EFFECTIVE FIELD THEORY OF COMPOSITE NUCLEONS IN HIGH-DENSITY MATTER*

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At very high baryon densities, such as in compact star cores, neighboring nucleons' quark–gluon wave functions overlap and delocalize, suggesting a percolation-like transition. Modeling this regime is difficult because nucleons gradually lose their identities and cease to be good degrees of freedom. We present a relativistic field theory that encodes nucleon internal structure via modified nucleon field operators. Instead of using complicated many-quark composite fields, the theory keeps nucleon fields whose anticommutation relations differ only slightly from the canonical fermionic form, with density-dependent deviations reflecting nucleon structure. Within a mean-field treatment, we compute the equation of state and the speed of sound for cold neutron matter.

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1. Introduction

A central challenge in nuclear physics is describing matter at densities where nucleons strongly overlap while quarks and gluons remain partially confined. In normal nuclear matter, with saturation density $\rho_0 = 0.16$ fm^{-3}, the mean nucleon separation is about 1.8 fm, so the system can be modeled using nucleon and meson degrees of freedom, such as in relativistic density functional theory (RDFT) [1].

However, in environments such as neutron star cores, where the baryon density ρ can be as high as $\rho \sim 5\ \rho_0$ or even higher, and the inter-nucleon distance d_{NN} drops to $d_{NN} \sim 1$ fm, the quark cores of neighboring nucleons begin to overlap and become delocalized, potentially leading to a percolation-like phenomenon [2]. In this regime, nucleons no longer constitute appropriate effective degrees of freedom, in the sense that they cannot be consistently represented by field operators obeying canonical anticommutation relations of the type used in RDFT.

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This regime poses significant challenges for traditional models, which often struggle to bridge the gap between canonical hadronic degrees of freedom and deconfined sub-structure. The present work addresses this by treating composite nucleons through the modification of their field algebra, reflecting the internal structure and wave-function overlap that occurs at high density.

2. Effective theory for overlapping nucleons

The breakdown of the composite-nucleon-field canonical anticommutation relations can be illustrated in a nonrelativistic framework. Consider a three-quark nucleon state $|n\rangle = \hat{b}_n^\dagger|0\rangle$, where $n = \{\boldsymbol{P}, m_\mathrm{S}, m_\mathrm{I}\}$ specifies the nucleon momentum $\boldsymbol{P}$ and its spin and isospin projections m_S and m_I. The vacuum state $|0\rangle$ is annihilated by the nucleon destruction operator, $\hat{b}_n|0\rangle = 0$. The nucleon creation operator can be written in terms of quark creation operators as (with repeated indices summed)

$$\hat{b}_n^\dagger = \frac{1}{\sqrt{3!}}\,\Psi_n^{\mu_1\mu_2\mu_3}\,\hat{q}_{\mu_1}^\dagger\hat{q}_{\mu_2}^\dagger\hat{q}_{\mu_3}^\dagger \quad \text{with} \quad \Psi_{n'}^{\mu_1\mu_2\mu_3 *}\Psi_n^{\mu_1\mu_2\mu_3} = \delta_{n'n}\,, \tag{1}$$

where μ collectively labels spatial (momentum) and internal (spin, color, flavor) quantum numbers. Using the fact that quark creation and annihilation operators satisfy the canonical anticommutation relations, one finds [3–6]

$$\left\{\hat{b}_{n'}, \hat{b}_n^\dagger\right\} = \delta_{n'n} - \hat{\Delta}_{n'n}\,, \qquad \left\{\hat{b}_{n'}, \hat{b}_n\right\} = 0\,, \tag{2}$$

$$\hat{\Delta}_{n'n} = 3\Psi_{n'}^{*\mu_1\mu_2\mu_3}\Psi_n^{\mu_1\mu_2\nu_3}\hat{q}_{\nu_3}^\dagger\hat{q}_{\mu_3} - \frac{3}{2}\Psi_{N'}^{*\mu_1\mu_2\mu_3}\Psi_N^{\mu_1\nu_2\nu_3}\hat{q}_{\nu_3}^\dagger\hat{q}_{\nu_2}^\dagger\hat{q}_{\mu_2}\hat{q}_{\mu_3}\,, \tag{3}$$

where we used $\langle n'|n\rangle = \langle 0|\hat{b}_{n'}\hat{b}_n^\dagger|0\rangle = \Psi_{n'}^{\mu\nu\sigma *}\Psi_n^{\mu\nu\sigma} = \delta_{n'n}$. The operator $\hat{\Delta}_{n'n}$ vanishes in the limit of point-like, nonoverlapping nucleons and reflects the internal structure of nucleons. In particular, it reflects the Pauli principle acting at the level of the quarks; the first term in $\hat{\Delta}_{n'n}$ corresponds to single-quark exchange, and the second to two-quark exchange between two spatially overlapping nucleons [6, 7]. Among other consequences, $\hat{\Delta}_{n'n}$ implies that the two-nucleon state $|nn'\rangle = \hat{b}_n^\dagger\hat{b}_{n'}^\dagger|0\rangle$ is no longer properly normalized; specifically, its norm is modified according to $N_{nn'} = \langle nn'|nn'\rangle = 1 - \delta_{nn'} - \Delta N_{nn'}$, where

$$\Delta N_{nn'} = 3\Psi_n^{*\mu_1\mu_2\mu_3}\Psi_{n'}^{*\nu_1\nu_2\nu_3}\left(\Psi_n^{\mu_1\mu_2\nu_3}\Psi_{n'}^{\nu_1\nu_2\mu_3} - \Psi_n^{\mu_1\nu_2\nu_3}\Psi_{n'}^{\nu_1\mu_2\mu_3}\right)\,. \tag{4}$$

These results highlight the inherent difficulties of working with composite field operators in many-body systems. Using a Gaussian form for the internal nucleon wave functions, one obtains [6, 7]

$$\Delta N_{nn'} = \left(\frac{r_0^3}{V}\right) \times \frac{9\sqrt{3}}{\pi^{3/2}}\left[A_{nn'}\,\mathrm{e}^{-(\boldsymbol{P}-\boldsymbol{P}')^2 r_0^2/12} - B_{nn'}\,\mathrm{e}^{-(\boldsymbol{P}-\boldsymbol{P}')^2 r_0^2/3}\right]\,, \tag{5}$$

where r_0^2 is the mean square radius of the nucleon, and $A_{nn'}$ and $B_{nn'}$ are numbers obtained by summing over color, spin, and isospin of the quarks for the different nucleon–nucleon channels (their explicit values are given in Table A1 of Ref. [7]). For N composite nucleons confined to a volume V, we can show that in the independent-pair approximation, the change in the norm scales as $\rho \times r_0^3$, where $\rho = N/V$ is the density[1]. This indicates that the composite structure of the nucleon becomes significant once the characteristic nucleon radius is comparable to the effective volume occupied by each nucleon in the medium.

The impact of nucleon compositeness in a high-density medium, particularly Pauli-exclusion effects at the quark level, can be effectively incorporated in a field-theoretic framework by introducing a *deformed* field algebra. This approach uses nucleon field operators whose anticommutation relations deviate minimally from canonical fermionic ones. Such operators, known as *quons*, were introduced by Greenberg in 1991 in nonrelativistic field theory to set experimental bounds on possible violations of Fermi and Bose statistics [8]. Extensions of this formalism to nonrelativistic bosonic systems, designed to encode the internal structure of atoms, appear in Refs. [9, 10]. Earlier developments are cited therein, and a recent relativistic generalization directly relevant to our work is given in Ref. [11]. In the present context, the quon field introduced to represent a composite nucleon field operator $\psi(x,t)$ is postulated to obey the generalized equal-time algebra

$$\psi(x,t)\psi^\dagger\left(x',t\right) + \lambda\,\psi^\dagger\left(x',t\right)\psi(x,t) = \delta\left(x-x'\right)\,, \tag{6}$$

where $\lambda(\rho)$ is a density-dependent parameter that characterizes the degree of nucleon compositeness. To clarify this, consider a two-nucleon state $|nn'\rangle$, constructed from the Fourier modes of ψ and the usual creation and annihilation operators. The deviation from the canonical norm is $\Delta N_{nn'} = x\,\delta_{nn'}$, where x is defined through $\lambda = 1 - x$. From Eq. (5), we identify $x \sim r_0^3/V$. This identification is interpreted as λ representing a coarse-grained average description of the deviation of the canonical composite-nucleon field anticommutation relations for a generic nucleon model; in the quark model illustration, a coarse-grained average of the joint effect of the single- and two-quark exchange contributions in Eq. (3). For a system of N composite nucleons confined within a volume V, in the low-density limit $\lambda \to 1$, we recover the standard canonical anticommutation relations of point-like nucleons. In contrast, at high densities, λ deviates from unity, effectively encoding the influence of Fermi–Dirac statistics at the quark substructure level for spatially overlapping nucleons. This class of generalized statistical

[1] In principle, r_0 may also depend on ρ; for "nucleon swelling", the norm deviation would remain an increasing function of ρ, but would no longer be strictly linear.

frameworks is not without precedent in physics, as it exhibits a close similarity to Haldane's fractional exclusion statistics (FES) [12] and to anyonic statistics in two-dimensional systems [13].

We specify the dynamics for the fields through an effective Lagrangian density that includes a Dirac kinetic term and contact-interaction terms of the form $C_{\rm s}\,(\bar\psi\psi)^2$, $C_{\rm v}\,(\bar\psi\gamma_\mu\psi)^2$, $C_{\rm ps}\,(\bar\psi\gamma_5\psi)^2$, $C_{\rm dv}\,(\bar\psi\partial\!\!\!/\psi)^2$, where the couplings $C_{\rm s}, C_{\rm v}, \ldots$ are dimensionful. Here, we present results using the simplest possible Lagrangian that saturates nuclear matter at the density ρ_0

$$\mathcal{L} = \bar\psi\left(i\partial\!\!\!/ - M\right)\psi - C_{\rm s}\left(\bar\psi\psi\right)^2 - C_{\rm v}\left(\bar\psi\gamma_\mu\psi\right)^2 , \tag{7}$$

where M is the nucleon mass. Within the mean-field approximation (MFA) (and no-sea), this Lagrangian with $\lambda = 1$ yields results that are identical to those obtained in the Walecka model [1] at the same level of approximation.

3. Equation of state and speed of sound

The grand-canonical potential per unit volume in the MFA is

$$\begin{aligned}\frac{\Omega}{V} &= -gT\int\frac{{\rm d}^3k}{(2\pi)^3}\left[\ln\left(1+{\rm e}^{-\beta(E(k)-\mu_\lambda)}\right)+\ln\left(1+{\rm e}^{-\beta(E(k)+\mu_\lambda)}\right)\right] \\ &\quad +\frac{1}{2}C_{\rm s}\rho_{\rm s}^2-\frac{1}{2}C_{\rm v}\rho^2+\rho\Sigma(\rho)\,,\end{aligned} \tag{8}$$

where the baryon density ρ and scalar density $\rho_{\rm s}$ are given by

$$\rho = g\int\frac{{\rm d}^3k}{(2\pi)^3}\left[\frac{1}{{\rm e}^{\beta(E(k)-\mu^*)}+\lambda}-\frac{1}{\lambda}\,\frac{1}{\lambda\,{\rm e}^{\beta(E(k)+\mu^*)}+1}\right], \tag{9}$$

$$\rho_{\rm s} = g\int\frac{{\rm d}^3k}{(2\pi)^3}\frac{M^*}{E(k)}\left[\frac{1}{{\rm e}^{\beta(E(k)-\mu^*)}+\lambda}+\frac{1}{\lambda}\,\frac{1}{\lambda\,{\rm e}^{\beta(E(k)+\mu^*)}+1}\right], \tag{10}$$

where $E(k) = \sqrt{\boldsymbol{k}^2+M^{*2}}$, with $M^* = M - C_{\rm s}\rho_{\rm s}$, and $\mu_\lambda = \mu^* + T\ln\lambda$ with $\mu^* = \mu - C_{\rm v}\rho + \Sigma(\rho)$. The term $\Sigma(\rho)$ represents a rearrangement term necessary for thermodynamic consistency; one needs such terms when using models where couplings or parameters, like λ, depend on the density [14].

A key ingredient of the framework is the density dependence of λ. In principle, one could seek guidance from nuclear many-body approaches that incorporate explicit quark degrees of freedom. However, for the purposes of gaining qualitative insight into the problem, we instead postulate a specific functional dependence guided by physical intuition, namely the requirement that the transition from a regime of well-separated (pure) nucleons to one of overlapping nucleons proceeds in a continuous manner. With such an ansatz, one does not anticipate a genuine phase transition; rather, the system is anticipated to exhibit smooth crossover behavior. In the inset of the left panel in Fig. 1, we present our model for $\lambda(\rho)$: it has a Gaussian form with an intrinsic scale equal to $12\rho_0$, it starts deviating from unity at $\rho/\rho_0 \sim 2.5$, and reaches $\lambda \sim 0.7$ at $\rho/\rho_0 = 10$. We note that our framework is not designed to treat quark matter when nucleons disappear completely. The other two free parameters are C_s and C_v, but these are known from the Walecka model. As usual, one has to solve two coupled self-consistent equations for M^* and μ^*.

Figure 1 shows the equation of state (EOS) and sound speed v_s^2/c^2 in cold neutron matter. The density-dependent deformation $\lambda(\rho)$ causes strong deviations from the canonical case $\lambda = 1$. Our results qualitatively resemble the quarkyonic matter predictions of Ref. [15]: the rightward bending of the EOS with $\lambda(\rho)$ produces a sharp rise of v_s^2/c^2 around $\epsilon \sim 0.5$ GeV/fm^3, followed by a rapid drop below the conformal bound $v_\mathrm{s}^2/c^2 = 1/3$, mirroring Figs. 3 and 2 of Ref. [15]. In the quarkyonic framework, the key role of the Pauli exclusion principle at both nucleonic and quark levels is mirrored in our approach, where an analogous mechanism is effectively encoded in the modified anticommutation relations of the nucleon fields.

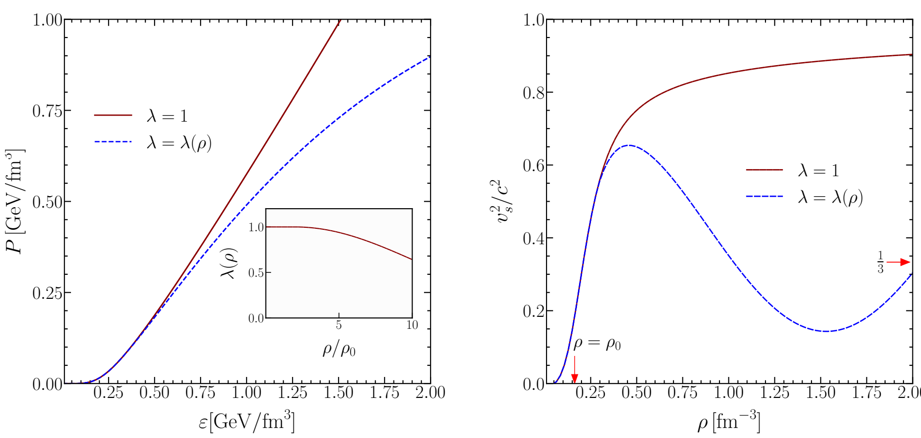

Fig. 1. EOS and speed of sound of cold neutron matter. The inset in the left panel shows our model for density dependence of λ.

In conclusion, this relativistic field theory provides a framework for tackling nucleon superposition in high-density nuclear matter by using deformed fields to model internal degrees of freedom. Future work will focus on constraining the density dependence of λ using a microscopic quark model.

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