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RHO MESONS AND THE COMPRESSIBILITY OF NUCLEAR MATTER IN THE SKYRME MODEL*

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In this paper, we briefly review some recent advances in a low-energy model of Quantum Chromodynamics, where nuclei and baryons are described by topological soliton solutions known as Skyrmions. Hence, following the great success of coupling the rho mesons to the Skyrme field (which provided more realistic nuclear binding energies for the model and the triggering of nuclear clustering), we consider the role that the leading $\rho\pi\pi$ interaction term can play in the study of dense nuclear matter. This contribution, motivated by a theoretical construction from a Yang–Mills theory in one higher dimension, astonishingly reduces the compression modulus that the Skyrme model provides from the very large value of $K_0 \simeq 1080$ MeV to a more physical $K_0 \simeq 351$ MeV.

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1. Introduction

In the last decades, topological solitons have gained importance in different realms of physics, from condensed matter systems to effective theories in nuclear and high-energy physics. These objects are particle-like solutions of non-linear field equations which appear as collective excitations of the relevant degrees of freedom of the theory underneath. Their topological nature comes from the topology of both base and target spaces, and as a result, they present a topological, then conserved, charge or winding number.

Among a plethora of topological solitons, we will focus on one type known as Skyrmions. They are the minimisers of the static energy functional of a low-energy model of Quantum Chromodynamics (QCD). It was proposed by the British physicist T.H.R. Skyrme in the sixties of the last century, and named the Skyrme model after him [1]. Hence, baryons and nuclei emerge as collective excitations of pionic fields, while the associated topological charge is identified with the (conserved) baryon number.

Although the model was abandoned for a time, a revival of Skyrme's ideas took place in the last decades, and important advances have been made. New extensions and related models have appeared, where a prominent role was played by theories with a lower topological bound for the energy known as the Bogomol'nyi–Prasad–Sommerfield (BPS) bound [2, 3]. In this way, some of the main drawbacks the theories posed were mitigated, namely, the largely unphysical binding energies and the lack of clustering structures [4, 5]. However, some problems remained, such as the large compression modulus. In this work, we provide a review of our recent work [6], showing how one of the previous extensions of the standard theory (the one introducing the coupling to vector mesons [3]) may help in bringing down this value from $K_0 \simeq 1080$ MeV to a more physical $K_0 \simeq 351$ MeV.

2. The rho meson revolution

We start by introducing the Lagrangian of the standard Skyrme model with a potential which gives mass to the pions [7]. It reads

$$L=\int\left\{-\frac{F_\pi^2}{16}\mathrm{Tr}\,(R_\mu R^\mu)+\frac{1}{32e^2}\mathrm{Tr}\,([R_\mu,R_\nu][R^\mu,R^\nu])+\frac{1}{8}m_\pi^2F_\pi^2\mathrm{Tr}\,(U-\mathbb{1})\right\}\mathrm{d}^3x\,, \tag{1}$$

where $R_\mu = (\partial_\mu U)U^\dagger$ is a right invariant $su(2)$-valued current written in terms of the Skyrme field $U \in \mathrm{SU}(2)$. Interestingly, we only have two coupling constants, the pion decay constant, F_π, which is usually considered as a free parameter in the model, and the dimensionless Skyrme coupling constant, e. Finally, the physical pion mass also contributes through the potential term.

For the energy of the system to be finite, the Skyrme field must approach the vacuum value $\mathbb{1}$ at infinity. This allows us to compactify the base space to the three sphere, S^3, and together with the fact that SU(2) is homeomorphic to S^3, one can see the Skyrme field as a map between three-spheres. For Skyrmions on strictly periodic domains, *i.e.* on a 3-torus, then Hopf's degree theorem ensures that the Skyrme field is characterized by a homotopy invariant (the topological degree), since the 3-torus is compact and without boundary. Hence, we have a topological, then conserved, charge, which we can identify with the baryon number, namely,

$$B = -\frac{1}{24\pi^2} \int \epsilon_{ijk} \mathrm{Tr}\,(R_i R_j R_k)\, \mathrm{d}^3x\,. \tag{2}$$

The model also presents the BPS bound known as the Skyrme–Faddeev bound, $E \geq \frac{3F_\pi}{e}\pi^2|B|$. However, it cannot be attained, and the nuclear binding energies the theory provides are too large. Among several attempts to mitigate this issue, one of the most successful approaches is where the Skyrme model emerges from a pure Yang–Mills theory in 4+1 dimensions [3]. Via a dimensional deconstruction, one ends with the standard Skyrme model (without the pion mass term) coupled to an infinite tower of vector mesons. Surprisingly, truncating the tower to keep the lightest of vector mesons, the rho, already provides an astonishing decrease in the binding energies as well as triggering the nuclear clustering when the pion mass potential is also considered.

On the other hand, the corresponding static energy functional is too involved and finding solutions becomes a highly numerically demanding problem. Nevertheless, we have recently found that keeping the only interaction term related to the $\rho \to \pi\pi$ decay can already improve another weakness the Skyrme model suffers: the too large compression modulus [6]. It should be noted that this restriction is physically natural since the branching ratio of this decay is of 99.9%.

The resulting static energy functional reads as follows:

$$\begin{aligned} E = \int \Big\{ & -\frac{1}{2}\mathrm{Tr}\,(R_i R_i) - \frac{1}{4}\mathrm{Tr}\,([R_i, R_j][R_i, R_j]) + M_\pi^2 F_\pi^2 \mathrm{Tr}\,(\mathbb{1} - U) \\ & - \mathrm{Tr}\,(\partial_i V_j - \partial_j V_i)^2 - 2M_\rho^2 \mathrm{Tr}\left(V_i^2\right) - 2c_\alpha \mathrm{Tr}\,([R_i, R_j]\,(\partial_i V_j - \partial_j V_i)) \Big\} \mathrm{d}^3x\,, \end{aligned} \tag{3}$$

with V_i the three $su(2)$-valued rho meson fields. The functional has been rescaled to be dimensionless by considering the usual Skyrme energy and length units, $F_\pi/(4e)$ and $2/(eF_\pi)$, respectively. Similarly, we have also introduced the rescaled pion and rho meson masses which, by taking the values of e and F_π proposed in [7], read: $M_\pi = \frac{2m_\pi}{eF_\pi} = 0.528$ and $M_\rho = \frac{2m_\rho}{eF_\pi} = 2.965$. Finally, c_α is related to the $\rho\pi\pi$ coupling constant and will be treated as a free parameter.

It is worth mentioning that the interaction term between the rho and Skyrme fields allows a new BPS energy bound for the minimal rho meson extension

$$E \geq \int \left\{ -\frac{1}{2}\mathrm{Tr}\,(R_i R_i) - \frac{1}{4}\left(1 - 16c_\alpha^2\right)\mathrm{Tr}\,([R_i, R_j][R_i, R_j]) \right\} \mathrm{d}^3x$$

$$\Rightarrow E \geq 12\pi^2\sqrt{1 - 16c_\alpha^2}|B|\,. \qquad (4)$$

Due to stability reasons, the second contribution cannot change sign, imposing the condition $c_\alpha \leq 1/4$. At the same time, this has an important effect on lowering the compression modulus, since it reduces the short-range interactions. Also, when $c_\alpha \to 1$, we recover the old Skyrme–Faddeev bound, which in the Skyrme units reads $E \geq 12\pi^2|B|$.

It should be noted that the beneficial effects of considering vector mesons have been further supported by recent works on the improvement in nuclear binding energies and compression modulus by the omega meson [8, 9].

3. Results

To describe nuclear matter, one needs to look at solutions for large baryon number, B, where the minimal configurations take the form of Skyrme crystals. In the case of the massive standard Skyrme model of Eq. (1), two minimal configurations with cubic symmetry were known for some time: a simple cubic crystal of half Skyrmions and an alpha-particle crystal. However, a recent detailed and rigorous study of the model has found out that other two configurations breaking this cubic geometry are also solutions [10]. These new lattices have non-cubic periods showing chain and multiwall structures. Among these four different configurations, the ground state in the massive Skyrme model was found to be the multiwall solution for all the densities and coupling constants considered.

We now want to see what the ground-state configuration looks like once we consider the minimal rho meson coupling as defined in Eq. (3). Interestingly, we observe that when the rho meson is included, the alpha-particle crystal turns out to be the global minimiser with the multiwall configuration being the second lowest energy solution (see [10] for details on the numerical scheme used for finding these solutions). Also, the greater c_α is, the bigger the energy difference between these two Skyrmion crystals becomes. Isosurfaces of the Skyrme field and rho meson energy densities on the unit cell of the alpha crystal are shown in Fig. 1 (a) for a high coupling constant c_α.

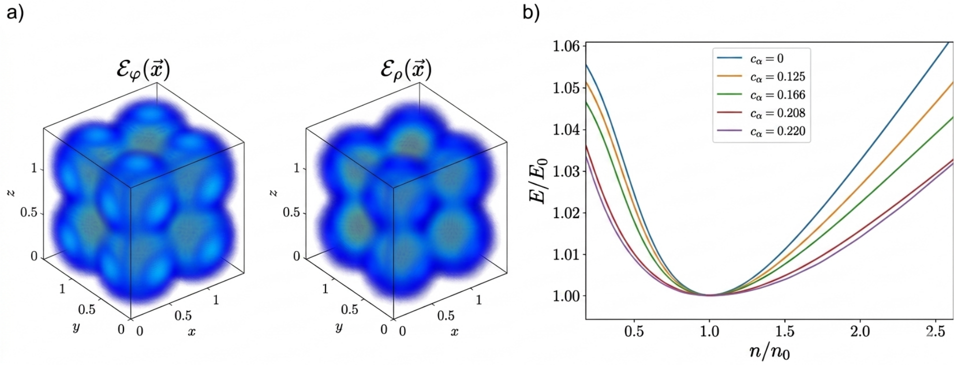

Fig. 1. (a) Skyrme field (left) and rho meson field (right) energy densities in the unit cell for the ground state with $c_\alpha = 0.225$. (b) Total energy normalised by the energy at saturation density as a function of baryon density. Figures are from [6].

Once we know the ground state of this theory, we can draw our attention to the large compression modulus problem that the Skyrme model poses. The compression modulus, K_0, is related to the energy of isospin symmetric nuclear matter around saturation density, n_0. Hence, if we consider a power series expansion in baryon density near n_0, we get

$$E(n_B) = E_0 + \frac{1}{2}K_0\frac{(n_B - n_0)^2}{9n_0^2} + O\left((n_B - n_0)^3\right) , \qquad (5)$$

with E_0 the energy at saturation density and K_0 appearing as a multipole of the expansion. In our model, this K_0 can be easily computed by considering

$$K_0 = 9n_0^2 \left.\frac{\partial^2 E}{\partial n_B^2}\right|_{n_0} . \qquad (6)$$

This quantity describes the stiffness of nuclear matter under external pressure at the saturation point and is expected to be $K_0 \sim 210$–240 MeV [11]. However, previous studies have shown values higher than 1000 MeV within Skyrme models [12]. In Table 1, we can see the effect of coupling the rho meson to the Skyrme model in the minimal way described in the previous section. By getting closer to the limiting value $c_\alpha = 1/4$, we manage to greatly improve the compression modulus from $K_0 = 1080$ MeV to a value $K_0 = 351$ MeV closer to the physical one. This effect can also be seen in Fig. 1 (b), where the total energy normalised by the energy at saturation density is depicted as a function of the baryon density. This demonstrates the importance that the rho meson has in describing nuclear matter in a topological soliton theory such as the Skyrme model.

Table 1. Compression modulus in MeV for different values of the coupling constant c_α.

c_α	0	0.125	0.166	0.208	0.220
K_0 [MeV]	1080	909	718	425	351

4. Conclusions

For a long time, Skyrmions have been proposed within a model of strong interactions at low energies, where nuclei are described as topological solitons. Despite a revival of Skyrme's ideas in the last decades, some problems and issues remained. However, some extensions and modifications of the theory have shed some light, mitigating weaknesses such as the unphysical large binding energies or the lack of clustering structures. Here, we have reviewed the importance of the coupling of the rho meson for a better description of nature in the Skyrme model, playing an important role in improving the compression modulus and paving the way for a more reliable description of nuclear matter.

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