PIONS RELOADED*

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We present a novel version of the pion Bethe–Salpeter equation in the chiral limit, solved using state-of-the-art QCD correlation functions as ingredients. The constraints imposed by the axial Ward–Takahashi identities are exactly fulfilled, both formally and numerically.

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1. Introduction

The wealth of information on the structure of the QCD correlation functions, amassed by functional methods and lattice simulations [1–4], may be profitably utilized in the physics of hadrons only within sophisticated frameworks that respect the constraints imposed by chiral symmetry. Recently, a theoretical approach was developed in [5], which allows for the self-consistent inclusion of dressed correlation functions in the dynamical equations describing the physics of mesons. This approach was judiciously simplified in [6], leading to a rather tractable set of dynamical equations. In particular, the quark gap and the pion Bethe–Salpeter equation (BSE)

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admit fully-dressed quark–gluon vertices, denoted by $\Gamma_\mu(q,r,-p)$, containing all possible tensorial structures. In this paper, we showcase the application of this method in the study of massless pions.

2. Main ingredients

The axial-vector vertex $\Gamma_5^\mu(P,p_2,-p_1)$, with $p_1 = p - P/2$, $p_2 = p + P/2$, satisfies the Ward–Takahashi identity (WTI) [7, 8]

$$-P_\mu \Gamma_5^\mu(P,p_2,-p_1) = S^{-1}(p_1)\gamma_5 + \gamma_5 S^{-1}(p_2)\,, \tag{1}$$

where $S^{-1}(q) = A(q^2)\not{q} - B(q^2)$ is the inverse quark propagator. The dynamical breaking of the chiral symmetry gives rise to a non-vanishing $B(q^2)$. Therefore, in the limit $P = 0$, $p_1 = p_2 = p$, the r.h.s. of Eq. (1) becomes: r.h.s. $= -2B(p^2)\gamma_5$. This, in turn, forces the axial-vector vertex on the l.h.s. of Eq. (1) to contain a longitudinally coupled massless pole, $(P^\mu/P^2)\,\chi(0,p,-p)\gamma_5$, associated with the corresponding massless Goldstone boson (pion). Then, equating both sides of Eq. (1) yields

$$\chi(0,p,-p) = \chi_1\left(p^2\right) + \chi_3\left(p^2\right)\not{p} = 2B\left(p^2\right)\,, \tag{2}$$

or, equivalently, for the individual components of the pion BS amplitude [7]

$$\chi_1\left(p^2\right) = 2B\left(p^2\right)\,, \qquad \chi_3\left(p^2\right) = 0\,. \tag{3}$$

The SDE governing $\Gamma_5^\mu(P,p_2,-p_1)$ is shown in Fig. 1 A [9]. The element crucial for effectuating the transition beyond the rainbow ladder (RL) is the axial-vector-gluon vertex, $G_5^{\mu\nu}(P,q,p_2,-q_1)$, denoted by a yellow circle; it is composed of "one-loop-dressed" diagrams, which, at the level of

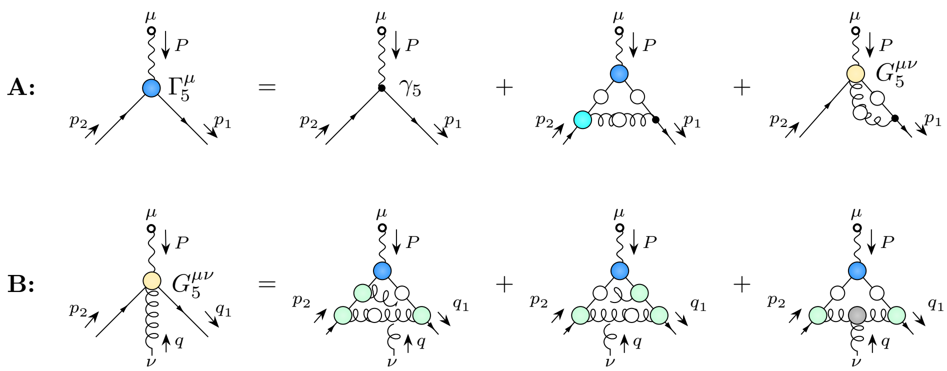

Fig. 1. (Colour on-line) Panel A: The exact SDE for the axial-vector Γ_5^μ. Panel B: The SDE for the vertex $G_5^{\mu\nu}$, after implementing the SV approximation.

$\Gamma_5^\mu(P, p_2, -p_1)$, correspond to "two-loop dressed" contributions. $G_5^{\mu\nu}(P, q, p_2, -q_1)$ obeys the WTI [10]

$$-iP_\mu G_5^{\mu\nu}(P, q, p_2, -q_1) = \Gamma^\nu(q, p_1, -q_1)\gamma_5 + \gamma_5 \Gamma^\nu(q, p_2, -q_2)\,, \tag{4}$$

where $q_i := p_i + q$, which guarantees that the diagrammatic representation of $\Gamma_5^\mu(P, p_2, -p_1)$, given by Fig. 1 A, satisfies the WTI of Eq. (1) exactly.

3. The symmetric-vertex approximation

As demonstrated in [5], the dynamical equation that determines $G_5^{\mu\nu}$ comprises two classes of diagrams: those that contain $G_5^{\mu\nu}$, and those where the $G_5^{\mu\nu}$ is replaced by Γ_5^μ. It turns out that a symmetry-preserving truncation may be implemented at this point, by keeping only the terms involving the Γ_5^μ, provided that a compensating modification takes place at the level of the quark–gluon vertex. Specifically, in the diagrams retained, shown in Fig. 1 B, one must implement the replacement $\Gamma_\mu(q, r, -p) \to V_\mu(q) = \gamma_\mu V(q)$, see Fig. 2. The same replacement must be carried out in the defining diagrams of the SDE for $\Gamma_\mu(q, r, -p)$ [11–13], which acquires the form shown in Fig. 3 A. Note that the WTIs are preserved provided that $V(q)$ be a function of a single kinematic variable, namely the momentum carried by the gluon. In practice, the $V(q)$ chosen coincides with the "symmetric limit", $q^2 = r^2 = p^2$, of the form factor λ_1 associated with the classical tensor γ_μ. Due to this particular characteristic, we coin this truncation as the "*symmetric-vertex*" (SV) approximation.

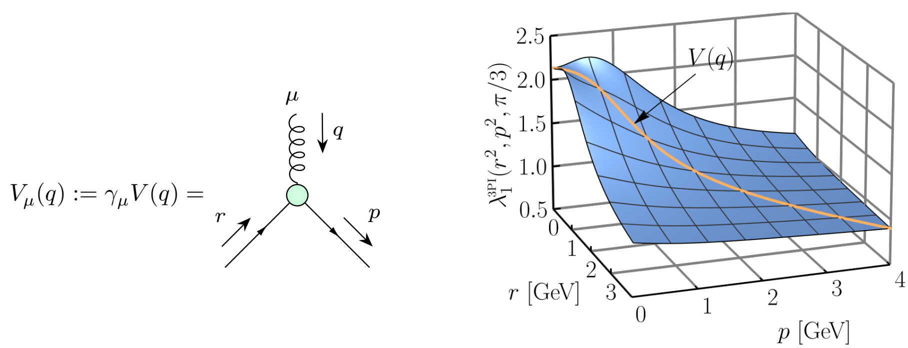

Fig. 2. (Colour on-line) Left panel: Schematic representation of the SV approximation. Right panel: The classical form factor λ_1 obtained in [17]; the orange/light grey line indicates the symmetric configuration $r^2 = p^2 = q^2$ used for $V(q)$.

We emphasize that the output of the SDE in Fig. 3 A, represented by the cyan/black circle, displays the *full* kinematic structure associated with a quark–gluon vertex, namely eight tensorial structures (Landau gauge). The corresponding form factors, λ_i, depend on three kinematic variables, *e.g.* q^2, r^2, and $q \cdot r$.

The new pion BSE consists of three diagrams, shown in Fig. 3 C. The first corresponds to the dressed version of the standard rainbow-ladder (RL) contribution [14–16], while the remaining two guarantee the symmetry-preserving nature of the approach.

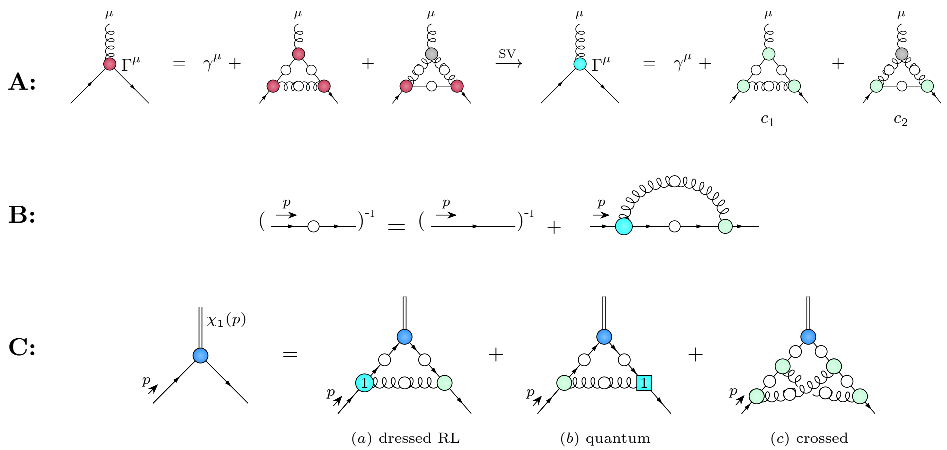

Fig. 3. (Colour on-line) Panel A: The SDE for the quark–gluon vertex in the SV approximation. Panel B: The corresponding gap equation. Panel C: The final pion BSE, after renormalization. The square cyan vertex stands for the "quantum" part of the quark–gluon vertex, composed of the sum of the graphs c_1 and c_2, shown in Panel A of this figure. The index "1" indicates that only the part containing an odd number of Dirac γ matrices survives.

4. Numerical analysis

The system of coupled equations shown in Fig. 3 (quark–gluon vertex SDE, quark-gap equation, and pion BSE) is solved using state-of-the-art ingredients for the correlation functions entering in the corresponding kernels. In particular, we use the gluon propagator obtained from the lattice simulation of [18]. As explained in [19, 20], the key dynamics associated with the emergence of a gluon-mass gap are due to the operation of the Schwinger mechanism in QCD. For the three-gluon vertex, $\Gamma_{\alpha\beta\gamma}(q, r, p)$, entering in graph c_2 of Fig. 3 A, we capitalize on the key property of the "planar-degeneracy" [21, 22]: the transversely projected $\Gamma_{\alpha\beta\gamma}(q,r,p)$ is very accurately described by the tree-level structure, multiplied by a special form factor $L_{sg}(s)$. An accurate fit for $L_{sg}(s)$ is given in Eq. (A1) of [23].

The two main highlights of the numerical analysis are presented in Fig. 4. On the left panel, we show the individual contribution of each of the three BSE diagrams to the eigenvalue $\lambda(P^2)$ of the BSE, for which, at $P^2 = 0$ (massless pion), we have $\lambda(0) = 1$. On the right panel, we demonstrate that the pion BS amplitude, $\chi_1(p)$, satisfies at a high degree of accuracy (better than 1%) the symmetry-induced relation given in Eq. (3). This result constitutes a clear numerical confirmation of the symmetry-preserving nature of the entire approach.

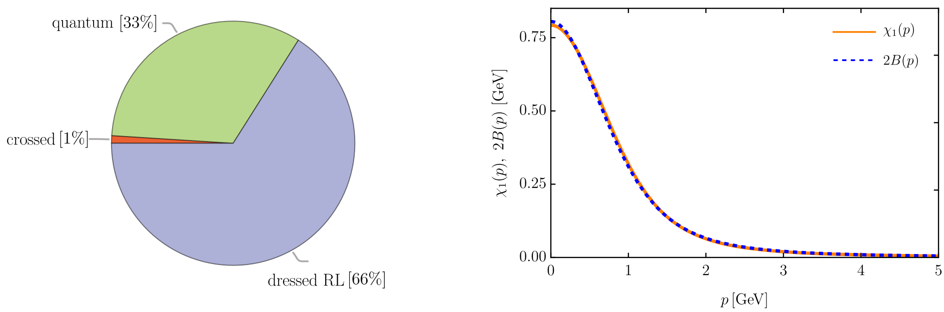

Fig. 4. Left panel: Contributions to the eigenvalue of the pion BSE, from each of the three diagrams shown in Fig. 3 C. Right panel: Numerical confirmation of the WTI-imposed relation $\chi_1(p) = 2B(p)$.

5. Conclusions

We have presented a brief review of the symmetric vertex approximation, which allows for the symmetry-preserving treatment of the pion dynamics using fully-dressed QCD correlation functions, and, in particular, fully-dressed quark–gluon vertices, with all form-factors dynamically evaluated.

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