

ANOMALOUS GLUE, η AND η' MESONS*

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Axial U(1) dynamics are characterised by large OZI violations. Here we review the phenomenology of η and η' production and decay processes, and its connection to the anomalous glue that generates a large part of the masses of these pseudoscalar mesons.

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1. Introduction

The flavour-singlet $J^P = 1^+$ channel is characterised by large OZI violation: the masses of the η and η' mesons are much greater than the values they would have if these mesons were pure Goldstone bosons associated with spontaneously broken chiral symmetry [1]. This extra mass is induced by non-perturbative gluon dynamics and the QCD axial anomaly [2]. How is this anomalous glue manifest in η and η' production and decay processes and in their interactions with nuclear matter? These processes are being studied in experiments from threshold [3] through to high-energy collisions where anomalously large branching ratios have been observed for D_s and B -meson decays to an η' plus additional hadrons [4, 5]. The QCD axial anomaly is also important in discussion of the proton spin puzzle [6].

Here we outline the key issues.

2. QCD considerations

Spontaneous chiral symmetry breaking in QCD is associated with a non-vanishing chiral condensate

$$\langle \text{vac} | \bar{q}q | \text{vac} \rangle < 0. \quad (1)$$

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The non-vanishing chiral condensate also spontaneously breaks the axial U(1) symmetry so, naively, in the two-flavour theory one expects an isosinglet pseudoscalar degenerate with the pion. The lightest mass isosinglet is the η meson, which has a mass of 547.75 MeV.

The puzzle deepens when one considers SU(3). Spontaneous chiral symmetry breaking suggests an octet of would-be Goldstone bosons: the octet associated with chiral SU(3)_L⊗SU(3)_R plus a singlet boson associated with axial U(1) — each with mass squared $m_{\text{Goldstone}}^2 \sim m_q$. The physical η and η' masses are about 300–400 MeV too big to fit in this picture. One needs extra mass in the singlet channel associated with non-perturbative topological gluon configurations and the QCD axial anomaly. The strange quark mass induces considerable η – η' mixing. For free mesons the η – η' mass matrix (at leading order in the chiral expansion) is

$$M^2 = \begin{pmatrix} \frac{4}{3}m_K^2 - \frac{1}{3}m_\pi^2 & -\frac{2}{3}\sqrt{2}(m_K^2 - m_\pi^2) \\ -\frac{2}{3}\sqrt{2}(m_K^2 - m_\pi^2) & [\frac{2}{3}m_K^2 + \frac{1}{3}m_\pi^2 + \tilde{m}_{\eta_0}^2] \end{pmatrix}. \quad (2)$$

Here $\tilde{m}_{\eta_0}^2$ is the gluonic mass term which has a rigorous interpretation through the Witten–Veneziano mass formula [7, 8] and which is associated with non-perturbative gluon topology, related perhaps to confinement [9] or instantons [10]. The masses of the physical η and η' mesons are found by diagonalizing this matrix, *viz.*

$$\begin{aligned} |\eta\rangle &= \cos\theta |\eta_8\rangle - \sin\theta |\eta_0\rangle, \\ |\eta'\rangle &= \sin\theta |\eta_8\rangle + \cos\theta |\eta_0\rangle, \end{aligned} \quad (3)$$

where

$$\eta_0 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}), \quad \eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}). \quad (4)$$

One obtains values for the η and η' masses:

$$\begin{aligned} m_{\eta',\eta}^2 &= \left(m_K^2 + \frac{\tilde{m}_{\eta_0}^2}{2} \right) \\ &\pm \frac{1}{2} \sqrt{(2m_K^2 - 2m_\pi^2 - \frac{1}{3}\tilde{m}_{\eta_0}^2)^2 + \frac{8}{9}\tilde{m}_{\eta_0}^4}. \end{aligned} \quad (5)$$

The physical mass of the η and the octet mass $m_{\eta_8} = \sqrt{\frac{4}{3}m_K^2 - \frac{1}{3}m_\pi^2}$ are numerically close, within a few percent. However, to build a theory of the η on the octet approximation risks losing essential physics associated with the

singlet component. Turning off the gluonic term, one finds the expressions $m_{\eta'} \sim \sqrt{2m_K^2 - m_\pi^2}$ and $m_\eta \sim m_\pi$. That is, without extra input from glue, in the OZI limit, the η would be approximately an isosinglet light-quark state $((1/\sqrt{2})|\bar{u}u + \bar{d}d\rangle)$ degenerate with the pion and the η' would be a strange-quark state $|\bar{s}s\rangle$ — mirroring the isoscalar vector ω and ϕ mesons.

Taking the value $\tilde{m}_{\eta_0}^2 = 0.73 \text{ GeV}^2$ in the leading-order mass formula, Eq. (5), gives agreement with the physical masses at the 10% level. This value is obtained by summing over the two eigenvalues in Eq. (5): $m_\eta^2 + m_{\eta'}^2 = 2m_K^2 + \tilde{m}_{\eta_0}^2$ and substituting the physical values of m_η , $m_{\eta'}$ and m_K [8]. The corresponding η - η' mixing angle $\theta \simeq -18^\circ$ is within the range -17° to -20° obtained from a study of various decay processes in [4, 11]¹. The key point of Eq. (5) is that mixing and gluon dynamics play a crucial role in both the η and η' masses.

3. The axial anomaly and $\tilde{m}_{\eta_0}^2$

The flavour-singlet part of η and η' mesons couples to the flavour-singlet axial-vector current $J_{\mu 5}$

$$J_{\mu 5} = (\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s) . \quad (6)$$

In classical field theory this current would be the partially conserved Noether current associated with axial U(1) symmetry. In QCD renormalization effects mean that $J_{\mu 5}$ satisfies the anomalous divergence equation

$$\partial^\mu J_{\mu 5} = 6\partial^\mu K_\mu + \sum_{k=1}^3 2im_k \bar{q}_k \gamma_5 q_k , \quad (7)$$

where

$$K_\mu = \frac{g^2}{32\pi^2} \varepsilon_{\mu\nu\rho\sigma} \left[A_a^\nu \left(\partial^\rho A_a^\sigma - \frac{1}{3} g f_{abc} A_b^\rho A_c^\sigma \right) \right] \quad (8)$$

is the gluonic Chern–Simons current. Here A_a^μ is the gluon field and

$$Q = \partial^\mu K_\mu = \frac{g^2}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu} \quad (9)$$

is the (gauge-invariant) topological charge density, $G_{\mu\nu}$ is the gluon field tensor and $\tilde{G}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta} G_{\alpha\beta}$. Its integral over space $\int d^4z Q = n$ measures

¹ Closer agreement with the physical masses can be obtained by introducing the singlet decay constant $F_0 \neq F_\pi$ and including higher-order mass terms in the chiral expansion [12, 13].

the gluonic winding number [14], which is an integer for (anti-)instantons and which vanishes in perturbative QCD. Eq. (7) allows us to define a partially conserved current $J_{\mu 5} = J_{\mu 5}^{\text{con}} + 2fK_\mu$, *viz.* $\partial^\mu J_{\mu 5}^{\text{con}} = \sum_{i=1}^3 2im_i \bar{q}_i \gamma_5 q_i$. When we make a gauge transformation U the gluon field transforms as

$$A_\mu \rightarrow UA_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}$$

and the operator K_μ transforms as

$$\begin{aligned} K_\mu \rightarrow & K_\mu + i\frac{g}{8\pi^2} \varepsilon_{\mu\nu\alpha\beta} \partial^\nu \left(U^\dagger \partial^\alpha U A^\beta \right) \\ & + \frac{1}{24\pi^2} \varepsilon_{\mu\nu\alpha\beta} \left[(U^\dagger \partial^\nu U)(U^\dagger \partial^\alpha U)(U^\dagger \partial^\beta U) \right]. \end{aligned} \quad (10)$$

In general, matrix elements of K_μ are gauge dependent. This means that one has to be careful writing matrix elements of $J_{\mu 5}$ as the sum of (measurable) “quark” and “gluonic” contributions.

3.1. The $U(1)$ effective Lagrangian for low-energy QCD

Independent of the detailed QCD dynamics one can construct low-energy effective chiral Lagrangians which include the effect of the anomaly and axial $U(1)$ symmetry, and use these Lagrangians to study low-energy processes involving the η and η' .

The physics of axial $U(1)$ degrees of freedom is described by the $U(1)$ -extended low-energy effective Lagrangian [8]. In its simplest form this reads

$$\begin{aligned} \mathcal{L} = & \frac{F_\pi^2}{4} \text{Tr}(\partial^\mu U \partial_\mu U^\dagger) + \frac{F_\pi^2}{4} \text{Tr} M (U + U^\dagger) \\ & + \frac{1}{2} iQ \text{Tr} [\log U - \log U^\dagger] + \frac{3}{\tilde{m}_{\eta_0}^2 F_0^2} Q^2. \end{aligned} \quad (11)$$

Here $U = \exp i(\phi/F_\pi + \sqrt{\frac{2}{3}}\eta_0/F_0)$ is the unitary meson matrix where $\phi = \sum \pi_a \lambda_a$ denotes the octet of would-be Goldstone bosons associated with spontaneous chiral $SU(3)_L \otimes SU(3)_R$ breaking and η_0 is the singlet boson. In Eq. (11) Q denotes the topological charge density; $M = \text{diag}[m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2]$ is the quark-mass induced meson mass matrix. The pion decay constant $F_\pi = 92.4 \text{ MeV}$ and F_0 is the flavour-singlet decay constant, $F_0 \sim F_\pi \sim 100 \text{ MeV}$ [11].

The flavour-singlet potential involving Q is introduced to generate the gluonic contribution to the η and η' masses and to reproduce the anomaly in the divergence of the gauge-invariantly renormalised flavour-singlet axial-vector current. The gluonic term Q is treated as a background field with

no kinetic term. It may be eliminated through its equation of motion to generate a gluonic mass term for the singlet boson, *viz.*

$$\frac{1}{2}iQ\text{Tr} \left[\log U - \log U^\dagger \right] + \frac{3}{\tilde{m}_{\eta_0}^2 F_0^2} Q^2 \mapsto -\frac{1}{2}\tilde{m}_{\eta_0}^2 \eta_0^2. \quad (12)$$

The most general low-energy effective Lagrangian involves a $U_A(1)$ invariant polynomial in Q^2 . Higher-order terms in Q^2 become important when we consider scattering processes involving more than one η' [15]. In general, couplings involving Q give OZI violation in physical observables.

4. Light-cone wavefunctions and fragmentation functions

In general, there are gluonic effects in η and η' phenomenology associated with the gluonic potential involving the topological charge density in the $U(1)$ -extended effective chiral Lagrangian for low energy QCD, OZI violation in the intermediate states of reactions involving flavour-singlet hadrons, and gluonic Fock components in the η and η' light-cone wavefunctions. At a theoretical level, technical issues include separating leading contributions associated with matrix elements of the singlet axial vector current $\bar{\psi}\gamma_\mu\gamma_5\psi$ and higher twist effects associated with $J^P = 1^+$ gauge invariant gluonic operators like $G_{\alpha\beta}iD_\mu\tilde{G}^{\alpha\beta}$ in the definition of the η' (light-cone) wavefunction². In the first case gluonic effects enter through the topological charge density in the anomalous divergence of the singlet current and in matrix elements involving the gauge-dependent anomalous Chern–Simons current K_μ making any quark–gluon separation subtle and, where meaningful, should be defined with respect to a certain choice of gauge. (In perturbation theory and in the light-cone gauge the forward matrix elements of K_μ are invariant under residual gauge degrees of freedom, allowing one to connect these matrix elements with polarised glue in the QCD parton model [6, 16]. The matrix elements of K_μ are gauge dependent as soon as one moves away from the forward direction.)

Consider the (leading twist) light-cone wavefunctions of the η and η' mesons [13, 17]. For the meson P (η or η'), let $\Psi_P^i(x, \vec{k}_\perp)$ denote the amplitude for finding a quark–antiquark pair carrying light-cone momentum fraction x and $(1-x)$ and transverse momentum \vec{k}_\perp ; i denotes the $SU(3)$ octet or singlet ($i = 8$ or 0) component of the wavefunction. These amplitudes are normalised via

$$\int \frac{d^2\vec{k}_\perp}{16\pi^3} \int_0^1 dx \Psi_P^i(x, \vec{k}_\perp) = \frac{f_P^i}{2\sqrt{6}}, \quad (13)$$

² There is no gauge-invariant twist-2, spin-one gluonic operator with $J^P = 1^+$.

where

$$\langle \text{vac} | J_{\mu 5}^i | P(p) \rangle = i f_P^i p_\mu \quad (14)$$

with f_P^i the corresponding decay constants [13,17]. Gauge dependence issues arise immediately that one tries to separate a “ K_μ contribution” from matrix elements of the singlet current $J_{\mu 5}$. If we calculate the hard (perturbative QCD) part of an η or η' production or decay process using a gauge-invariant scheme like $\overline{\text{MS}}$, then the anomalous glue associated with the QCD axial anomaly will be included in the “quark–antiquark part” of the η or η' wavefunction with the quark–antiquark pair feeling the effect of the OZI violating gluonic potential associated with Q and (possible) strong coupling to glue in the intermediate state of the reaction. The η – η' mixing angle is built into the light-cone wavefunction. Separate to glue associated with the QCD axial anomaly, one might also consider mixing with the lightest mass 0^- glueball. Possible candidates for this state include the $\eta(1405)$ and a glueball predicted by lattice QCD with mass above 2 GeV [18]. Studies of possible gluonic components in the meson wavefunctions have been carried out in fits to data on exclusive η and η' production and decay processes [13, 19].

Semi-inclusive η production in high-energy collisions has been a topical issue since the pioneering work of Field and Feynman [20]. One finds the interesting result that the ratio of η to π^0 production rises rapidly with the transverse momentum of the produced meson and levels off at $R_{\eta/\pi^0} \sim 0.4$ – 0.5 above $p_t \sim 3$ GeV in hadron–hadron collisions (proton–proton, proton–ion and ion–ion) independent of the colliding hadron species [21], consistent with the expectations from string fragmentation models. Studies of η and η' production in hadron jets at LEP were performed [22]. While the L3 analysis claims to observe an excess of η production in gluon jets, neither OPAL nor ALEPH found an excess. The ratio of η to π^0 multiplicities in quark and gluon jets was measured over the range $x = E/E_{\text{beam}}$ between 0.1 and 0.5. Good fits to these ratios are $R_{\eta/\pi^0} = 1.1x^{0.94}$ in quark jets and $3.4x^{1.01}(1-x)$ in gluon jets over the measured region. η' production was observed to be anomalously suppressed compared to the expectations of string fragmentation models without an additional “ η' suppression factor”, possibly associated with the mass of the produced η' .

5. Low energy η and η' hadron interactions

5.1. Light-mass exotic meson production

The interactions of the η and η' with other mesons and with nucleons can be studied by coupling the Lagrangian Eq. (11) to other particles. For example, the OZI violating interaction $\lambda Q^2 \partial_\mu \pi_a \partial^\mu \pi_a$ is needed to generate the leading (tree-level) contribution to the decay $\eta' \rightarrow \eta \pi \pi$ [15]. When

iterated in the Bethe–Salpeter equation for meson–meson rescattering this interaction yields a dynamically generated exotic state with quantum numbers $J^{PC} = 1^{-+}$ and mass about 1400 MeV [23]. This suggests a dynamical interpretation of the lightest-mass 1^{-+} exotic observed at BNL [24] and CERN [25].

5.2. Proton–nucleon collisions

For proton–nucleon collisions one finds a gluon-induced contact interaction in the $pp \rightarrow pp\eta'$ reaction [26]:

$$\mathcal{L}_{\text{contact}} = -\frac{i}{F_0^2} g_{QNN} \tilde{m}_{\eta_0}^2 \mathcal{C}_{\eta_0} (\bar{p}\gamma_5 p) (\bar{p}p). \quad (15)$$

Here g_{QNN} is the 1PI coupling of Q to the nucleon and \mathcal{C} is a second OZI violating coupling. The physical interpretation of the contact term (15) is a “short distance” ($\sim 0.2\text{fm}$) interaction where glue is excited in the interaction region of the proton–proton collision and then evolves to become an η' in the final state. This gluonic contribution to the cross-section for $pp \rightarrow pp\eta'$ is extra to the contributions associated with meson exchange models. There is no reason, *a priori*, to expect it to be small. Since glue is flavour-blind the contact interaction (15) has the same size in both the $pp \rightarrow pp\eta'$ and $pn \rightarrow pn\eta'$ reactions. The ratio $R_\eta = \sigma(pn \rightarrow pn\eta)/\sigma(pp \rightarrow pp\eta)$ has been measured for quasi free η production from a deuteron target up to 100 MeV above threshold [27]. One finds that R_η is approximately energy-independent ~ 6.5 over the energy range 20–100 MeV signifying a strong isovector exchange contribution to the η production mechanism. In the extreme scenario that the glue-induced production saturated the η' production cross-section, the ratio $R_{\eta'} = \sigma(pn \rightarrow pn\eta')/\sigma(pp \rightarrow pp\eta')$ would go to one after we correct for the final state interaction between the two outgoing nucleons. Proton–proton data is available from COSY-11 [3]; the proton–neutron process has been measured and the data is being analysed [28].

5.3. η and η' interactions with the nuclear medium

Measurements of the pion, kaon and eta meson masses and their interactions in finite nuclei provide new constraints on our understanding of dynamical symmetry breaking in low energy QCD [29]. For the η the in-medium mass m_η^* is sensitive to the flavour-singlet component in the η , and hence to the non-perturbative glue associated with axial U(1) dynamics. An important source of the in-medium mass modification comes from light-quarks coupling to the scalar σ mean-field in the nucleus. Increasing the flavour-singlet component in the η at the expense of the octet component

gives more attraction, more binding and a larger value of the η -nucleon scattering length, $a_{\eta N}$. Since the mass shift is approximately proportional to the η -nucleon scattering length, it follows that the physical value of $a_{\eta N}$ should be larger than if the η were a pure octet state.

Meson masses in nuclei are determined from the scalar induced contribution to the meson propagator evaluated at zero three-momentum, $\vec{k} = 0$, in the nuclear medium. Let $k = (E, \vec{k})$ and m denote the four-momentum and mass of the meson in free space. Then, one solves the equation

$$k^2 - m^2 = \text{Re} \Pi(E, \vec{k}, \rho) \quad (16)$$

for $\vec{k} = 0$ where Π is the in-medium s -wave meson self-energy. Contributions to the in-medium mass come from coupling to the scalar σ field in the nucleus in mean-field approximation, nucleon-hole and resonance-hole excitations in the medium. The s -wave self-energy can be written as [30]

$$\Pi(E, \vec{k}, \rho) \Big|_{\{\vec{k}=0\}} = -4\pi\rho \left(\frac{b}{1 + b\langle 1/r \rangle} \right). \quad (17)$$

Here ρ is the nuclear density, $b = a(1 + m/M)$ where a is the meson-nucleon scattering length, M is the nucleon mass and $\langle 1/r \rangle$ is the inverse correlation length, $\langle 1/r \rangle \simeq m_\pi$ for nuclear matter density [30]. (m_π is the pion mass.) Attraction corresponds to positive values of a . The denominator in Eq. (17) is the Ericson-Ericson-Lorentz-Lorenz double scattering correction.

What should we expect for the η and η' ?

To investigate what happens to $\tilde{m}_{\eta_0}^2$ in the medium we first couple the σ (correlated two-pion) mean-field in nuclei to the topological charge density Q by adding the Lagrangian term

$$\mathcal{L}_{\sigma Q} = Q^2 g_\sigma^Q \sigma, \quad (18)$$

where g_σ^Q denotes coupling to the σ mean field — that is, we consider an in-medium renormalization of the coefficient of Q^2 in the effective chiral Lagrangian [31]. We can eliminate Q through its equation of motion (following Eq. (12)). The gluonic mass term for the singlet boson then becomes

$$\tilde{m}_{\eta_0}^2 \mapsto \tilde{m}_{\eta_0}^{*2} = \tilde{m}_{\eta_0}^2 \frac{1 + 2x}{(1 + x)^2} < \tilde{m}_{\eta_0}^2, \quad (19)$$

where

$$x = \frac{1}{3} g_\sigma^Q \sigma \tilde{m}_{\eta_0}^2 F_0^2. \quad (20)$$

That is, *the gluonic mass term decreases in-medium* independent of the sign of g_σ^Q and the medium acts to partially neutralise axial U(1) symmetry breaking by gluonic effects.

The above discussion is intended to motivate the *existence* of medium modifications to $\tilde{m}_{\eta_0}^2$ in QCD³. However, a rigorous calculation of m_η^* from QCD is beyond present theoretical technology. Hence, one has to look to QCD motivated models and phenomenology for guidance about the numerical size of the effect. The physics described in Eqs. (2)–(5) tells us that the simple octet approximation may not suffice.

This physics has been investigated by Bass and Thomas [31]. Phenomenology is used to estimate the size of the effect in the η using the Quark Meson Coupling model (QMC) of hadron properties in the nuclear medium [33]. Here one uses the large η mass (which in QCD is induced by mixing and the gluonic mass term) to motivate taking an MIT Bag description for the η wavefunction, and then coupling the light (up and down) quark and antiquark fields in the η to the scalar σ field in the nucleus working in mean-field approximation [33]. The strange-quark component of the wavefunction does not couple to the σ field and η – η' mixing is readily built into the model.

Increasing the mixing angle increases the amount of singlet relative to octet components in the η . This produces greater attraction through increasing the amount of light-quark compared to strange-quark components in the η and a reduced effective mass. Through Eq. (17), increasing the mixing angle also increases the η -nucleon scattering length $a_{\eta N}$. The model results are shown in Table I. The key observation is that η – η' mixing leads to a factor of two increase in the mass-shift and in the scattering length

TABLE I

Physical masses fitted in free space, the bag masses in medium at normal nuclear-matter density, $\rho_0 = 0.15 \text{ fm}^{-3}$, and corresponding meson–nucleon scattering lengths (calculated at the mean-field level with the Ericson–Ericson–Lorentz–Lorenz factor switched off).

| | m (MeV) | m^* (MeV) | $\text{Re } a$ (fm) |
|-------------------------|-----------|-------------|---------------------|
| η_8 | 547.75 | 500.0 | 0.43 |
| η (-10°) | 547.75 | 474.7 | 0.64 |
| η (-20°) | 547.75 | 449.3 | 0.85 |
| η_0 | 958 | 878.6 | 0.99 |
| η' (-10°) | 958 | 899.2 | 0.74 |
| η' (-20°) | 958 | 921.3 | 0.47 |

³ In the chiral limit the singlet analogy to the Weinberg–Tomozawa term does not vanish because of the anomalous glue terms. Starting from the simple Born term one finds anomalous gluonic contributions to the singlet-meson nucleon scattering length proportional to $\tilde{m}_{\eta_0}^2$ and $\tilde{m}_{\eta_0}^4$ [32].

obtained in the model. This result may explain why values of $a_{\eta N}$ extracted from phenomenological fits to experimental data where the η - η' mixing angle is unconstrained give larger values than those predicted in theoretical models where the η is treated as a pure octet state.

The density dependence of the mass-shifts in the QMC model is discussed in Ref. [33]. Neglecting the Ericson–Ericson term, the mass-shift is approximately linear. For densities ρ between 0.5 and 1 times ρ_0 (nuclear matter density) we find

$$\frac{m_{\eta}^*}{m_{\eta}} \simeq 1 - 0.17 \frac{\rho}{\rho_0} \quad (21)$$

for the mixing angle -20° . The scattering lengths extracted from this analysis are density independent to within a few percent over the same range of densities.

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