the results shown in this section were obtained using HERWIG7 [14, 15] with the NNPDF40MC sets transformed into different schemes and stored in LHAPDF6 format.

We use a setup appropriate for LHC Run 2, with a center-of-mass energy of 13 TeV and cuts similar to ATLAS and CMS for $pp \to Z \to ll$, with the invariant mass of the lepton pair as the dynamical renormalisation

[5] We consider negativity lower than 10^{-4} to be negligible, as it is smaller than the LHAPDF6 target interpolation precision (10^{-3}).

[6] For an NLO calculation, such a choice would induce only NNLO modifications.

and factorisation scale. Figure 5 presents the differential cross section for $Z + j$ production at the LHC for different schemes. Distributions in both Z transverse momentum and jet rapidity calculated in other schemes are consistently smaller than the $\overline{\text{MS}}$ ones, with the variation going as far as 20% for the Krk and MPOS/MPOSδ schemes, exceeding the $\overline{\text{MS}}$ scale variation (shown by the shaded band) at low transverse momentum or at the extremes of the jet rapidity.

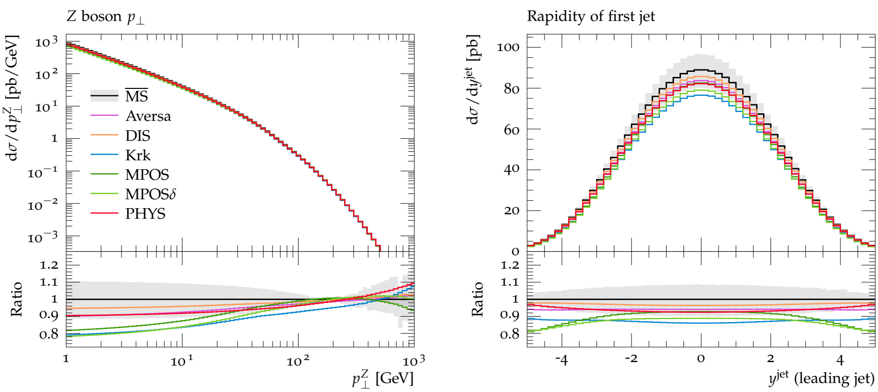

Fig. 5. Factorisation-scheme dependence of differential cross sections for $Z + j$ production at the LHC. Leading-order predictions for these observables correspond to an NLO calculation of the Drell–Yan process.

5. Conclusion

We performed the first-ever systematic comparison of many of the proposed factorisation schemes for next-to-leading order QCD calculations, decomposing their transformation kernels on a common basis, allowing for easier comparison. At the parton distribution functions level, we observed several features, among which the deviation from the usual number sum rules or the negativity of heavy flavor PDFs near the mass thresholds for specific schemes. We also studied the impact of the choice of factorisation scheme to LHC phenomenology and found that the uncertainty coming from this choice could be as large as 20%, sometimes even exceeding the scale uncertainty of the $\overline{\text{MS}}$ calculation. This work was done by transforming the input $\overline{\text{MS}}$ PDFs at each scale, the natural extension of this work will be to determine the splitting functions in the schemes considered in this work. This will allow one to transform the input PDFs at an initial scale and evolve them through the DGLAP evolution in a given scheme.

S.D. is grateful for the support of the National Science Centre (NCN), Poland under MAESTRO grant No. 2023/50/A/ST2/00224. A.K. and A.S. are grateful for the support of the National Science Centre (NCN), Poland under the OPUS grant No. 2025/57/B/ST2/04034.

REFERENCES

[1] W.A. Bardeen, A.J. Buras, D.W. Duke, T. Muta, *Phys. Rev. D* **18**, 3998 (1978).

[2] S. Delorme, A. Kusina, A. Siódmok, J. Whitehead, *Eur. Phys. J. C* **85**, 505 (2025); *Erratum ibid.* **85**, 1151 (2025).

[3] G. Altarelli, G. Parisi, *Nucl. Phys. B* **126**, 298 (1977).

[4] V.N. Gribov, L.N. Lipatov, *Sov. J. Nucl. Phys.* **15**, 438 (1972).

[5] Y.L. Dokshitzer, *Sov. Phys. JETP* **46**, 641 (1977).

[6] S. Jadach *et al.*, *J. High Energy Phys.* **2015**, 052 (2015).

[7] S. Jadach *et al.*, *Eur. Phys. J. C* **76**, 649 (2016).

[8] A. Candido, S. Forte, F. Hekhorn, *J. High Energy Phys.* **2020**, 129 (2020).

[9] P. Sarmah, A. Siódmok, J. Whitehead, *J. High Energy Phys.* **2025**, 062 (2025).

[10] T.-J. Hou *et al.*, *Phys. Rev. D* **103**, 014013 (2021).

[11] A. Buckley *et al.*, *Eur. Phys. J. C* **75**, 132 (2015).

[12] J. Cruz-Martinez *et al.*, *J. High Energy Phys.* **2024**, 088 (2024).

[13] NNPDF Collaboration (R.D. Ball *et al.*), *Eur. Phys. J. C* **82**, 428 (2022).

[14] J. Bellm *et al.*, *Eur. Phys. J. C* **76**, 196 (2016).

[15] G. Bewick *et al.*, *Eur. Phys. J. C* **84**, 1053 (2024).