CHIRAL AND U(1)$_{\mathrm{A}}$ RESTORATION IN EFFECTIVE FIELD THEORIES*

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This paper reviews the interplay between chiral and axial U(1)$_{\mathrm{A}}$ symmetry restoration in QCD at finite temperature, using effective field theories and Ward identities. We analyse how degeneration patterns of scalar and pseudoscalar partners signal symmetry restoration, relating lattice results and phenomenology. Emphasis is placed on pseudoscalar susceptibilities, their relation to quark condensates and the topological susceptibility, and their implications for $O(4)$ *versus* $O(4) \times \mathrm{U}(1)_{\mathrm{A}}$ patterns. Results from Chiral Perturbation Theory are compared with lattice data, highlighting the role of explicit symmetry breaking, the approach to the chiral limit, and the impact of the strange sector.

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1. Introduction

At low energies, QCD is governed by an approximate chiral symmetry, which becomes exact in the limit of vanishing quark masses. For N_f light flavours, the massless QCD Lagrangian is invariant under $\mathrm{SU}(N_f)_{\mathrm{L}} \times \mathrm{SU}(N_f)_{\mathrm{R}}$ flavour rotations of the left- and right-handed projections of the quark fields [1]. For two light quarks, this symmetry is expected to be an almost perfect approximation, whereas, for three flavours, sizeable corrections are expected to be induced by the strange-quark mass, value of which is comparable to Λ_{QCD}. If this symmetry were realized in the Wigner–Weyl mode, states of opposite parity would be nearly degenerate in mass.

In nature, however, this symmetry is not manifest in the hadron spectrum: the absence of parity doublets and the non-degeneracy of vector and axial-vector correlators signal a spontaneous chiral symmetry breaking. As a consequence, the light pseudoscalar mesons π, K, and η emerge as pseudo-Nambu–Goldstone bosons. This breaking is reflected in the non-vanishing

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pion-decay constant f_π and in the light-quark condensate $\langle(\bar{q}q)_l\rangle_T$, which acts as an order parameter of the broken phase. Finally, the singlet axial symmetry $\mathrm{U}(1)_\mathrm{A}$ is anomalously broken, effects of which are reflected in the anomalous divergence of the singlet axial current and in the unusually large mass of the η' meson.

The restoration of chiral symmetry at finite temperature T is a central topic in the physics of strongly interacting matter. However, this regime is not easily accessible experimentally and is best studied using lattice QCD. In this framework, the order parameter is usually expressed through the subtracted condensate $\Delta_{l,s}$, to remove ultraviolet and finite-size divergences:

$$\Delta_{l,s} = \frac{\langle\bar{q}q\rangle_T - (2m_l/m_s)\langle\bar{s}s\rangle_T}{\langle\bar{q}q\rangle_0 - (2m_l/m_s)\langle\bar{s}s\rangle_0}\,, \tag{1}$$

with $\langle\ldots\rangle_{T/0}$ denoting thermal and vacuum expectation values, respectively

Chiral symmetry is also explicitly broken by the light-quark masses, which, in particular, gives rise to a crossover transition, so that $\langle(\bar{q}q)_l\rangle_T$ does not vanish at the physical point. A complementary probe is the light scalar susceptibility χ_l,

$$\chi_l(T) = \int_E \mathrm{d}^4x\left[\langle\mathcal{T}(\bar{q}q)_l(x)(\bar{q}q)_l(0)\rangle_T - \langle(\bar{q}q)_l\rangle_T^2\right] = -\frac{\partial}{\partial m_l}\langle(\bar{q}q)_l\rangle_T\,, \tag{2}$$

which corresponds to the zero-momentum correlator of the scalar density, and is sensitive to fluctuations of the order parameter across the transition. Thus, $\chi_l(T)$ is expected to peak at T_c.

Lattice QCD results for $\Delta_{l,s}$ and χ_l show a rapid crossover around $T_\mathrm{c} \sim 155$ MeV for physical quark masses and $N_f = 2+1$ [2, 3]. In the chiral limit, however, the transition is expected to become critical [4], with lattice evidence pointing to $T_\mathrm{c} \simeq 132^{+3}_{-6}$ MeV for $m_l \to 0$, consistent with a possible second-order $O(4)$ phase transition [5].

The possible restoration of $\mathrm{U}(1)_\mathrm{A}$ near the chiral transition is of particular interest. While instanton suppression suggests restoration at high temperature, lattice and experimental indications point to a more subtle picture near T_c. Whether $\mathrm{U}(1)_\mathrm{A}$ is effectively restored close to the transition has important consequences, as it would imply an enlarged $O(4)\times\mathrm{U}(1)_\mathrm{A}$ symmetry and potentially modify the order of the phase transition [4]. These questions can be addressed through the study of chiral partner degeneration, which we consider below.

2. Chiral partners and symmetry restoration patterns

A convenient way to analyse chiral and $\mathrm{U}(1)_\mathrm{A}$ restoration is through quark bilinears in the scalar and pseudoscalar channels and their associated susceptibilities. For light quarks, one can define the isotriplet and isosinglet operators

$$\begin{aligned} \pi^a &= i\bar{q}_l\gamma_5\tau^a q_l\,, \qquad \delta^a = \bar{q}_l\tau^a q_l\,, \\ \eta_l &= i\bar{q}_l\gamma_5 q_l\,, \qquad d\sigma_l = \bar{q}_l q_l\,, \end{aligned} \tag{3}$$

as well as their strange counterparts and mixed light–strange operators. The corresponding susceptibilities are obtained from the Euclidean correlators at zero momentum,

$$\chi^{ab}_{\mathrm{P,S}} = \int_T \mathrm{d}^4x \left\langle \mathcal{T}\, O^a_{\mathrm{P,S}}(x)\, O^b_{\mathrm{P,S}}(0) \right\rangle\,, \tag{4}$$

with $O^a_{\mathrm{P,S}}$ any of the scalar or pseudoscalar bilinears defined in (3), that encode the relevant information about the thermal behaviour of the system.

Under $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R} \equiv O(4)$ transformations, the pairs (π, σ) and (η_l, δ) are connected by chiral rotations, whereas $\mathrm{U(1)_A}$ transformations connect (π, δ) and (η_l, σ). Chiral restoration implies $\chi^\pi_{\mathrm{P}} \sim \chi^\sigma_{\mathrm{S}}$ and $\chi^{\eta_l}_{\mathrm{P}} \sim \chi^\delta_{\mathrm{S}}$, while the effective restoration of $\mathrm{U(1)_A}$ would additionally require $\chi^\pi_{\mathrm{P}} \sim \chi^\delta_{\mathrm{S}}$ and $\chi^{\eta_l}_{\mathrm{P}} \sim \chi^\sigma_{\mathrm{S}}$.

On the one hand, lattice QCD results for $N_f = 2+1$ and physical quark masses show a clear degeneration of the (π, σ) partners around the crossover temperature T_{c} [6], signalling the restoration of $O(4)$ symmetry. However, the corresponding $\mathrm{U(1)_A}$ partners, such as (π, δ) and (η, σ), do not show degeneration in the same temperature region, indicating that the axial symmetry remains effectively broken at T_{c} [6]. This supports a restoration pattern consistent with $O(4)$ rather than $O(4) \times \mathrm{U(1)_A}$ at physical masses. At smaller pion masses, $N_f = 2+1$ lattice results indicate that $\mathrm{U(1)_A}$ remains effectively broken near T_{c}, even in the chiral limit [7, 8], pointing to a persistent breaking of the axial symmetry near the transition temperature.

On the other hand, simulations with two degenerate flavours ($N_f = 2$) provide evidence that the $\mathrm{U(1)_A}$ symmetry may be effectively restored close to the critical temperature in the chiral limit [9, 10], even for physical quark masses [11]. In this case, the degeneration of π and δ channels, as well as the suppression of topological effects, suggest a restoration pattern compatible with $O(4) \times \mathrm{U(1)_A}$. As we will see, this difference between $N_f = 2$ and $N_f = 2+1$ emphasises the significant impact of the strange-quark mass.

3. Effective field theory and Ward identity analysis

Effective field theories (EFTs) provide a framework to study chiral symmetry restoration at low energies. In particular, Chiral Perturbation Theory (ChPT) incorporates the symmetry-breaking pattern of QCD, and describes the dynamics of the pseudo-Goldstone bosons through a controlled expansion in momenta, quark masses, and temperature. Within this approach, thermal corrections to the quark condensate and scalar susceptibilities can be computed.

ChPT predicts a smooth decrease of the light-quark condensate with temperature and a corresponding growth of the scalar susceptibility [12, 13], signalling the approach to chiral symmetry restoration. Moreover, it provides a consistent description of partner degeneration. For instance, the pseudoscalar susceptibility in the pion channel is directly related to the quark condensate at NLO ChPT [14],

$$\chi_{\mathrm{P}}^{\pi}(T) = -\frac{\langle \bar{q}q\rangle_l(T)}{m_q}\,, \tag{5}$$

which explains the observed scaling of lattice screening masses and supports the interpretation of chiral restoration in terms of partner degeneration.

This relationship is a particular example of a family of Ward identities (WIs) that relate pseudoscalar susceptibilities and quark condensates [15]. These WIs provide a complementary and model-independent tool for partner degeneration analysis. In particular, in the singlet channel, they connect the pseudoscalar susceptibilities with quark condensates and the topological susceptibility $\chi_{\mathrm{top}}(T)$ [16], which encodes the effects of the $\mathrm{U}(1)_{\mathrm{A}}$ anomaly. In particular, a key quantity to characterize the interplay between chiral and axial restoration is the difference between pseudoscalar susceptibilities, which measures the breaking of the $\mathrm{U}(1)_{\mathrm{A}} \times O(4)$ symmetry. WIs relate this observable to the topological susceptibility and to mixed light–strange correlators,

$$\chi_{5,\mathrm{disc}}(T) = \frac{1}{4}\left(\chi_{\mathrm{P}}^{\pi}(T) - \chi_{\mathrm{P}}^{\eta_l}(T)\right) = \frac{1}{m_l^2}\chi_{\mathrm{top}}(T) = -\frac{2m_l}{m_s}\chi_P^{l,s}(T)\,, \tag{6}$$

with m_l and m_s the light and strange quark masses, respectively. For exact $O(4)$ restoration, *i.e.*, for $N_f = 2$ flavours in the chiral limit, parity conservation implies $\chi_{\mathrm{P}}^{l,s}$ to vanish, hence showing exact $O(4) \times \mathrm{U}(1)_{\mathrm{A}}$ restoration in this limit [17, 18].

In the physical case, though, explicit chiral symmetry breaking distorts the exact degeneration of partners, but WIs and EFTs still provide robust relations among observables. In particular, ChPT predicts that $\chi_{5,\mathrm{disc}}(T)$ scales similarly to the subtracted condensate $\Delta_{l,s}(T)$, in agreement with lattice results. Furthermore, the analysis can be extended to the $I = 1/2$ sector, where K–κ degeneration is controlled by the same symmetry-breaking parameters, providing additional constraints on the restoration pattern [19, 20], and suggesting that the effective restoration pattern depends sensitively on the quark masses. In the limit of $m_s \gg m_l$, results for $N_f = 2$ and $N_f = 2+1$ can be reconciled, while for physical quark masses, the strange sector plays a crucial role in delaying the effective restoration of $\mathrm{U(1)_A}$. Overall, these approaches provide a unified description that connects lattice data, symmetry arguments, and hadronic degrees of freedom in the study of QCD at finite temperature.

4. Summary

In this paper, we have reviewed chiral and $\mathrm{U(1)_A}$ restoration in QCD at finite temperature, combining lattice results, symmetry arguments, and effective field theories.

While chiral symmetry is effectively restored around T_c through partner degeneration, consistent with an $O(4)$ pattern for physical quark masses, the fate of $\mathrm{U(1)_A}$ is more subtle; lattice results for $N_f = 2+1$ indicate that it remains broken near T_c, while $N_f = 2$ studies suggest a possible effective restoration. This highlights the role of quark masses, particularly the strange quark.

Ward identities relate susceptibilities, condensates, and the anomalous sector of QCD, providing robust constraints on symmetry restoration. Chiral Perturbation Theory complements this picture by describing the thermal behaviour of hadronic observables near the transition. Our results indicate that exact chiral symmetry restoration implies an effective restoration of $\mathrm{U(1)_A}$. At the physical point, however, explicit chiral symmetry breaking and the strange-quark sector enhance $\mathrm{U(1)_A}$ breaking, accounting for the differences observed between two- and three-flavour lattice results.

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