PRECISE DISPERSIVE DESCRIPTION OF $\pi\pi$ INTERACTIONS AND MODEL-INDEPENDENT DETERMINATION OF RESONANCES*

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In this paper, we review our recent work on $\pi\pi$ scattering. We present new global fits that describe all experimental data up to 1.8 GeV while satisfying dispersive constraints, within uncertainties, up to 1.6 GeV. Using continued fractions to continue forward dispersion relations analytically, we extract the pole parameters of the $f_0(500)$, $\rho(770)$, $f_0(980)$, $f_2(1270)$, $f_0(1370)$, $\rho(1450)$, $f_0(1500)$, and $\rho_3(1690)$ in a model-independent way.

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1. Introduction

A precise description of low-energy $\pi\pi$ interactions is central in hadronic physics, as pion rescattering plays a role in many processes. Interest in $\pi\pi$ scattering has grown in recent decades, driven by high-precision data from experiments such as ALICE, Belle, and LHCb, as well as constraints from lattice QCD at increasingly low pion masses. Moreover, these interactions provide access to hadronic resonances in the non-perturbative QCD regime.

Regarding $\pi\pi \to \pi\pi$ data [1–8], most of them were extracted indirectly in the 1970s from $\pi N \to \pi\pi N'$ reactions using a number of approximations. As a result, the various datasets suffer from large systematic uncertainties, are often mutually inconsistent, and do not satisfy dispersive constraints. In addition, several experiments admit multiple solutions for the same data [1, 5, 6, 8], which become incompatible above 0.9 GeV.

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A powerful tool to address this "data problem" is Dispersion Theory, which follows from the analyticity properties of the scattering amplitude and, ultimately, from causality. It leads to integral equations, usually referred to as dispersion relations, which can also incorporate other fundamental principles such as Lorentz invariance, unitarity, and crossing symmetry.

Dispersive constraints not only allow us to decide among different datasets and data solutions, but also provide a solution for the so-called "model problem" [9]. Resonances are defined by the associated pole position $\sqrt{s_{\rm pole}} = M_{\rm R} - i\Gamma_{\rm R}/2$ in the scattering amplitude, and this definition is process-independent. The residue of the pole is related to the coupling of the resonance to a given channel. Extracting resonance masses, widths, and couplings is therefore challenging, since it requires a reliable analytic continuation from the physical real-s axis into the complex plane. In practice, these quantities are often obtained from models such as isobar models or Breit–Wigner-like parametrizations, but these approaches do not provide reliable analytic continuations when the resonance lies deep in the complex plane or is not well isolated from other singularities. This "model problem" can be addressed with dispersion relations, which provide a model-independent continuation into the complex plane below the first inelastic threshold. Even for resonances above inelastic thresholds, dispersive constraints can be used to restrict the amplitude on the real axis, and then continue it analytically with a reliable and stable method.

Here, we review our results from [10, 11]. In the first work, we provided dispersively constrained $\pi\pi$ global parametrizations that addressed the "data problem", whereas in the very recent [11], we have extracted the resonance parameters in a model-independent way from the dispersive output of the global parametrizations, using continued fractions [12] for the analytic continuation.

2. Dispersively constrained Global Fits

In [10], our main goal was to improve previous dispersive analyses from our group [13–17]. In particular, a set of partial waves (CFD) with $\ell \leq 3$ was given in [16]. They described $\pi\pi$ scattering data up to 1.4 GeV, while satisfying forward dispersion relations (FDRs) up to that energy, and Roy [18] and GKPY [16] dispersion relations up to 1.1 GeV for the S and P partial waves. Then, $S0$ and P global parametrizations (where global means that they describe all the available data) were provided in [17]. They were obtained by fitting the CFD parametrizations and the three data solutions I, II, and III [1, 5, 6, 8] that exist above 1.4 GeV.

Nevertheless, some aspects of these dispersive analyses could be improved. First, a revision of the inelasticities for the $S2$, P, and $D0$ waves was necessary close to the inelastic thresholds. Second, we had to provide global parametrizations for the $S2$, D, and F waves, valid up to at least 1.8 GeV and describing all the available data. Third, in [16], there was a mismatch between the Regge regime and the partial-wave amplitudes, which introduced some artifacts in the FDRs close to 1.4 GeV. Finally, by introducing new global G waves, the FDR fulfillment could be extended beyond 1.4 GeV for some of the amplitudes. In [10], we presented our new sets of dispersively constrained Global Fits I, II, and III (fitting data solutions I, II, and III, respectively), which address all those caveats and present additional improvements. The most significant results can be seen in Fig. 1. They were obtained via a constrained fit to the data, imposing the dispersive constraints as penalty functions. The fulfillment of the FDRs containing the P wave was extended to 1.6 GeV, while Roy and GKPY equations were also imposed. The results for the Global Fit I, which is the one that satisfies the dispersive constraints slightly better, can be seen in Fig. 2.

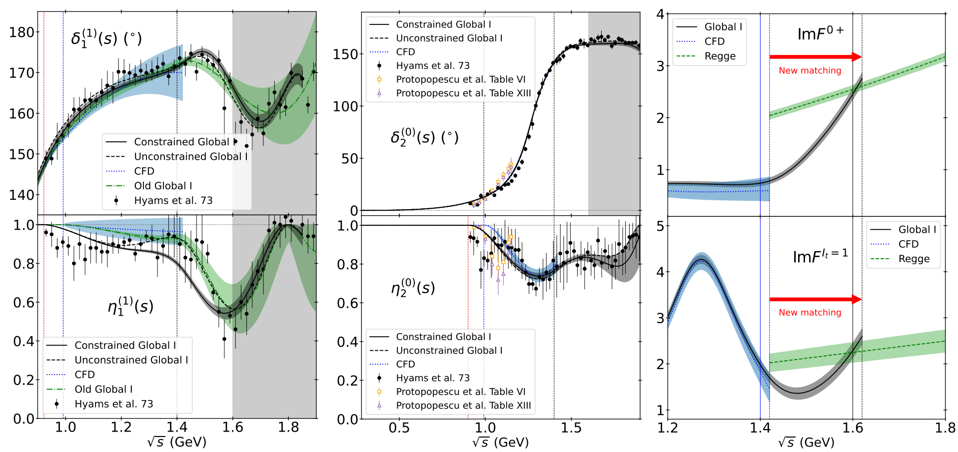

Fig. 1. Some of the improvements from our new $\pi\pi$ dispersive analysis [10]: updated global P wave (left), new global $D0$ wave (center), and the new matching with the Regge regime (right), which has been improved and extended up to 1.6 GeV for the amplitudes containing the P wave. The results correspond to the Global Fit I and improve upon the previous CFD [16] and previous Global Fits [17].

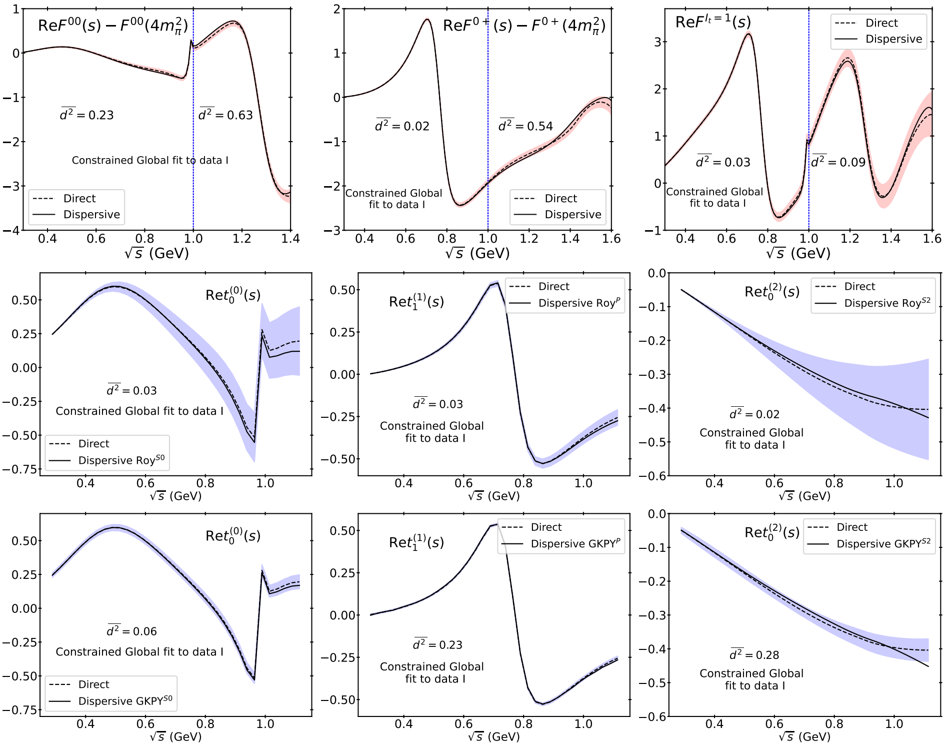

Fig. 2. Global Fit I [10] fulfillment of the dispersive constraints: forward (upper row), Roy (central row), and GKPY (lower row) dispersion relations. The continuous lines mark the real part of the amplitudes obtained from dispersive integrals, whereas the dashed lines are obtained directly from the global parametrizations. The bands attached to the dashed lines are the error of their differences.

3. Determination of $\pi\pi$ resonances

In our recent work [11], we have presented model-independent and dispersive determination of resonances. Below 1.7 GeV, we have obtained pole parameters that we associated with the following resonances: $f_0(500)$, $\rho(770)$, $f_0(980)$, $f_2(1270)$, $f_0(1370)$, $\rho(1450)$, $f_0(1500)$, and $\rho_3(1690)$. To do that, we have continued the FDR output analytically, using the dispersively constrained Global Fits from [10] as input, by means of continued fractions [12]. This method consists of interpolating the output of the FDRs at N points along a real-s segment with a set of nested fractions, yielding a Padé approximant of order $((N-1)/2,(N-1)/2)$ that contains poles. We then search for poles in the complex plane and compute a weighted average after varying N, the Global Fit parameters, and the interpolated segments. We refer to this procedure as the FDR$_{C_N}$ method. From the F^{00} amplitude, we have extracted isoscalar resonance values compatible and competitive with those obtained in [19, 20]. In the isovector sector, using the continuation of the F^{0+} amplitude (see Fig. 3), we have obtained two resonances inaccessible to us before: the $\rho(1450)$ and the $\rho_3(1690)$. For the $\rho(770)$, our results again agree with previous work from our group. However, we have found no stable pole that could be associated with a putative $\rho(1250)$, recently claimed in [21] from the same data that we analyzed [1, 6]. As shown in [11], we have also checked several features of our method, including the stability of the results under changes in N and the reliability of the continuation.

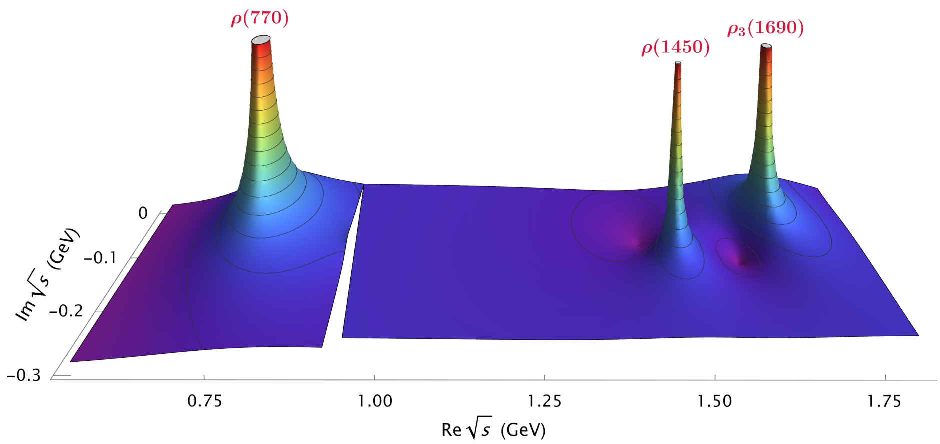

Fig. 3. Representative analytic continuation of the F^{0+} FDR output, using the FDR$_{C_N}$ method with the Global Fit I parametrizations from [10]. We extract the pole parameters of the $\rho(770)$, $\rho(1450)$, and $\rho_3(1690)$ resonances.

4. Summary

In this paper, we have reviewed our recent results from [10, 11]. They provide an improved dispersive and model-independent description of $\pi\pi$ interactions, their uncertainties, and the lightest resonances produced in them. The new Global Fits describe all the available data while satisfying dispersive constraints up to 1.6 GeV. Regarding the resonances, our results are not only compatible with previous works, but also include the description of the $\rho(1450)$ and the $\rho_3(1690)$ resonances.

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REFERENCES

[1] B. Hyams *et al.*, *Nucl. Phys. B* **64**, 134 (1973).
[2] D.H. Cohen, T. Ferbel, P. Slattery, B. Werner, *Phys. Rev. D* **7**, 661 (1973).
[3] M.J. Losty *et al.*, *Nucl. Phys. B* **69**, 185 (1974).
[4] N.B. Durusoy *et al.*, *Phys. Lett. B* **45**, 517 (1973).
[5] G. Grayer *et al.*, *Nucl. Phys. B* **75**, 189 (1974).
[6] B. Hyams *et al.*, *Nucl. Phys. B* **100**, 205 (1975).
[7] W. Hoogland *et al.*, *Nucl. Phys. B* **126**, 109 (1977).
[8] R. Kamiński, L. Leśniak, K. Rybicki, *Z. Phys. C* **74**, 79 (1997).
[9] J.R. Peláez, A. Rodas, J. Ruiz de Elvira, *Eur. Phys. J. Spec. Top.* **230**, 1539 (2021).
[10] J.R. Peláez, P. Rabán, J. Ruiz de Elvira, *Phys. Rev. D* **111**, 074003 (2025).
[11] J.R. Peláez, P. Rabán, J. Ruiz de Elvira, *Phys. Rev. D* **113**, 034018 (2026).
[12] L. Schlessinger, *Phys. Rev.* **167**, 1411 (1968).
[13] J.R. Peláez, F.J. Ynduráin, *Phys. Rev. D* **71**, 074016 (2005).
[14] R. Kamiński, J.R. Peláez, F.J. Ynduráin, *Phys. Rev. D* **74**, 014001 (2006); *Erratum ibid.* **74**, 079903 (2006).
[15] R. Kamiński, J.R. Peláez, F.J. Ynduráin, *Phys. Rev. D* **77**, 054015 (2008).
[16] R. García-Martín *et al.*, *Phys. Rev. D* **83**, 074004 (2011).
[17] J.R. Pelaez, A. Rodas, J. Ruiz de Elvira, *Eur. Phys. J. C* **79**, 1008 (2019).
[18] S.M. Roy, *Phys. Lett. B* **36**, 353 (1971).
[19] R. García-Martín, R. Kamiński, J.R. Peláez, J. Ruiz de Elvira, *Phys. Rev. Lett.* **107**, 072001 (2011).
[20] J.R. Peláez, A. Rodas, J. Ruiz de Elvira, *Phys. Rev. Lett.* **130**, 051902 (2023); *Erratum ibid.* **132**, 239901 (2024).
[21] N. Hammoud, R. Kamiński, V. Nazari, G. Rupp, *Phys. Rev. D* **102**, 054029 (2020).