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AUTOMATED COMPUTATION OF MULTIPLE MELLIN–BARNES INTEGRALS HAVING POLYGAMMA FUNCTIONS IN THEIR INTEGRAND*

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The conic hulls/triangulation method is extended to the analytic computation of multiple Mellin–Barnes having polygamma functions of arbitrary order in their integrand. This computational approach is automatized in the `MBConicHulls.wl` Mathematica package, which we give a brief description of the new corresponding functionalities, basing our discussion on the example of a simple two-loop sunset integral.

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1. Introduction

This contribution presents some of the results of the recent work [1] where Mellin–Barnes (MB) integrals having polygamma functions in their integrand are studied. As the talk was given at the MTTD 2025 conference a few months before completion of [1], we will focus only on the part of the work which was presented at the conference. In particular, since we did not yet consider the straight contour case in full generality at that time, it will not be mentioned here and we refer the reader to [1] for this case and for more results. We only mention that the examples shown in the present paper do not appear in [1].

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In QFT, the Mellin–Barnes (MB) representation method [2, 3] has often been used in the past as a powerful technique to compute Feynman integrals (see [4–7], *etc.*). Although having today only the runner-up position in the computational tool-kit of particle physicists [8], it is still largely used (see, for example, even if only the talks [9, 10] at MTTD 2025 and [1] for more references), and new approaches to improve this method go on to be developed, as evidenced in recent years by the works [11–16].

In this context, the recent Mathematica package `MBConicHulls.wl` [17], allowing for the automated analytic computation of multiple MB integrals, has been developed by the present authors and other collaborators in [11] after which it has been improved to include new functionalities in [12] and a better computational efficiency in [13]. In the present work, we go one step further by adding the possibility to use this package for the automatic computation of multiple MB integrals whose integrand, usually composed of a ratio of products of gamma functions, involves also products of polygamma functions of arbitrary order appearing in the ratio's numerator. This new functionality can be used, for instance, in order to compute series representations of MB representations of Feynman integrals which have previously been expanded in powers of ϵ, the dimensional regularization parameter, at the integral level.

The content of this paper reads as follows. In a first short section, we recall basic facts about multiple MB integrals, and we also present their extended expression when including polygamma functions of arbitrary order in their integrand. A second section presents the computational strategy, while a third section gives a detailed example of application using the `MBConicHulls.wl` package. Then follow our conclusions.

2. MB integrals with polygamma functions

Multifold MB integrals have the following form:

$$I(x_1,\cdots,x_N) \equiv \int\limits_{-i\infty}^{+i\infty} \frac{\mathrm{d}z_1}{2\pi i} \cdots \int\limits_{-i\infty}^{+i\infty} \frac{\mathrm{d}z_N}{2\pi i} \frac{\prod\limits_{i=1}^{k} \Gamma^{a_i}(s_i(\boldsymbol{z}))}{\prod\limits_{j=1}^{l} \Gamma^{b_j}(t_j(\boldsymbol{z}))} x_1^{z_1}\cdots x_N^{z_N}\,, \tag{1}$$

where $\boldsymbol{z}=(z_1,\cdots,z_N)$, a_i and b_j are positive integers, $k \geq N$ (cancellations between gamma functions in the integrand are tacitely excluded), and the variables $x_1,\cdots,x_N$ can be real or complex-valued. The s_i and t_j argument functions of the gamma functions in the MB integrand are

$$s_i(\boldsymbol{z}) = \sum_{k=1}^{N} e_{ik} z_k + f_i \,, \qquad t_j(\boldsymbol{z}) = \sum_{k=1}^{N} g_{jk} z_k + h_j \,, \tag{2}$$

where the f_i and h_j constants are real or complex numbers, and the coefficients e_{ik} and g_{jk} are in general integers.

If not explicitly defined, the contours of integration in Eq. (1) are such that they do not split the set of poles of each of the gamma functions of the numerator into different subsets. We also note that it is sometimes convenient to rewrite Eq. (1) in the canonical form [12]

$$I(x_1, \cdots, x_N) = \int\limits_{-i\infty}^{+i\infty} \frac{\mathrm{d}z_1}{2\pi i} \cdots \int\limits_{-i\infty}^{+i\infty} \frac{\mathrm{d}z_N}{2\pi i}$$

$$\times \frac{\mathit{\Gamma}(-z_1) \cdots \mathit{\Gamma}(-z_N) \prod\limits_{i=N+1}^{k'} \mathit{\Gamma}^{a'_i}(s'_i(\boldsymbol{z}))}{\prod\limits_{j=1}^{l} \mathit{\Gamma}^{b'_j}(t'_j(\boldsymbol{z}))} x_1'^{z_1} \cdots x_N'^{z_N} \,, \tag{3}$$

where

$$s'_i(\boldsymbol{z}) = \sum_{k=1}^{N} e'_{ik} z_k + f'_i \,, \qquad t'_j(\boldsymbol{z}) = \sum_{k=1}^{N} g'_{jk} z_k + h'_j \,. \tag{4}$$

In the rest of this paper, the integrals that we will be interested in belong to the more general class where polygamma functions $\psi(m, z)$ also appear in the numerator of the integrand

$$J(x_1, \cdots, x_N) = \int\limits_{-i\infty}^{+i\infty} \frac{\mathrm{d}z_1}{2\pi i} \cdots \int\limits_{-i\infty}^{+i\infty} \frac{\mathrm{d}z_N}{2\pi i}$$

$$\times \frac{\mathit{\Gamma}(-z_1) \cdots \mathit{\Gamma}(-z_N) \prod\limits_{i=1}^{m} \mathit{\Gamma}^{a_i}(s_i(\boldsymbol{z})) \prod\limits_{p=1}^{n} \psi^{c_p}(m_p, u_p(\boldsymbol{z}))}{\prod\limits_{j=1}^{l} \mathit{\Gamma}^{b_j}(t_j(\boldsymbol{z}))} x_1^{z_1} \cdots x_N^{z_N} \,, \tag{5}$$

where $u_p(\boldsymbol{z}) = \sum_{k=1}^{N} q_{pk} z_k + r_p$ and, as in the case of $s_i(\boldsymbol{z})$ and $t_j(\boldsymbol{z})$, r_p are real or complex numbers, while the coefficients q_{pk} are in general integers.

3. Computational strategy

As the gamma function $\mathit{\Gamma}(z)$, the polygamma functions $\psi(m, z)$ have poles at all nonpositive integers. However, their multiplicity depends on the order of the function under consideration.

Moreover, in a similar way, the relation

$$\Gamma(z) = \frac{\Gamma(z+1)}{z} \tag{6}$$

can be applied iteratively to obtain the well-known reflection formula which can be slightly modified to explicitly extract singularities when $z \to 0$ as follows:

$$\Gamma(z-n) = \frac{\Gamma(z+1)\Gamma(1-z)(-1)^n}{z\,\Gamma(n+1-z)}\,. \tag{7}$$

One can get, from the relation

$$\psi(m,z) = \psi(m,z+1) + \frac{(-1)^{m+1}m!}{z^{m+1}} \tag{8}$$

or by the logarithmic derivative of Eq. (7), the following result:

$$\begin{aligned}\psi(m,z-n) \;=\; & \frac{(-1)^{m+1}m!}{z^{m+1}} + \psi(m,z+1) + (-1)^{m+1}\psi(m,1-z) \\ & + (-1)^m\psi(m,1+n-z)\end{aligned} \tag{9}$$

which allows us to extract singularities in a similar way as Eq. (7) when polygamma functions appear in the integrand. Note that, contrary to Eq. (7), there will not be only one term to consider but four when using this expression.

To fix ideas, let us consider a simple example by computing the one-fold MB integral

$$I_0 = \int_{-i\infty}^{+i\infty} \frac{\mathrm{d}s}{2\pi i} x^s \Gamma(-s)\Gamma(1+s)\psi(0,-s)\,, \tag{10}$$

where, as usual, it is understood that the contour of integration separates the left-handed poles (those of $\Gamma(1+s)$) from the right-handed ones (those of $\Gamma(-s)$ and $\psi(0,-s)$).

If one closes the contour of integration to the right, one can sum the residues over the double poles located at all non-negative integers n in order to compute I_0.

For this, it is convenient to rewrite the residues

$$\mathrm{Res}_n = \mathrm{Lim}_{s\to n}\frac{\mathrm{d}}{\mathrm{d}s}\left[(s-n)^2 x^s \Gamma(-s)\Gamma(1+s)\psi(0,-s)\right] \tag{11}$$

as

$$\begin{aligned}\mathrm{Res}_n = \mathrm{Lim}_{s\to n}\frac{\mathrm{d}}{\mathrm{d}s}\Bigg[&(s-n)^2 x^s \frac{\Gamma(1-s+n)\Gamma(1-n+s)(-1)^n}{(n-s)} \\ &\times\left(\frac{1}{n-s} + \psi(0,1-s+n) - \psi(0,1+s-n) + \psi(0,1+s)\right)\Bigg]\end{aligned} \tag{12}$$

using Eqs. (7) and (9).

Simplifying the brackets and performing the derivative and the limit, one then obtains

$$\begin{aligned} I_0 &= -\sum_{n=0}^{\infty} \mathrm{Res}_n = -\sum_{n=0}^{\infty} (-x)^{n+1} [\log x - \psi(0, 1+n)] \\ &= \frac{\log x - \gamma_{\mathrm{E}} - \log(1+x)}{1+x} \,. \end{aligned} \tag{13}$$

We note here that the same final expression can be obtained by closing the contour of integration to the left and by summing the residues over the simple poles of $\Gamma(1+s)$. In fact, I_0 has two different series representations which converge for different ranges of x (both yielding the right-hand side of Eq. (13) once resummed). This means that I_0 belongs to the degenerate class: polygamma functions can be ignored when calculating the Δ vector associated with MB integrals (see [18, 19] for more details about this vector which, in the case at hand, reduces to a number). This is due to the fact that one can also write I_0 as

$$I_0 = \left(\frac{\mathrm{d}}{\mathrm{d}a} \int_{-i\infty}^{i\infty} \frac{\mathrm{d}s}{2\pi i} \, x^s \, \Gamma(a-s)\Gamma(1+s) \right) \Bigg|_{a \to 0} \,, \tag{14}$$

where the polygamma function has disappeared and where clearly $\Delta = 0$, reflecting the fact that the MB integral indeed belongs to the degenerate class.

It is easy to check that the computation of Eq. (14), where the integral is calculated before taking the derivative with respect to the auxiliary parameter a, yields the same result as in Eq. (13). This alternative approach is generalized in [1] to solve the straight contour case, whereas the first strategy detailed above is used for MB integrals with non-straight contours, both approaches have been automatized in the `MBConicHulls.wl` Mathematica package [17] whose use will be explained in the next section.

Within the conic hulls or triangulation approaches, in the general multifold case, one can treat polygamma functions in the same way as gamma functions in order to identify the relevant sets of poles associated with the various series representations of multiple MB integrals: one defines vectors or points configurations from their argument.

4. An example of application

In order to illustrate the use of the new functionality of our package, we have chosen a simple example taken from [20] where a sunset Feynman integral denoted as **J011**$(1, 2, 2)$ has been considered. We will compute its ϵ expansion at third order from its dimensionally regularized one-fold MB representation.

Sunsets (or sunrises) are two-loop integrals of the type

$$H_{\sigma,\nu_1,\nu_2}\left(m_1,m_2,m_3;p^2\right)$$
$$\equiv\int\frac{\mathrm{d}^dk_1\mathrm{d}^dk_2}{\left[(k_1-p)^2-m_1^2+i0\right]^{\nu_1}\left[k_2^2-m_2^2+i0\right]^{\sigma}\left[(k_1-k_2)^2-m_3^2+i0\right]^{\nu_2}}\,, \quad (15)$$

where $d=4-2\epsilon$.

In our case of interest,

$$\mathbf{J011}(1,2,2)=\left(i\pi^{2-\epsilon}\Gamma\left(1+\epsilon\right)\right)^{-2}H_{1,2,2}(m,0,m;m^2)|_{m\to1}.$$

As shown in [20], after some manipulations, the analytic expression of this particular sunset can be reduced to the following very simple form:

$$\mathbf{J011}(1,2,2)=\frac{1}{3\epsilon^2}\left(\frac{\Gamma(1+2\epsilon)\Gamma(1-\epsilon)}{\Gamma(1+\epsilon)}-1\right) \quad (16)$$

whose ϵ expansion reads

$$\mathbf{J011}(1,2,2)=\frac{\pi^2}{9}+\frac{1}{3}\psi(2,1)\epsilon+\frac{\pi^4}{30}\epsilon^2+\mathcal{O}\left(\epsilon^3\right) \quad (17)$$

which can also be written as [20]

$$\mathbf{J011}(1,2,2)=\frac{2}{3}\zeta_2-\frac{2}{3}\zeta_3\epsilon+3\zeta_4\epsilon^2+\mathcal{O}\left(\epsilon^3\right)\,. \quad (18)$$

As one can see, the ϵ expansion of **J011**(1,2,2) is very simple and does not involve $1/\epsilon$ terms. This will simplify the analysis that we wish to follow now in order to check that the first three terms of this expansion can be obtained from the ϵ expanded MB representation of **J011**(1,2,2), using results given by the new functionality of our `MBConicHulls.wl` package.

From `AMBRE` [21, 22], one can obtain for **J011**(1,2,2) the following MB representation:

$$\begin{aligned}\mathbf{J011}(1,2,2) &= -\frac{\sqrt{\pi}}{2^{1+2\epsilon}}\frac{\Gamma(1-\epsilon)}{\Gamma(1+\epsilon)^2}\int\limits_{-i\infty}^{+i\infty}\frac{\mathrm{d}z}{2\pi i}\left(-\frac{1}{4}\right)^z\\ &\quad\times\frac{\Gamma(-z)\Gamma(1+z)\Gamma(1+\epsilon+z)\Gamma(1+2\epsilon+z)}{\Gamma\left(\frac{3}{2}+\epsilon+z\right)\Gamma(2-\epsilon+z)}\,, \quad (19)\end{aligned}$$

where in passing, we recognize the MB representation of a ${}_3F_2$ generalized hypergeometric function of argument $\frac{1}{4}$.

Performing the ϵ expansion at the integral level, we get, for the first three terms

$$\begin{aligned}
&\mathbf{J011}(1,2,2) = -\frac{\sqrt{\pi}}{2}\int\limits_{-i\infty}^{+i\infty}\frac{\mathrm{d}z}{2\pi i}\left(-\frac{1}{4}\right)^{z}\frac{\Gamma(-z)\Gamma(1+z)^{3}}{\Gamma\left(\frac{3}{2}+z\right)\Gamma(2+z)}\\
&\times\Bigg[1+\epsilon\Bigg(-2+4\gamma_{\mathrm{E}}+\psi\left(0,\frac{3}{2}\right)+3\psi(0,1+z)-\psi\left(0,\frac{3}{2}+z\right)\\
&+\psi(0,2+z)\Bigg)+\epsilon^{2}\Bigg(2-8\gamma_{\mathrm{E}}+8\gamma_{\mathrm{E}}^{2}-\frac{\pi^{2}}{12}-(2-4\gamma_{\mathrm{E}})\psi\left(0,\frac{3}{2}\right)\\
&\quad+\frac{1}{2}\psi\left(0,\frac{3}{2}\right)^{2}+\left[-2+4\gamma_{\mathrm{E}}+\psi\left(0,\frac{3}{2}\right)\right]\Bigg[3\psi\left(0,1+z\right)-\psi\left(0,\frac{3}{2}+z\right)\\
&+\psi\left(0,2+z\right)\Bigg]+\frac{9}{2}\psi\left(0,1+z\right)^{2}-3\psi\left(0,1+z\right)\psi\left(0,\frac{3}{2}+z\right)\\
&+\frac{1}{2}\psi\left(0,\frac{3}{2}+z\right)^{2}+3\psi\left(0,1+z\right)\psi\left(0,2+z\right)-\psi\left(0,\frac{3}{2}+z\right)\psi\left(0,2+z\right)\\
&+\frac{1}{2}\psi\left(0,2+z\right)^{2}+\frac{5}{2}\psi\left(1,1+z\right)-\frac{1}{2}\psi\left(1,\frac{3}{2}+z\right)-\frac{1}{2}\psi\left(1,2+z\right)\Bigg)+\mathcal{O}\left(\epsilon^{3}\right)\Bigg].
\end{aligned}\tag{20}$$

Our aim is now to compute Eq. (20) with the `MBConicHulls.wl` package and to compare with the result shown in Eq. (17) (or (18)). As one can see, a non-trivial combination of polygamma functions of order zero and one appears in the integrand of the ϵ expanded MB integral, which, if once integrated, reproduces the expected result, will provide a convincing check that our implementation of polygamma functions in the package is correct.

The leading term in the expansion is easily computed using the standard procedure of the `MBConicHulls.wl` package (see, for instance, [11]) which yields

$$-\frac{\sqrt{\pi}}{2}\int\limits_{-i\infty}^{+i\infty}\frac{\mathrm{d}z}{2\pi i}\left(-\frac{1}{4}\right)^{z}\frac{\Gamma(-z)\Gamma(1+z)^{3}}{\Gamma\left(\frac{3}{2}+z\right)\Gamma(2+z)}=-\frac{\pi^{2}}{9}\tag{21}$$

in accordance with Eq. (17), up to an overall sign[1].

---

[1] We have numerically checked by computing **J011**(1,2,2) with FIESTA [23] that our sign is correct. It is probably due to a different convention choice in the Feynman integral definition of [20], the latter being unfortunately not explicitly given in this reference.

We note here in passing that a naive use of the relations

$$\mathit{\Gamma}(2+z)=(1+z)\mathit{\Gamma}(1+z) \quad \text{and} \quad \mathit{\Gamma}(-z)=-(1+z)\mathit{\Gamma}(-1-z) \tag{22}$$

cannot be done to simplify this integral as

$$\begin{aligned} &-\frac{\sqrt{\pi}}{2}\int\limits_{-i\infty}^{+i\infty}\frac{\mathrm{d}z}{2\pi i}\left(-\frac{1}{4}\right)^{z}\frac{\mathit{\Gamma}(-z)\mathit{\Gamma}(1+z)^{3}}{\mathit{\Gamma}\left(\frac{3}{2}+z\right)\mathit{\Gamma}(2+z)}\\ &=\frac{\sqrt{\pi}}{2}\int\limits_{-i\infty}^{+i\infty}\frac{\mathrm{d}z}{2\pi i}\left(-\frac{1}{4}\right)^{z}\frac{\mathit{\Gamma}(-1-z)\mathit{\Gamma}(1+z)^{2}}{\mathit{\Gamma}\left(\frac{3}{2}+z\right)} \end{aligned} \tag{23}$$

without having first explicitly specified the contour of integration, otherwise this would lead to some overlap of left and right-handed poles which would pinch the contour (see [2, p. 85] for a discussion about this point in another example). We recall that by default the `MBConicHulls.wl` package supposes that the contours are separating the left- and the right-handed poles. In order to specify the contours with the package, one can follow the procedure described in [12].

Let us now focus on the subleading term in the ϵ expansion. Looking at Eq. (20), one sees that it is made of one trivial part which is just equal to

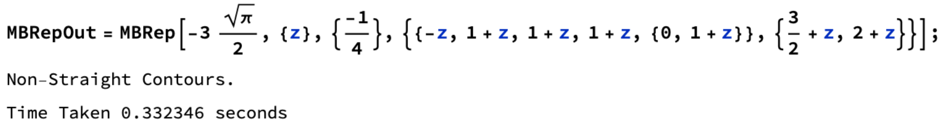
$$\epsilon\left(-2+4\gamma_{\mathrm{E}}+\psi\left(0,\frac{3}{2}\right)\right)\left(-\frac{\pi^{2}}{9}\right) \tag{24}$$

and a non-trivial one involving three polygamma functions in the MB integrand. The sum of the latter cannot be simplified straightforwardly, therefore, we have to compute all three of them separately. Let us see how to do that on the example of the first one, namely

$$-3\frac{\sqrt{\pi}}{2}\int\limits_{-i\infty}^{+i\infty}\frac{\mathrm{d}z}{2\pi i}\left(-\frac{1}{4}\right)^{z}\frac{\mathit{\Gamma}(-z)\mathit{\Gamma}(1+z)^{3}}{\mathit{\Gamma}\left(\frac{3}{2}+z\right)\mathit{\Gamma}(2+z)}\psi(0,1+z) \tag{25}$$

which we detail now, the computation with the `MBConicHulls.wl` package.

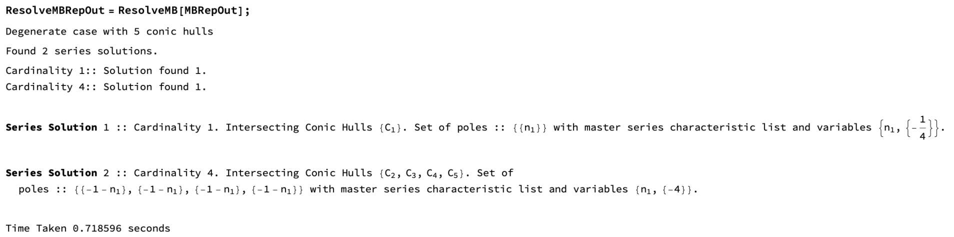
One begins by defining the MB representation as follows (see [11] for additional details):



```
MBRepOut = MBRep[-3 √π/2, {z}, {-1/4}, {{-z, 1 + z, 1 + z, 1 + z, {0, 1 + z}}, {3/2 + z, 2 + z}}];
Non-Straight Contours.
Time Taken 0.332346 seconds
```

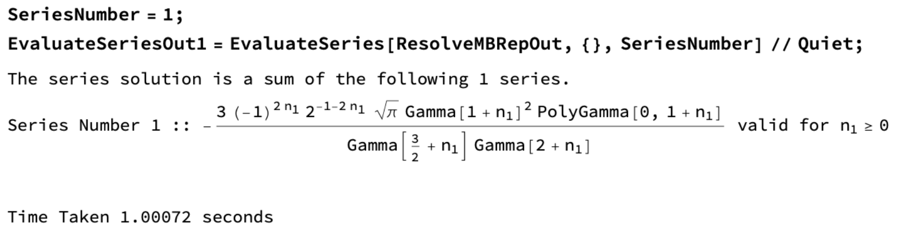
One can see on the Mathematica screenshot above that what differentiates gamma functions from polygamma functions in the syntax of `MBConicHulls.wl` is that the latter include some additional brackets, in order to input not only their argument but also their order (in the present case, the order is zero).

Solving this integral is next performed as usual with the `ResolveMB` command:



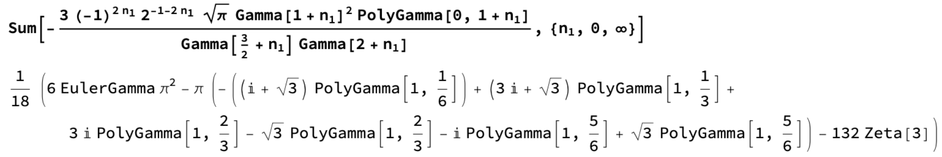
```
ResolveMBRepOut = ResolveMB[MBRepOut];
Degenerate case with 5 conic hulls
Found 2 series solutions.
Cardinality 1:: Solution found 1.
Cardinality 4:: Solution found 1.

Series Solution 1 :: Cardinality 1. Intersecting Conic Hulls {C1}. Set of poles :: {{n1}} with master series characteristic list and variables {n1, {-1/4}}.

Series Solution 2 :: Cardinality 4. Intersecting Conic Hulls {C2, C3, C4, C5}. Set of
  poles :: {{-1 - n1}, {-1 - n1}, {-1 - n1}, {-1 - n1}} with master series characteristic list and variables {n1, {-4}}.

Time Taken 0.718596 seconds
```

where one notices that two different series representations of the integral can be obtained. Focusing on the first one, we get



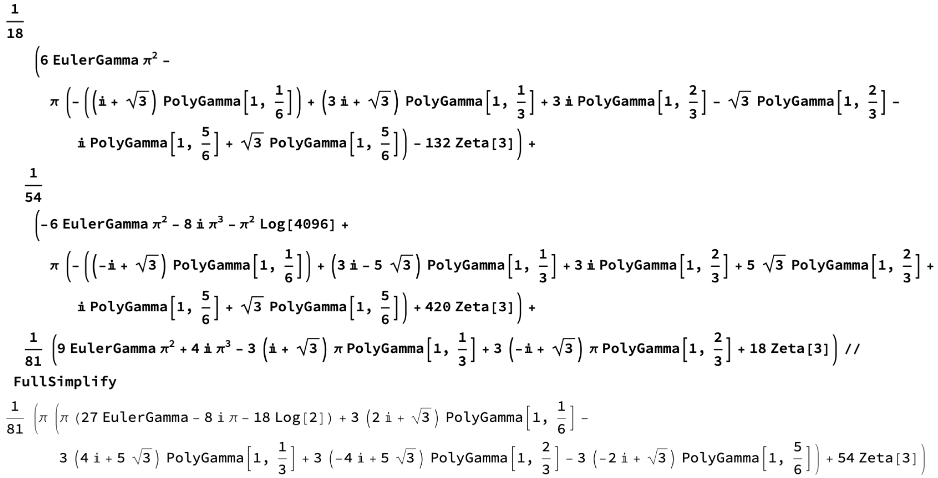
```
SeriesNumber = 1;
EvaluateSeriesOut1 = EvaluateSeries[ResolveMBRepOut, {}, SeriesNumber] // Quiet;
The series solution is a sum of the following 1 series.
Series Number 1 :: - (3 (-1)^(2 n1) 2^(-1-2 n1) √π Gamma[1 + n1]^2 PolyGamma[0, 1 + n1]) / (Gamma[3/2 + n1] Gamma[2 + n1])  valid for n1 ≥ 0

Time Taken 1.00072 seconds
```

which can be summed as



```
Sum[- (3 (-1)^(2 n1) 2^(-1-2 n1) √π Gamma[1 + n1]^2 PolyGamma[0, 1 + n1]) / (Gamma[3/2 + n1] Gamma[2 + n1]), {n1, 0, ∞}]
1/18 (6 EulerGamma π^2 - π (-((ⅈ + √3) PolyGamma[1, 1/6]) + (3 ⅈ + √3) PolyGamma[1, 1/3] +
      3 ⅈ PolyGamma[1, 2/3] - √3 PolyGamma[1, 2/3] - ⅈ PolyGamma[1, 5/6] + √3 PolyGamma[1, 5/6]) - 132 Zeta[3])
```

Doing the same kind of analysis for the two other MB integrals involving the polygamma functions $\psi\left(0, \frac{3}{2}+z\right)$ and $\psi(0, 2+z)$), and adding the three contributions, one ends up with the following result which can be a bit simplified using the in-built `FullSimplify` command of Mathematica:



```
1/18
  (6 EulerGamma π^2 -
    π (-((ⅈ + √3) PolyGamma[1, 1/6]) + (3 ⅈ + √3) PolyGamma[1, 1/3] + 3 ⅈ PolyGamma[1, 2/3] - √3 PolyGamma[1, 2/3] -
       ⅈ PolyGamma[1, 5/6] + √3 PolyGamma[1, 5/6]) - 132 Zeta[3]) +
 1/54
  (-6 EulerGamma π^2 - 8 ⅈ π^3 - π^2 Log[4096] +
    π (-((-ⅈ + √3) PolyGamma[1, 1/6]) + (3 ⅈ - 5 √3) PolyGamma[1, 1/3] + 3 ⅈ PolyGamma[1, 2/3] + 5 √3 PolyGamma[1, 2/3] +
       ⅈ PolyGamma[1, 5/6] + √3 PolyGamma[1, 5/6]) + 420 Zeta[3]) +
 1/81 (9 EulerGamma π^2 + 4 ⅈ π^3 - 3 (ⅈ + √3) π PolyGamma[1, 1/3] + 3 (-ⅈ + √3) π PolyGamma[1, 2/3] + 18 Zeta[3]) //
FullSimplify
1/81 (π (π (27 EulerGamma - 8 ⅈ π - 18 Log[2]) + 3 (2 ⅈ + √3) PolyGamma[1, 1/6] -
     3 (4 ⅈ + 5 √3) PolyGamma[1, 1/3] + 3 (-4 ⅈ + 5 √3) PolyGamma[1, 2/3] - 3 (-2 ⅈ + √3) PolyGamma[1, 5/6]) + 54 Zeta[3])
```

Interestingly enough, the expected final result $-\frac{2\zeta(3)}{3}$, given in Eq. (18), appears at the very end of the simplified expression above (up to an overall sign, see footnote 1), which means that the rest of this expression, multiplied by ϵ, must be canceled by the result given in Eq. (24). Although Mathematica does not succeed in simplifying the latter to zero analytically, a numerical calculation yields indeed a null result when adding these two contributions, and it is, in fact, also possible to prove this null result analytically by hand using

$$\psi\left(0,\frac{3}{2}\right)=-\gamma_{\mathrm{E}}-2(\log 2-1) \tag{26}$$

and

$$\psi\left(m,nz\right)=\frac{1}{n^{m+1}}\sum_{k=0}^{n-1}\psi\left(m,z+\frac{k}{n}\right)\qquad m>0,n\in\mathbb{N}\,. \tag{27}$$

We will not present the details of the calculation of the $\mathcal{O}(\epsilon^2)$ term, which are too lengthy to be given here. However, as can be seen in the `Examples_Polygamma.nb` Mathematica notebook [17], the various different contributions (some of them involving products of polygamma functions of order zero, some others having polygamma functions of order one, see Eq. (20)) add up to give the expected simple result given in Eq. (18) (still up to an overall sign, see footnote 1). We could check this only numerically because, contrary to what happened with the $\mathcal{O}(\epsilon)$ term that we have detailed above, some of the sums involved at $\mathcal{O}(\epsilon^2)$ cannot be rewritten in closed form.

We conclude from the computation of the ϵ expansion of the **J011**$(1,2,2)$ sunset that it gives a good hint that our implementation of polygamma functions in the `MBConicHulls.wl` package has been correctly performed.

This pedagogical example also shows that hidden cancellations which cannot be seen by Mathematica may appear when computing the ϵ expansion of MB representations of Feynman integrals in the traditional way, *i.e* by solving ϵ singularities and expanding MB integrals in powers of ϵ. Obviously, the higher the order in ϵ, the more the number of these intricate cancellations blows up and may be difficult to find. It would thus be interesting, in a more complicated case than the simple sunset considered here, to focus on the simplification of the exact ϵ expressions, in order to try to bypass the search of hidden cancellations in the lengthy expressions of ϵ expanded results. This, perhaps, could yield drastic simplifications of the final expressions. Such statements have already been suggested in the past [24–26] although, to our knowledge, their direct derivation from the MB method is still lacking.

5. Conclusion

The MB computational technique is a powerful tool used in high-energy physics and other domains of theoretical physics and mathematics to compute complicated integrals. To achieve this goal, the studied integrals have to be transformed into MB integral representations which can then be evaluated. In order to ease the computation of such MB integrals, the present authors and collaborators have developed, in a series of papers [11–13], a Mathematica package, called `MBConicHulls.wl` [17] which allows their automatic analytic evaluation. The implementation of polygamma functions in the `MBConicHulls` package, that we have described in the present paper, extends its computational possibilities, in particular when one is interested in the ϵ expansion of Feynman integrals. We have shown in one example how to use the new functionalities of the package. The details of the corresponding calculations have been described partly in the main core of the text and partly in a Mathematica notebook provided in [17].

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