UNQUENCHED RADIALLY EXCITED P-WAVE CHARMONIA*

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The ground-state positive-parity charmonia $\chi_{c0}(1P)$, $\chi_{c1}(1P)$, $h_c(1P)$, and $\chi_{c2}(1P)$ are generally well described in static ("quenched") quark models, in which dynamical effects of actual or virtual strong decay are neglected. In contrast, the five PDG candidates for P-wave charmonia in the energy region of 3.85–3.95 GeV, probably including the first radial excitations of the above ones, display a totally different and quite disparate mass pattern. Moreover, two scalar states are listed, *viz.* $\chi_{c0}(3860)$ and $\chi_{c0}(3915)$, the former one apparently being very broad. Preliminary results will be presented here for the first radial excitations of the lowest P-wave $c\bar{c}$ states, obtained with the Resonance-Spectrum Expansion while including in the calculation all OZI-allowed decay channels of the most relevant charm-meson pairs. Employing a generalised scheme of computing coupling constants for decays based on the $^3P_0$ model ensures that no distortion of the spectra will occur due to the different classes of allowed decay channels for the various positive-parity charmonia.

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1. Introduction

Charmonium spectroscopy is undoubtedly the most fertile testing ground for QCD-inspired quark models in view of the many observed states [1], which allow one to fine-tune model parameters and make further predictions, also in the bottomonium sector. In particular, a good description of the lowest positive-parity charmonia $\chi_{c0}(1P)$, $\chi_{c1}(1P)$, $h_c(1P)$, and $\chi_{c2}(1P)$ with a spin-independent scalar linear-plus-Coulombic confining potential is found, provided it is complemented with perturbative spin–spin, spin–orbit, and tensor forces [2, 3]. The status of the five PDG [1] candidates for radially excited P-wave charmonia $\chi_{c0}(3860)$, $\chi_{c1}(3872)$, $\chi_{c0}(3915)$, $\chi_{c2}(3930)$, and $X(3940)$ is completely different (see Table 1). For instance, two scalars instead of only one are listed, having wildly disparate decay widths. Also,

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Table 1. $2P$ charmonia candidates with listed [1] mass, width, and decays.

PDG entry	$I^G\,(J^{PC})$	M [MeV]	Γ [MeV]	(Main) Decays
$\boldsymbol{\chi_{c0}(3860)}$	$0^+(0^{++})$	3862^{+26+40}_{-32-13}	$201^{+154+88}_{-67-82}$	$D^0\bar{D}^0$, D^+D^-
$\boldsymbol{\chi_{c1}(3872)}$	$0^+(1^{++})$	3871.64 ± 0.06	1.19 ± 0.21	$D^0\bar{D}^0\pi^0$, $\bar{D}^{*0}D^0$
$\boldsymbol{\chi_{c0}(3915)}$	$0^+(0^{++})$	3922.1 ± 1.8	20 ± 4	D^+D^-, $D_s^+D_s^-$, $\omega J/\psi$
$\boldsymbol{\chi_{c2}(3930)}$	$0^+(2^{++})$	3922.5 ± 1.0	35.2 ± 2.2	D^+D^-, $D^0\bar{D}^0$
$\boldsymbol{X(3940)}$	$?^?(?^{??})$	3942 ± 9	43^{+28}_{-18}	D^+D^-, $D\bar{D}^* + \text{c.c.}$

$\chi_{c1}(3872)$ is surprisingly lighter than $\chi_{c0}(3915)$. Finally, if $X(3940)$ is the $2\,{}^1P_1$ state, as compatible with its observed [1] $D\bar{D}^\star$ decay, being heavier than $\chi_{c2}(3930)$ is equally unexpected. We can compare these irregularities to the situation in bottomonium, where the corresponding $2P$ states all lie below the thresholds of open-bottom meson pairs. Writing symbolically $\chi_{bi}(n)$ for $M(\chi_{bi}(nP))$ and $h_b(n)$ for $M(h_b(nP))$, we get [1] the following ratios of P-wave mass splittings:

$$\frac{\chi_{b1}(2)-\chi_{b0}(2)}{\chi_{b1}(1)-\chi_{b0}(1)} = 0.69\,, \quad \frac{h_b(2)-\chi_{b1}(2)}{h_b(1)-\chi_{b1}(1)} = 0.67\,, \quad \frac{\chi_{b2}(2)-h_b(2)}{\chi_{b2}(1)-h_b(1)} = 0.69\,.$$

These numbers show a remarkably regular pattern of mass-splitting reductions for the first P-wave $b\bar{b}$ radial excitations, which may very well be the consequence of a node in the corresponding meson wave functions. In contrast, the equivalent ratios among P-wave charmonia are

$$\frac{\chi_{c1}(2)-\chi_{c0}(2)}{\chi_{c1}(1)-\chi_{c0}(1)} = -0.53\,, \quad \frac{h_c(2)-\chi_{c1}(2)}{h_c(1)-\chi_{c1}(1)} = 4.79\,, \quad \frac{\chi_{c2}(2)-h_c(2)}{\chi_{c2}(1)-h_c(1)} = -0.63\,.$$

So clearly, the fact that the $2P$ charmonium states all lie *above* their lowest open-charm thresholds completely changes the picture, resulting in complex mass shifts and threshold effects not governed by simple formulae.

Before investigating such phenomena explicitly for all $2P$ charmonia, I shall first review some old results for the $\chi_{c1}(3872)$ and $\chi_{c0}(3915)$ states.

2. $\boldsymbol{\chi_{c0}(3915)}$ and $\boldsymbol{\chi_{c1}(3872)}$

In order to determine the nature of mesonic resonances in unitarised models, pole trajectories as a function of coupling constant are often studied (see *e.g.* some examples in Ref. [4] and the $\chi_{c1}(3872)$ case below). However, much can be learned as well from trajectories as a function of quark and decay-product masses. In Ref. [5], a very simple model of this type was employed to compute bound-state and resonance mass-width trajectories of a dynamical, ground-state, and radially excited strange scalar meson. This allowed one to directly connect a variety of scalar mesons to one another, including the $K_0^\star(700)$ and the $\chi_{c0}(3915)$ [1], the latter resonance having first been observed by the Belle Collaboration in 2004 [6] at a mass of 3943 ± 11 MeV and found at 3946 MeV in Ref. [5]. The corresponding trajectory is depicted in Fig. 1, together with several others. Concerning coupling-constant pole trajectories, in Ref. [7] the $\chi_{c1}(3872)$ meson was studied in a simple unitarised model. Besides computing and plotting its

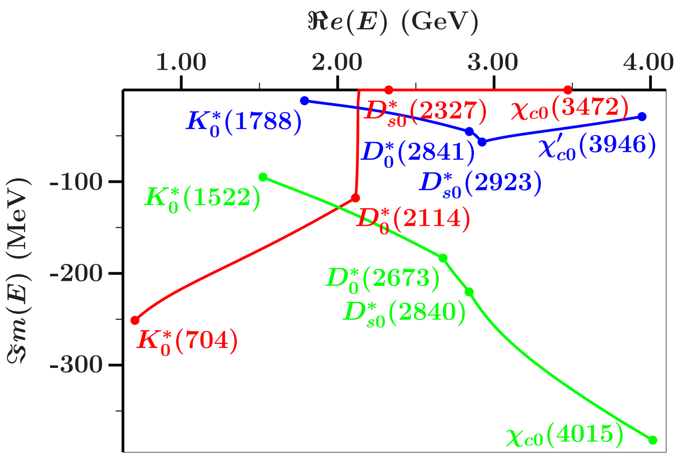

Fig. 1. Masses *versus* widths of scalar mesons as a function of quark and decay-product masses. Figure reprinted from Ref. [5] (`arXiv` version).

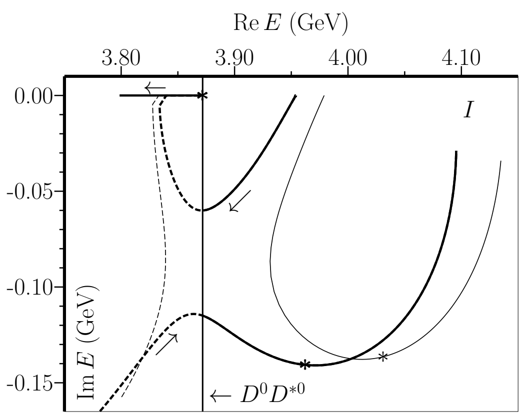

Fig. 2. Pole trajectories as a function of decay coupling in a simple r-space model. Boldface curves: $\chi_{c1}(3872)$ as a mass-shifted intrinsic P-wave $c\bar{c}$ state; other curves: dynamical state. Reprinted from Ref. [7].

two-component wave function for varying parameters, it was also shown that a small parameter change can make the state become a dynamical resonance instead of an intrinsic one, as depicted in Fig. 2. Finally, I recall the $\chi_{c1}(3872)$ wave function as obtained in a more realistic unitarised model [8], with different classes of decay channels, as shown in Fig. 3. One can see that at large r values, the $\bar{D}^0 D^{*0}$ component dominates just like the overall probability, but in the interior region, the state is predominantly $c\bar{c}$.

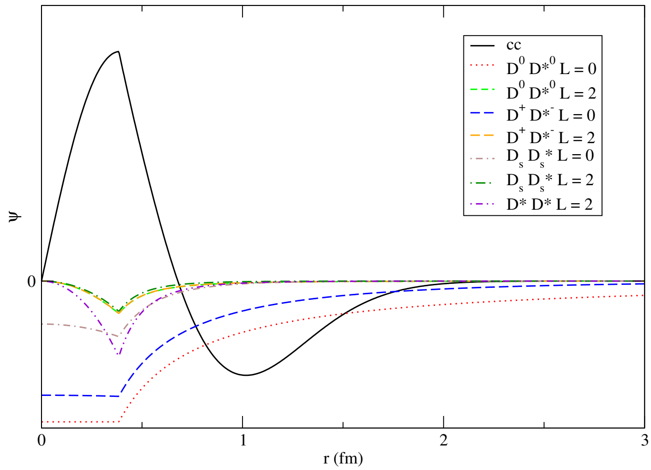

Fig. 3. $\chi_{c1}(3872)$ wave function in a multichannel model; reprinted from Ref. [8].

3. Charmonia with $J^{PC} = 0^{++}$, 1^{++}, 1^{+-}, 2^{++} in the RSE model

Now I will proceed with the actual calculation of $2P$ charmonia employing the Resonance-Spectrum Expansion (RSE) [9], very similarly to the $\chi_{c1}(3872)$ modelling in Ref. [10], the main difference being here the additional inclusion of the $D_s^* \bar{D}_s^*$ channel. For the multichannel T-matrix in the RSE approach, see Appendix A of Ref. [10]. A crucial point in simultaneously studying resonances with different quantum numbers is to ensure that no spectrum distortions will occur owing to including different sets of decay channels. This can be guaranteed by employing the formalism for computing decay couplings of any ground-state or radially excited meson developed in Ref. [11]. The tables for $^3P_0$, $^3P_1$, $^1P_1$, and $^3P_2$ charmonia together with the squares of their couplings to the considered two-meson decay channels are presented in Tables 2 and 3. Indeed, the sums of the squares of the ground state $g_i^{(0)}$ are equal to $1/3$ in all four cases. The same equal sum of squares also holds for arbitrary n, except for $(n{+}1)^3P_2$ charmonia, which is probably related to mixing with the n^3F_2 states (see below).

Table 2. Decay couplings squared of χ_{c0} (left part) and χ_{c1} (right part) states.

$\boldsymbol{\chi_{c0}}$	L	$\left[g_i^{(0)}\right]^2$	$\left[g_i^{(n)}\right]^2 \times 4^n$	$\boldsymbol{\chi_{c1}}$	L	$\left[g_i^{(0)}\right]^2$	$\left[g_i^{(n)}\right]^2 \times 4^n$
$D\bar{D}$	0	1/36	$(n{+}1)/36$	$D^0\bar{D}^{*0}$	0	1/54	$(n{+}1)/54$
$D_s\bar{D}_s$	0	1/72	$(n{+}1)/72$	$D^\pm\bar{D}^{*\mp}$	0	1/54	$(n{+}1)/54$
$D^*\bar{D}^*$	0	1/108	$(n{+}1)/108$	$D_s\bar{D}_s^*$	0	1/54	$(n{+}1)/54$
$D_s^*\bar{D}_s^*$	0	1/216	$(n{+}1)/216$	$D^0\bar{D}^{*0}$	2	5/216	$(2n{+}5)/216$
$D^*\bar{D}^*$	2	5/27	$(2n{+}5)/27$	$D^\pm\bar{D}^{*\mp}$	2	5/216	$(2n{+}5)/216$
$D_s^*\bar{D}_s^*$	2	5/54	$(2n{+}5)/54$	$D_s\bar{D}_s^*$	2	5/216	$(2n{+}5)/216$
				$D^*\bar{D}^*$	2	5/36	$(2n{+}5)/36$
				$D_s^*\bar{D}_s^*$	2	5/72	$(2n{+}5)/72$

Table 3. Decay couplings squared of h_c (left part) and χ_{c2} (right part) states.

$\boldsymbol{h_c}$	L	$\left[g_i^{(0)}\right]^2$	$\left[g_i^{(n)}\right]^2 \times 4^n$	$\boldsymbol{\chi_{c2}}$	L	$\left[g_i^{(0)}\right]^2$	$\left[g_i^{(n)}\right]^2 \times 4^n$
$D\bar{D}^*$	0	1/54	$(n+1)/54$	$D^*\bar{D}^*$	0	1/27	$(n{+}1)/27$
$D_s\bar{D}_s^*$	0	1/108	$(n+1)/108$	$D_s^*\bar{D}_s^*$	0	1/54	$(n{+}1)/54$
$D^*\bar{D}^*$	0	1/54	$(n+1)/54$	$D\bar{D}$	2	1/36	$(2n{+}5)/180$
$D_s\bar{D}_s^*$	0	1/108	$(n+1)/108$	$D_s\bar{D}_s$	2	1/72	$(2n{+}5)/360$
$D\bar{D}^*$	2	5/54	$(2n+5)/54$	$D\bar{D}^*$	2	1/12	$(2n{+}5)/60$
$D_s\bar{D}_s^*$	2	5/108	$(2n+5)/108$	$D_s\bar{D}_s^*$	2	1/24	$(2n{+}5)/120$
$D^*\bar{D}^*$	2	5/54	$(2n+5)/54$	$D^*\bar{D}^*$	2	2/27	$(n{+}1)/108+$ $7(2n{+}5)/540$
$D_s^*\bar{D}_s^*$	2	5/108	$(2n+5)/108$	$D_s^*\bar{D}_s^*$	2	1/27	$(n{+}1)/216+$ $7(2n{+}5)/1080$

The preliminary results of this calculation should not be taken at face value regarding the precise numbers, in view of the omission so far of spin–orbit and tensor splittings, but rather as an indication of unitarisation effects for the different $2P$ $c\bar{c}$ states. Masses of the found poles (in MeV) are

$$
{}^3P_0\colon \begin{cases} 3871.4 - i \times 89.5\,, \\ 3900.5 - i \times 36.5\,, \end{cases} \qquad {}^3P_1\colon 3871.5 - i \times 0.7\,,
$$

$$
{}^1P_1\colon \quad 3877.0 - i \times 3.0\,, \qquad {}^3P_2\colon 3892.1 - i \times 0.3\,.
$$

These numbers are obtained for the overall coupling $\lambda = 3.1$ and the decay radius $r = 3.0$ GeV^{-1}, so are equal or very close to the values used in Ref. [10]. Note that results are generally not very sensitive to r, contrary to λ, which parameter is nonetheless not at odds with prior work [4]. Thus, the $\chi_{c1}(3872)$ pole comes out almost on top of the average experimental result [1], while the $\chi_{c0}(3871.4 - i \times 89.5)$ pole is compatible with a $\chi_{c0}(3860)$ [1] having a large width $\sim$200 MeV, albeit with enormous error bars. But more important than the precise model numbers is the fact that two scalar $c\bar{c}$ resonances are found in this energy region. As for the present $h_c(2P)$ and $\chi_{c2}(2P)$ results, it seems hard to explain the $X(3940)$ [1] without also considering spin–orbit and tensor splittings, besides possible ${}^3P_2/{}^3F_2$ mixing [12]. Especially the scalar and tensor states may undergo sizeable shifts of their bare masses, of negative or positive sign, respectively, albeit of largely unknown magnitude for radial excitations. These extensions are already being studied in detail [13], including a careful tracking of poles in the complex energy and momentum planes in order to determine their nature, as either dominantly intrinsic or dynamically generated positive-parity $c\bar{c}$ resonances.

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