NNLOCAL: COMPLETELY LOCAL SUBTRACTIONS FOR COLOR-SINGLET PRODUCTION IN HADRON COLLISIONS*

Vittorio Del Duca [a], Claude Duhr [b]
Levente Fekésházy [c,d], Flavio Guadagni [e]
Pooja Mukherjee [c], Gábor Somogyi [f]
Francesco Tramontano [g,h], Sam Van Thurenhout [f]

[a]INFN, Laboratori Nazionali di Frascati, 00044 Frascati (RM), Italy
[b]Bethe Center for Theoretical Physics, Universität Bonn, 53115, Germany
[c]II. Institut für Theoretische Physik, Universität Hamburg
Luruper Chaussee 149, 22761, Hamburg, Germany
[d]Institute for Theoretical Physics, ELTE Eötvös Loránd University
Pázmány Péter sétány 1/A, 1117, Budapest, Hungary
[e]Physik-Institut, Universität Zürich, 8057 Zürich, Switzerland
[f]HUN-REN Wigner Research Centre for Physics
Konkoly-Thege Miklós u. 29-33, 1121 Budapest, Hungary
[g]Dipartimento di Fisica Ettore Pancini, Università di Napoli Federico II
Via Cintia, Edificio 6, 80126 Napoli, Italy
[h]INFN — Sezione di Napoli, Via Cintia, Edificio 6, 80126 Napoli, Italy

Received 9 January 2026, accepted 15 January 2026,
published online 22 April 2026

In this contribution, we present the extension of the CoLoRFulNNLO subtraction scheme to the production of color-singlet final states in hadronic collisions. We also showcase the NNLOCAL code, a publicly available proof-of-concept implementation of the method, and report on the current directions of code development.

DOI:10.5506/APhysPolBSupp.19.2-A7

1. Introduction

The Standard Model (SM) of particle physics provides a very successful description of elementary particles and their interactions. Indeed, data

* Presented at the XLVI International Conference of Theoretical Physics "Matter to the Deepest", Katowice, Poland, 15–19 September, 2025.

collected at the LHC have given us a spectacular confirmation of the SM up to the TeV scale. Nevertheless, despite this great success, it is by now rather obvious that the SM cannot be the ultimate theory of fundamental interactions, since it fails to account for phenomena such as dark matter, baryogenesis, and neutrino masses. Despite this, no direct signals of physics beyond the SM have been observed at the LHC so far and first indications of New Physics may well be indirect. This state of affairs has highlighted the role of precision experiments as crucial tests of the validity of the SM at the TeV scale and beyond.

From a theoretical point of view, in order to fully exploit the physics potential of the LHC, quantum chromodynamics (QCD), as well as electroweak interactions (EW) must be understood and modelled as best as possible. In this regard, a plethora of issues must be addressed, from better determination of parton density functions (PDFs), to the computation of exact higher-order corrections, to refined descriptions of parton evolution and hadronization, among many others. In this contribution, we focus on the evaluation of exact higher-order perturbative corrections. Although straightforward in principle, computing perturbative results beyond leading order (LO) in practice runs into a host of technical difficulties. One such issue is the presence of infrared (IR) divergences in intermediate steps of the calculation. Indeed, higher-order corrections are sums of several pieces which involve extra radiation (as compared to the LO process), which can be real or virtual. In both cases, the presence of unresolved partons leads to the appearance of IR singularities, although the sum of all contributions is finite for properly defined (infrared and collinear-safe) observables. Clearly, these divergences must be regularized and rendered finite before any numerical calculation can take place. One fruitful approach to handling IR divergences is to employ a local subtraction method. The essential idea is to use appropriately chosen subtraction terms to reshuffle divergences between each contribution of the full higher-order correction in such a way that they are rendered finite after the reshuffling. Since the construction of the subtraction terms is not unique, several explicit schemes have been pursued in the literature [1–7]. It must be pointed out that approaches other than local subtraction [8, 9] have also been successfully applied to compute higher-order corrections.

In this contribution, we give a brief overview of the CoLoRFulNNLO method [4] for computing QCD corrections at next-to-next-to-leading order (NNLO). In particular, we address the recent extension of the method to deal with the production of color-singlet final states in hadron–hadron interactions. We also showcase the NNLOCAL code [10], a proof-of-concept public implementation of the method, and provide an update on its status.

2. Local subtraction at NNLO

Hadronic cross sections in QCD perturbation theory are computed as convolutions of PDFs with hard scattering partonic cross sections

$$\sigma(p_A,p_B) = \sum_{a,b} \int_0^1 \mathrm{d}x_a\, f_a\left(x_a,\mu_\mathrm{F}^2\right) \int_0^1 \mathrm{d}x_b\, f_b\left(x_b,\mu_\mathrm{F}^2\right)\, \sigma_{ab}\left(p_a,p_b;\mu_\mathrm{F}^2\right) . \quad (1)$$

Here, $f_a(x_a,\mu_\mathrm{F}^2)$ and $f_b(x_b,\mu_\mathrm{F}^2)$ are non-perturbative PDFs, while the hard partonic cross section $\sigma_{ab}(p_a,p_b;\mu_\mathrm{F}^2)$ can be computed in perturbation theory, *i.e.*, as an expansion in the strong coupling α_s

$$\sigma_{ab}\left(p_a,p_b;\mu_\mathrm{F}^2\right) = \alpha_\mathrm{s}^\ell \sum_{k=0}^{\infty} \left(\frac{\alpha_\mathrm{s}}{2\pi}\right)^k \sigma_{ab}^{\mathrm{N}^k\mathrm{LO}}\left(p_a,p_b;\mu_\mathrm{R}^2,\mu_\mathrm{F}^2\right) . \quad (2)$$

The precision of the prediction in Eq. (1) can be increased by retaining more terms in the perturbative expansion. However, as mentioned above, beyond leading order, we must account for extra radiation, both real and virtual. Hence, higher-order corrections are sums of several terms, corresponding to different patterns (real and/or virtual) of extra emissions. At NNLO, the correction for a process with m jets at the Born level is a sum of three terms,

$$\begin{aligned}\sigma_{ab}^{\mathrm{NNLO}} &= \int_{m+2} \mathrm{d}\sigma_{ab}^{\mathrm{RR}} J_{m+2} + \int_{m+1} \left(\mathrm{d}\sigma_{ab}^{\mathrm{RV}} + \mathrm{d}\sigma_{ab}^{\mathrm{C}_1}\right) J_{m+1} \\ &\quad + \int_{m} \left(\mathrm{d}\sigma_{ab}^{\mathrm{VV}} + \mathrm{d}\sigma_{ab}^{\mathrm{C}_2}\right) J_m , \end{aligned} \quad (3)$$

corresponding to double real emission ($\mathrm{d}\sigma_{ab}^{\mathrm{RR}}$), real-virtual emission ($\mathrm{d}\sigma_{ab}^{\mathrm{RV}}$), and double virtual emission ($\mathrm{d}\sigma_{ab}^{\mathrm{VV}}$). The collinear remnants $\mathrm{d}\sigma_{ab}^{\mathrm{C}_1}$ and $\mathrm{d}\sigma_{ab}^{\mathrm{C}_2}$ arise due to the need to renormalize the PDFs, while J_n is the value of some collinear and infrared-safe observable on an n-parton final state. As is well-known, the three contributions above are separately IR divergent, even though their sum is finite.

One approach to dealing with the IR singularities in Eq. (3) involves first regularizing all expressions using dimensional regularization in $d = 4 - 2\epsilon$ dimensions, and then reshuffling divergences between individual terms by subtracting and adding back suitably chosen approximate cross sections. The CoLoRFulNNLO method [4] represents a particular practical realization of this idea. In this setup, the full NNLO cross section is written as

$$
\begin{aligned}
\sigma_{ab}^{\mathrm{NNLO}} = & \int\limits_{m+2} \left[\mathrm{d}\sigma_{ab}^{\mathrm{RR}} J_{m+2} - \mathrm{d}\sigma_{ab}^{\mathrm{RR,A_1}} J_{m+1} - \mathrm{d}\sigma_{ab}^{\mathrm{RR,A_2}} J_m + \mathrm{d}\sigma_{ab}^{\mathrm{RR,A_{12}}} J_m \right] \\
& + \int\limits_{m+1} \Bigg\{ \left[\mathrm{d}\sigma_{ab}^{\mathrm{RV}} + \mathrm{d}\sigma_{ab}^{\mathrm{C_1}} + \int\limits_1 \mathrm{d}\sigma_{ab}^{\mathrm{RR,A_1}} \right] J_{m+1} \\
& - \left[\mathrm{d}\sigma_{ab}^{\mathrm{RV,A_1}} + \mathrm{d}\sigma_{ab}^{\mathrm{C_1,A_1}} + \left(\int\limits_1 \mathrm{d}\sigma_{ab}^{\mathrm{RR,A_1}} \right)^{\mathrm{A_1}} \right] J_m \Bigg\} \\
& + \int\limits_m \Bigg\{ \mathrm{d}\sigma_{ab}^{\mathrm{VV}} + \mathrm{d}\sigma_{ab}^{\mathrm{C_2}} + \int\limits_2 \left[\mathrm{d}\sigma_{ab}^{\mathrm{RR,A_2}} - \mathrm{d}\sigma_{ab}^{\mathrm{RR,A_{12}}} \right] \\
& + \int\limits_1 \left[\mathrm{d}\sigma_{ab}^{\mathrm{RV,A_1}} + \mathrm{d}\sigma_{ab}^{\mathrm{C_1,A_1}} \right] + \int\limits_1 \left(\int\limits_1 \mathrm{d}\sigma_{ab}^{\mathrm{RR,A_1}} \right)^{\mathrm{A_1}} \Bigg\} J_m \,.
\end{aligned} \tag{4}
$$

In Eq. (4), the various approximate cross sections play the following roles:

(i) $\mathrm{d}\sigma_{ab}^{\mathrm{RR,A_1}}$ approximates $\mathrm{d}\sigma_{ab}^{\mathrm{RR}}$ in all single unresolved limits;

(ii) $\mathrm{d}\sigma_{ab}^{\mathrm{RR,A_2}}$ approximates $\mathrm{d}\sigma_{ab}^{\mathrm{RR}}$ in all double unresolved limits;

(iii) $\mathrm{d}\sigma_{ab}^{\mathrm{RR,A_{12}}}$ approximates $\mathrm{d}\sigma_{ab}^{\mathrm{RR,A_2}}$ in all single unresolved limits *and* $\mathrm{d}\sigma_{ab}^{\mathrm{RR,A_1}}$ in all double unresolved limits;

(iv) $\mathrm{d}\sigma_{ab}^{\mathrm{RV,A_1}}$ approximates $\mathrm{d}\sigma_{ab}^{\mathrm{RV}}$ in all single unresolved limits;

(v) $\mathrm{d}\sigma_{ab}^{\mathrm{C_1,A_1}}$ approximates $\mathrm{d}\sigma_{ab}^{\mathrm{C_1}}$ in all single unresolved limits, and

(vi) $(\int_1 \mathrm{d}\sigma_{ab}^{\mathrm{RR,A_1}})^{\mathrm{A_1}}$ approximates $\int_1 \mathrm{d}\sigma_{ab}^{\mathrm{RR,A_1}}$ in all single unresolved limits.

The subtraction terms appearing in Eq. (4) are built starting from IR factorization formulae, which capture the universal (*i.e.*, process and observable independent) behavior of QCD squared matrix elements in the different IR limits. Although the relevant expressions are well-known at NNLO accuracy [11–23], the limit formulae cannot directly be used as subtraction terms for the following reasons. First, the IR singular regions in phase space overlap, hence care must be taken to avoid multiple subtraction in overlapping regions. Second, the limit formulae are only well-defined in the strict IR limit they describe, thus their definitions must be carefully extended over the full phase space, away from the limits. Finally, the subtraction terms must be integrated over the momenta of unresolved emissions such that they can be combined with the virtual contributions. This leads to many complicated phase-space integrals to be evaluated.

The issue of overlapping singularities can be addressed simply by the inclusion–exclusion principle, where after subtracting each limit, we add back the pairwise overlap of limits, then subtract again the triple overlaps, and so on. The necessary overlapping limit formulae can be computed systematically. Limit formulae are then extended over the full phase space by defining suitable momentum mappings and specifying all quantities, such as momentum fractions and transverse momenta in collinear splitting, precisely in terms of the original kinematics of the event. In this way, we obtain true subtraction terms that are fully local in phase space and account for all spin and color correlations, hence the cancellation of singularities in any IR limit can be tested point-by-point in phase space. The full details of our construction can be found in [24].

Then, in order to finish the definition of the scheme, we must integrate the subtraction terms over the momenta of unresolved emissions. The integrated subtraction terms turn out to be not simple functions, but rather distributions in the momentum fractions of incoming partonic momenta. To compute the required phase-space integrals, we deployed a host of methods, choosing the ones most suited to the particular integral at hand. *E.g.*, when computing $\int_2 \mathrm{d}\sigma_{ab}^{\mathrm{RR,A}_2}$, we first exploit reverse unitarity [25] to perform an integration-by-parts reduction [26, 27], then use the method of differential equations [28–30] to obtain the results [31]. However, the evaluation of $\int_2 \mathrm{d}\sigma_{ab}^{\mathrm{RR,A}_{12}}$ is more straightforward starting from an explicit parametric representation [32, 33], which is then directly integrated in terms of generalized polylogarithms (GPLs) [34]. Finally, certain elements of the computation are easiest to perform using Mellin–Barnes (MB) methods [35].

In order to highlight the importance of choosing the correct method for each contribution, consider the following integral which appears when computing the integrated versions of certain double unresolved subtraction terms

$$\begin{aligned} I(\xi_a) \;=\; & 2^{-1-3\epsilon}\xi_a^{-1-3\epsilon}(1-\xi_a)^{-1-3\epsilon}(1+\xi_a)^{\epsilon}\int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \\ & \times x_1^{-\epsilon}\left(1-x_1\right)^{-1-3\epsilon} x_2^{-1-2\epsilon}\left(1-x_2\right)^{-1+\epsilon}\left(2\xi_a+x_1-x_1\xi_a\right)^{\epsilon} \\ & \times \left(2x_1x_2\xi_a^2-2x_2\xi_a^2-x_1^2\xi_a+2x_1\xi_a+x_1^2x_2\xi_a-4x_1x_2\xi_a\right. \\ & \left.+2x_2\xi_a+x_1^2-x_1^2x_2\right)^{\epsilon}. \end{aligned} \tag{5}$$

This integral is singular in ϵ and has an overlapping singularity in the integration variables (*i.e.*, the last factor vanishes when both $x_1 = 0$ and $x_2 = 0$, but not when only one of them is zero). Moreover, the last factor is quadratic in x_1 and resolving the ϵ-poles by sector decomposition [36] will make this factor quadratic also in x_2. This makes the direct integration of the resulting representation very difficult if not outright impossible. Nevertheless, $I(\xi_a)$ can be evaluated very easily using MB methods. In particular, it turns out that it is possible to write the integral in Eq. (5) as an only 2-dimensional MB integral

$$\begin{aligned} I(\xi_a) &= (1+\xi_a)^\epsilon \int_{-i\infty}^{+i\infty} \frac{\mathrm{d}z_1}{2\pi i} \int_{-i\infty}^{+i\infty} \frac{\mathrm{d}z_2}{2\pi i}\, 2^{-1-2\epsilon-z_1+z_2} \xi_a^{-1-2\epsilon-z_1+z_2} \\ &\quad \times (1-\xi_a)^{-1-\epsilon-z_2} \Gamma(-z_1)\,\Gamma(1+2z_1-z_2)\,\Gamma(-z_2) \\ &\quad \times \frac{\Gamma(-2\epsilon-z_1)\,\Gamma(-\epsilon+z_1)\,\Gamma(-\epsilon-z_1+z_2)\,\Gamma(\epsilon+z_1)}{\Gamma(-\epsilon)\Gamma(1-2\epsilon+z_1-z_2)}\,. \end{aligned} \tag{6}$$

To be able to find straight-line integration contours for this MB integral, we must perform an auxiliary regularization, *e.g.*, by introducing a factor of $(1-x_2)^\delta$ into the original integral. After this, Eq. (6) may be treated by standard MB tools and methods. In fact, after analytically continuing to $\epsilon \to 0$, resolving the ϵ-poles and analytically continuing to $\delta \to 0$, we find a 0-dimensional MB integral, *i.e.*, there are no actual integrations left to perform at all. Hence, we immediately obtain the result

$$I(\xi_a) = 2^{-1-\epsilon}\xi_a^{-1-\epsilon}(1-\xi_a)^{-1-\epsilon}(1+\xi_a)^\epsilon \frac{\Gamma(1-2\epsilon)\Gamma(-2\epsilon)\Gamma(\epsilon)}{\Gamma(1-3\epsilon)}\,. \tag{7}$$

To finish, we note that, with the appropriate combination of methods, we were able to obtain fully analytic expressions for all integrated subtraction terms. The final results, after simplification, can be expressed in terms of classical polylogarithms with algebraic arguments. By combining all integrated approximate cross sections, we were able to establish the cancellation of all double virtual poles in Eq. (4) analytically [10].

3. The NNLOCAL code

The CoLoRFulNNLO method, described above, has been implemented in the proof-of-concept parton-level Monte Carlo code NNLOCAL [10]. Our code provides the first public implementation of a fully local and analytic subtraction scheme at NNLO accuracy, and includes facilities that allow the user to check the cancellation of kinematic singularities and virtual poles explicitly (the former through dedicated phase-space generation routines). It can be used to compute any infrared and collinear-safe observable via a user-defined analysis routine. Support for parallel running, managed via scripts, is provided, as are tools for the efficient building and monitoring of Monte Carlo integration grids. The current public version of the code deals with Higgs boson production in hadron collisions in the Higgs effective field theory (HEFT) with no light quarks. In this configuration, it has been validated by performing tuned comparisons of the inclusive cross section to the n3loxs code [37], modified to exclude the quark channels.

By way of illustration, we present the rapidity distribution of a Higgs boson of mass $m_H = 125$ GeV at the 13 TeV LHC (in HEFT with $n_f = 0$) in Fig. 1. The bottom panels show the estimated Monte Carlo integration uncertainty. We observe a very good numerical convergence and stability of the prediction throughout the rapidity range for both the total distribution as well as the NNLO contribution. The relative uncertainties of the total distribution and NNLO contribution are seen to be generally around $\sim 0.5\%$ and $\sim 1\%$ over the central rapidity range of $|y_H| < 2.5$. The shown results were obtained on a MacBook Pro laptop with an M2 processor with 8 CPU cores with a runtime of around 1 hour and 15 minutes.

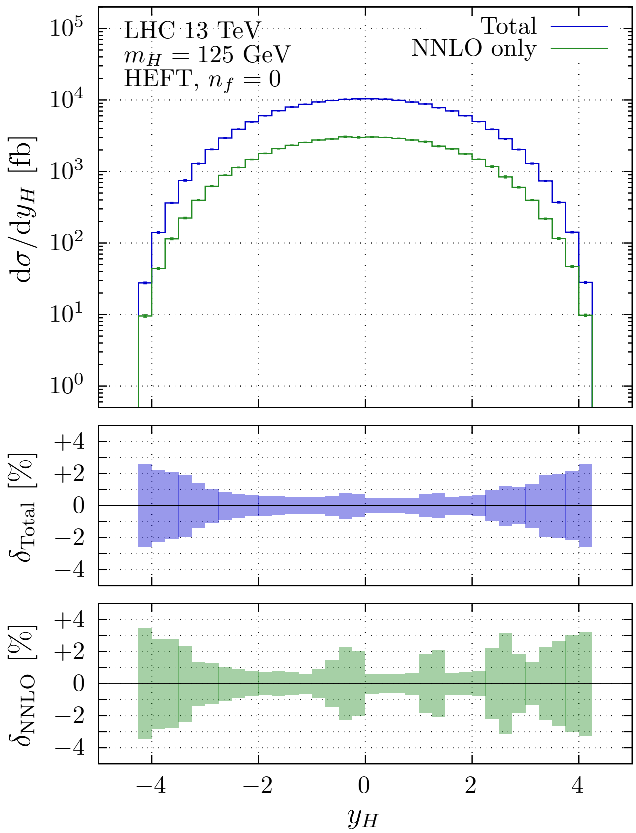

Fig. 1. The rapidity distribution of the Higgs boson at NNLO in HEFT without light quarks at the 13 TeV LHC. Figure taken from Ref. [10].

Currently, development of the code is underway to include all partonic channels for Higgs boson production as well as other processes, such as vector boson production in hadron collisions. In fact, the full set of double real emission partonic subprocesses, along with the required subtractions, have been implemented for $pp \to H$ production. This allows us to compute the regularized double real contribution to the total Higgs production cross section. In Table 1, we present the breakdown of this contribution in terms of the various partonic subprocesses. Although non-physical, the proper convergence and numerical stability of these results is a crucial test of the complete setup and implementation.

Table 1. The double real contributions to the total Higgs boson production cross section at the 13 TeV LHC for the various partonic subprocesses.

Subprocess	$\sigma^{\mathrm{RR,reg}}$ [fb]	%
gg	180.7 ± 3.6	40.06
gq	166.7 ± 3.4	36.96
$g\bar{q}$	55.87 ± 0.48	12.39
qq	27.49 ± 0.01	6.09
$q\bar{q}$	18.17 ± 0.01	4.03
$\bar{q}\bar{q}$	2.121 ± 0.001	0.47
$\sum$	451.1 ± 7.5	100

4. Conclusions

In this contribution, we have reported on the extension of the CoLoRFulNNLO subtraction method to the production of color-singlet final states in hadronic collisions. IR singularities are regularized by subtraction terms built by carefully extending the known QCD IR limit formulae over the full real emission phase space. By applying various integration methods, it is possible to obtain fully analytic results for the integrated subtraction terms and check the cancellation of virtual poles explicitly. In this regard, we have stressed the importance of the choice of integration method in terms of computational efficiency. Finally, we reported on NNLOCAL, the first public implementation of a completely local analytic subtraction scheme at NNLO accuracy. We have validated our code by computing the cross section of Higgs boson production at the LHC in HEFT with no light quarks, and developments are underway that will turn it into a useful tool for computing NNLO QCD corrections to a plethora of important LHC processes.

This work has been supported by grant K143451 of the National Research, Development and Innovation Fund in Hungary and the Bolyai Fellowship program of the Hungarian Academy of Sciences. The work of C.D. was funded by the European Union (ERC Consolidator Grant LoCoMotive 101043686). Views and opinions expressed are, however, those of the author(s) only and do not necessarily reflect those of the European Union or the European Research Council. Neither the European Union nor the granting authority can be held responsible for them. The work of L.F. was supported by the German Academic Exchange Service (DAAD) through its Bi-Nationally Supervised Scholarship program. The work of F.G. is supported in part by the Swiss National Science Foundation (SNSF) under contract 200020_219367. The work of G.S. is supported in part by the National Science Center (NCN), Poland under grant 2023/50/A/ST2/00224.

REFERENCES

[1] A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover, *J. High Energy Phys.* **2005**, 056 (2005).

[2] M. Czakon, D. Heymes, *Nucl. Phys. B* **890**, 152 (2015).

[3] M. Cacciari *et al.*, *Phys. Rev. Lett.* **115**, 082002 (2015); *Erratum ibid.* **120**, 139901 (2018).

[4] V. Del Duca *et al.*, *Phys. Rev. D* **94**, 074019 (2016).

[5] F. Caola, K. Melnikov, R. Röntsch, *Eur. Phys. J. C* **77**, 248 (2017).

[6] L. Magnea *et al.*, *J. High Energy Phys.* **2018**, 107 (2018); *Erratum ibid.* **2019**, 013 (2019).

[7] F. Herzog, *J. High Energy Phys.* **2018**, 006 (2018).

[8] S. Catani, M. Grazzini, *Phys. Rev. Lett.* **98**, 222002 (2007).

[9] J. Gaunt, M. Stahlhofen, F.J. Tackmann, J.R. Walsh, *J. High Energy Phys.* **2015**, 058 (2015).

[10] V. Del Duca *et al.*, *J. High Energy Phys.* **2025**, 151 (2025).

[11] Z. Bern, L.J. Dixon, D.C. Dunbar, D.A. Kosower, *Nucl. Phys. B* **425**, 217 (1994).

[12] Z. Bern, V. Del Duca, C.R. Schmidt, *Phys. Lett. B* **445**, 168 (1998).

[13] Z. Bern, V. Del Duca, W.B. Kilgore, C.R. Schmidt, *Phys. Rev. D* **60**, 116001 (1999).

[14] J.M. Campbell, E.W.N. Glover, *Nucl. Phys. B* **527**, 264 (1998).

[15] S. Catani, M. Grazzini, *Phys. Lett. B* **446**, 143 (1999).

[16] S. Catani, M. Grazzini, *Nucl. Phys. B* **570**, 287 (2000).

[17] S. Catani, M. Grazzini, *Nucl. Phys. B* **591**, 435 (2000).

[18] V. Del Duca, A. Frizzo, F. Maltoni, *Nucl. Phys. B* **568**, 211 (2000).

[19] D.A. Kosower, P. Uwer, *Nucl. Phys. B* **563**, 477 (1999).

[20] D.A. Kosower, *Phys. Rev. D* **67**, 116003 (2003).

[21] M. Czakon, *Nucl. Phys. B* **849**, 250 (2011).

[22] O. Braun-White, N. Glover, *J. High Energy Phys.* **2022**, 059 (2022).

[23] T. Gehrmann, M. Löchner, arXiv:2511.09691 [hep-ph].

[24] V. Del Duca, G. Somogyi, F. Tramontano, arXiv:2512.05192 [hep-ph].

[25] C. Anastasiou, K. Melnikov, *Nucl. Phys. B* **646**, 220 (2002).

[26] F.V. Tkachov, *Phys. Lett. B* **100**, 65 (1981).

[27] K.G. Chetyrkin, F.V. Tkachov, *Nucl. Phys. B* **192**, 159 (1981).

[28] A.V. Kotikov, *Phys. Lett. B* **259**, 314 (1991).

[29] E. Remiddi, *Nuovo Cim. A* **110**, 1435 (1997).

[30] T. Gehrmann, E. Remiddi, *Nucl. Phys. B* **580**, 485 (2000).

[31] V. Del Duca *et al.*, *PoS* **(EPS-HEP2025)**, 243 (2026), arXiv:2601.02033 [hep-ph].

[32] S. Van Thurenhout *et al.*, *Acta Physica Pol. B Proc. Suppl.* **18**, 6-A31 (2025).

[33] L. Fekésházy, G. Somogyi, S. Van Thurenhout, arXiv:2512.24403 [hep-ph].

[34] A.B. Goncharov, arXiv:math/0103059.

[35] I. Dubovyk, J. Gluza, G. Somogyi, «Mellin-Barnes Integrals: A Primer on Particle Physics Applications», *Springer*, 2022.

[36] G. Heinrich, *Int. J. Mod. Phys. A* **23**, 1457 (2008).

[37] J. Baglio, C. Duhr, B. Mistlberger, R. Szafron, *J. High Energy Phys.* **2022**, 066 (2022).