η–η′ MIXING FROM $V \to P\gamma$ AND $J/\psi \to VP$ DECAYS*

Rafel Escribano

Grup de Física Teòrica and IFAE, Universitat Autònoma de Barcelona
08193 Bellaterra (Barcelona), Spain

The η–η′ pseudoscalar mixing angle and the gluonium content of the η′ meson are deduced from an updated phenomenological analysis of $V \to P\gamma$ and $J/\psi \to VP$ decays. In absence of gluonium, the value of the mixing angle in the quark-flavour basis is found to be $\phi_P = (41.5 \pm 1.2)°$ from $V \to P\gamma$ and $\phi_P = (40.5 \pm 2.4)°$ from $J/\psi \to VP$. In presence of gluonium, the values for the mixing angle and the gluonic content of the η′ wave function are $\phi_P = (41.4 \pm 1.3)°$ and $Z^2_{\eta'} = 0.04 \pm 0.09$ from $V \to P\gamma$ and $\phi_P = (44.5 \pm 4.3)°$ and $Z^2_{\eta'} = 0.28 \pm 0.21$ from $J/\psi \to VP$, respectively.

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1. Introduction

Is η′ partially made of gluonium? To answer this question we perform a phenomenological analysis of radiative $V \to P\gamma$ and $J/\psi \to VP$ decays, with $V = \rho, K^*, \omega, \phi$ and $P = \pi, K, \eta, \eta'$, in order to determine the gluonic content of the η′ wave function. Similar analyses were driven in the seminal work by Rosner [1], where the allowed gluonic admixture in the η′ could not be established due to the lack of data on $\phi \to \eta'\gamma$, and, later on, by Kou who pointed out that the η′ gluonic component might be as large as 26% [2]. More recently, the study by Thomas over a large number of different processes concludes that while the data hint at a small gluonic component in the η′, the results depend sensitively on unknown form factors associated with exclusive dynamics [3].

From the experimental side, the KLOE Collaboration, combining the new measurement of $R_\phi \equiv B(\phi \to \eta'\gamma)/B(\phi \to \eta\gamma)$ with other constraints, has estimated the gluonium content of the η′ meson as $Z^2_{\eta'} = 0.14 \pm 0.04$ [4]. This new result contrasts with the former value $Z^2_{\eta'} = 0.06^{+0.09}_{-0.06}$, which was compatible with zero and consistent with a gluonium fraction below 15% [5].

A more extensive version of the present work including a detailed analysis also for the case of the η, a comparison with other approaches, and a complete list of references can be found in Refs. [6, 7].

2. Notation

We work in a basis consisting of the states [1]

$$|\eta_q\rangle \equiv \frac{1}{\sqrt{2}}|u\bar{u} + d\bar{d}\rangle\,, \qquad |\eta_s\rangle \equiv |s\bar{s}\rangle\,, \qquad |G\rangle = |\text{gluonium}\rangle\,. \qquad (1)$$

The physical states η and η' are assumed to be the linear combinations

$$|\eta\rangle = X_\eta|\eta_q\rangle + Y_\eta|\eta_s\rangle + Z_\eta|G\rangle\,, \qquad |\eta'\rangle = X_{\eta'}|\eta_q\rangle + Y_{\eta'}|\eta_s\rangle + Z_{\eta'}|G\rangle\,, \quad (2)$$

with $X_{\eta(\eta')}^2 + Y_{\eta(\eta')}^2 + Z_{\eta(\eta')}^2 = 1$. A significant gluonic admixture in a state is possible only if $Z_{\eta(\eta')}^2 = 1 - X_{\eta(\eta')}^2 - Y_{\eta(\eta')}^2 > 0$. The implicit assumptions in Eq. (2) are the following: *(i)* no mixing with π^0 — isospin symmetry, and *(ii)* no mixing with radial excitations or η_c states. Assuming the absence of gluonium for the η, the coefficients $X_{\eta(\eta')}$, $Y_{\eta(\eta')}$ and $Z_{\eta(\eta')}$ are described in terms of two angles (see Ref. [6] for details),

$$\begin{aligned} X_\eta &= \cos\phi_P\,, & X_{\eta'} &= \sin\phi_P\cos\phi_{\eta'G}\,, \\ Y_\eta &= -\sin\phi_P\,, & Y_{\eta'} &= \cos\phi_P\cos\phi_{\eta'G}\,, \\ Z_\eta &= 0\,, & Z_{\eta'} &= -\sin\phi_{\eta'G}\,, \end{aligned} \qquad (3)$$

where ϕ_P is the η–η' mixing angle and $\phi_{\eta'G}$ weights the amount of gluonium in the η' wave-function. For a comprehensive treatment of η–η' mixing in absence of gluonium see Ref. [8].

3. Phenomenological model

3.1. $VP\gamma$ $M1$ transitions

Our model for the $VP\gamma$ $M1$ transitions is based on three characteristic ingredients: *(i)* A $VP\gamma$ magnetic dipole transition proceeds via quark or antiquark spin-flip amplitudes proportional to $\mu_q = e_q/2m_q$. This effective magnetic moment breaks SU(3) in a well defined way and distinguishes photon emission from strange or non-strange quarks via $m_s > \bar{m}$; *(ii)* The spin-flip $V \leftrightarrow P$ conversion amplitude has then to be corrected by the relative overlap between the P and V wave functions [9]; *(iii)* Indeed, the OZI-rule reduces considerably the possible transitions and their respective VP wave-function overlaps: C_s, C_q and C_π characterize the $\langle\eta_s|\phi_s\rangle$, $\langle\eta_q|\omega_q\rangle = \langle\eta_q|\rho\rangle$

and $\langle\pi|\omega_q\rangle = \langle\pi|\rho\rangle$ spatial overlaps, respectively. Notice that distinction is made between the $|\pi\rangle$ and $|\eta_q\rangle$ spatial extension due to the gluon or $U(1)_A$ anomaly.

The relevant $VP\gamma$ couplings are written in terms of a $g \equiv g_{\omega_q\pi\gamma}$ as

$$
\begin{aligned}
g_{\rho\eta^{(\prime)}\gamma} &= g\, z_q\, X_\eta^{(\prime)}, \\
g_{\omega\eta^{(\prime)}\gamma} &= \tfrac{1}{3}g\left(z_q\, X_\eta^{(\prime)}\cos\phi_V + 2\tfrac{\bar{m}}{m_s}z_s\, Y_\eta^{(\prime)}\sin\phi_V\right), \\
g_{\phi\eta^{(\prime)}\gamma} &= \tfrac{1}{3}g\left(z_q\, X_\eta^{(\prime)}\sin\phi_V - 2\tfrac{\bar{m}}{m_s}z_s\, Y_\eta^{(\prime)}\cos\phi_V\right),
\end{aligned}
\tag{4}
$$

where we have redefined $z_q \equiv C_q/C_\pi$ and $z_s \equiv C_s/C_\pi$.

3.2. $J/\psi \to VP$ transitions

The amplitudes for the $J/\psi \to VP$ decays are expressed in terms of an $SU(3)$-symmetric coupling strength g (SOZI amplitude) which comes from a three-gluon annihilation diagram, an electromagnetic coupling strength e (with phase θ_e relative to g) which comes from the electromagnetic interaction diagram [10], an $SU(3)$-symmetric coupling strength which is written by g with suppression factor r contributed from the doubly disconnected diagram (nonet-symmetry-breaking DOZI amplitude) [11], where the vector and the pseudoscalar exchange an extra gluon, and r' which is the relative gluonic production amplitude representing the diagram connected to a pure glueball state. The SU(3) violation is accounted for by a factor $(1-s)$ for

TABLE I

General parametrization of amplitudes for $J/\psi \to VP$ decays.

Process	Amplitude
$\rho\pi$	$g + e$
$K^{*+}K^- +$ c.c.	$g(1-s) + e(2-x)$
$K^{*0}\bar{K}^0 +$ c.c.	$g(1-s) - e(1+x)$
$\omega_q\eta$	$(g+e)X_\eta + \sqrt{2}\,rg[\sqrt{2}X_\eta + Y_\eta] + \sqrt{2}\,r'gZ_\eta$
$\omega_q\eta'$	$(g+e)X_{\eta'} + \sqrt{2}\,rg[\sqrt{2}X_{\eta'} + Y_{\eta'}] + \sqrt{2}\,r'gZ_{\eta'}$
$\phi_s\eta$	$[g(1-2s) - 2ex]Y_\eta + rg[\sqrt{2}X_\eta + Y_\eta] + r'gZ_\eta$
$\phi_s\eta'$	$[g(1-2s) - 2ex]Y_{\eta'} + rg[\sqrt{2}X_{\eta'} + Y_{\eta'}] + r'gZ_{\eta'}$
$\rho\eta$	$3eX_\eta$
$\rho\eta'$	$3eX_{\eta'}$
$\omega_q\pi^0$	$3e$
$\phi_s\pi^0$	0

every strange quark contributing to g and a factor x for a strange quark contributing to e. The general parametrization of amplitudes for $J/\psi \to VP$ decays is written in Table I.

4. Results

4.1. $V \to P\gamma$ analysis

We proceed to fit our theoretical expressions for the amplitudes comparing the available experimental information on $\Gamma(V \to P\gamma)$ and $\Gamma(P \to V\gamma)$ taken exclusively from Ref. [12]. In the following, we leave the z's free and allow for gluonium in the η' wave function only. This will permit us to fix the gluonic content of the η' in a way identical to the experimental measurement by KLOE. However, as a matter of comparison, we first consider the absence of gluonium in both mesons, *i.e.* $\phi_{\eta G} = \phi_{\eta' G} = 0$. The result of the fit gives $\chi^2/\text{d.o.f.} = 4.4/5$ with

$$g = 0.72\pm0.01 \text{ GeV}^{-1}, \quad \phi_P = (41.5\pm1.2)^\circ, \quad \phi_V = (3.2\pm0.1)^\circ,$$
$$\frac{m_s}{\bar{m}} = 1.24\pm0.07, \quad\quad z_q = 0.86\pm0.03 \quad z_s = 0.78\pm0.05. \quad (5)$$

Now we assume $\phi_{\eta G} = 0$, *i.e.* $Z_\eta = 0$ and proceed to fit the gluonic content of the η' wave function under this assumption. The results of the new fit are[1]

$$g = 0.72 \pm 0.01 \text{ GeV}^{-1}, \quad \frac{m_s}{\bar{m}} = 1.24 \pm 0.07, \quad \phi_V = (3.2 \pm 0.1)^\circ,$$
$$\phi_P = (41.4 \pm 1.3)^\circ, \quad\quad |\phi_{\eta' G}| = (12 \pm 13)^\circ,$$
$$z_q = 0.86 \pm 0.03, \quad\quad z_s = 0.79 \pm 0.05, \quad\quad\quad (6)$$

with $\chi^2/\text{d.o.f.} = 4.2/4$. The result obtained for $\phi_{\eta' G}$ suggests a very small amount of gluonium in the η' wave function, $|\phi_{\eta' G}| = (12 \pm 13)^\circ$ or $Z_{\eta'}^2 = 0.04 \pm 0.09$. Our values contrast with those reported by KLOE recently, $\phi_P = (39.7 \pm 0.7)^\circ$ and $|\phi_{\eta' G}| = (22 \pm 3)^\circ$ — or $Z_{\eta'}^2 = 0.14 \pm 0.04$ — see [4].

Our main results can also be displayed graphically following Refs. [1,2,4]. In Fig. 1, we plot the regions for the $X_{\eta'}$ and $Y_{\eta'}$ parameters which are allowed by the experimental couplings of the $\eta' \to \rho\gamma$, $\eta' \to \omega\gamma$ and $\phi \to \eta'\gamma$ transitions. The limits of the bands are given at 68% C.L. or 1σ. The remaining parameters are taken from Eq. (6). In addition to the bands, we have also plotted the circular boundary denoting the constraint $X_{\eta'}^2 + Y_{\eta'}^2 \leq 1$ as well as the favoured region for the η–η' mixing angle assuming the *absence*

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[1] There is a sign ambiguity in $\phi_{\eta' G}$ that cannot be decided since this angle enters into $X_{\eta'}$ and $Y_{\eta'}$ through a cosine.
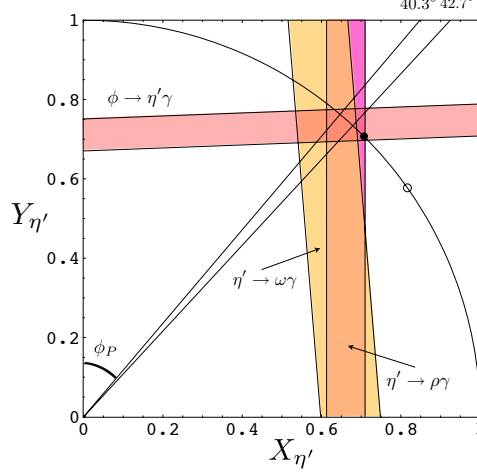
Fig. 1. Constraints on non-strange ($X_{\eta'}$) and strange ($Y_{\eta'}$) quarkonium mixing coefficients in the η'. The vertical and inclined bands are the regions for $X_{\eta'}$ and $Y_{\eta'}$ allowed by the experimental couplings of the $\eta' \to (\rho, \omega)\gamma$ and $\phi \to \eta'\gamma$ transitions.

of gluonium, $40.3° \leq \phi_P \leq 42.7°$, obtained at 1σ from the corresponding fitted value in Eq. (5). There exists an intersection region of the three bands inside and on the circumference. As most of this region is interior but close to the circular boundary it may well indicate a small but non necessarily zero gluonic content of the η'. Indeed, we have found $Z_{\eta'}^2 = 0.04 \pm 0.09$ (or $|Z_{\eta'}| = 0.2 \pm 0.2$) or using the angular description $|\phi_{\eta'G}| = (12 \pm 13)°$.

4.2. $J/\psi \to VP$ analysis

Given the large number of parameters to be fitted, 13 in the most general case for 11 observables (indeed 10 because there is only an upper limit for $\phi\pi^0$), we perform the following simplifications. First, we fix the parameters $x = m_{u,d}/m_s$ and the vector mixing angle ϕ_V to the values obtained from a recent fit to the most precise data on $V \to P\gamma$ decays [6], that is $m_s/m_{u,d} = 1.24 \pm 0.07$ which implies $x = 0.81 \pm 0.05$ and $\phi_V = (3.2 \pm 0.1)°$. Second, we do not allow for gluonium in the η wave function, thus the mixing pattern of η and η' is given by the mixing angle ϕ_P and the coefficient $Z_{\eta'}$ (see Notation).

We proceed to present the results of the fits. To describe data without considering the contribution from the doubly disconnected diagram (terms proportional to rg) has been shown to be unfeasible [13]. Therefore, it is required to take into account nonet-symmetry-breaking effects. We have also tested that it is not possible to get a reasonable fit setting the SU(3)-breaking

correction s to its symmetric value, *i.e.* $s = 0$. If gluonium is not allowed in the η' wave function, the result of the fit gives $\phi_P = (40.4 \pm 2.4)°$ — or $\theta_P = (-14.3 \pm 2.4)°$ — with $\chi^2/$d.o.f. $= 3.6/4$, in disagreement at the 2σ level with $\theta_P = (-19.1 \pm 1.4)°$ [14], $\theta_P = (-19.2 \pm 1.4)°$ [13], and $\theta_P \simeq -20°$ [15], but in correspondence with $\phi_P = (39.9 \pm 2.9)°$ [16] and $\phi_P = (40 \pm 2)°$ [3]. In some analyses, x is kept fixed to one since it always appears multiplying e and hence also considered as a second order contribution. In this case, our fit gives $\phi_P = (40.2 \pm 2.4)°$ with $\chi^2/$d.o.f. $= 3.4/4$. However, none of the former analyses include the effects of a vector mixing angle different from zero. It was already noticed in Ref. [17] that these effects, which were considered there for the first time, turn out to be crucial to find a less negative value of the η–η' mixing angle. If we take now the fitted value $\phi_V = +3.2°$ (see above), one gets $\phi_P = (40.5 \pm 2.4)°$ with $\chi^2/$d.o.f. $= 4.2/4$ and $\phi_P = (40.3 \pm 2.4)°$ with $\chi^2/$d.o.f. $= 3.8/4$ for $x = 0.81$ and $x = 1$, respectively. These new fits seem to refute the strong correlation between the two mixing angles found in Ref. [17]. One interesting feature of the present analysis is the effect produced in the fits by the new averaged value of the $\rho\pi$ branching ratio. For instance, if $B(\rho\pi) = (16.9 \pm 1.5)\%$ [18] is replaced by its old value $(12.8 \pm 1.0)\%$ [19] one gets $\phi_P = (37.7 \pm 1.5)°$ with $\chi^2/$d.o.f. $= 8.8/4$ for $x = 0.81$ and $\phi_V = 3.2°$, *i.e.* the central value and the error of the mixing angle become smaller and the quality of the fit worse. However, this value is now in agreement with that found in Ref. [17].

As stated, the former fits are performed assuming the absence of gluonium in η'. Now, we redo some of the fits accepting a gluonic content in the η' wave function. For $\phi_V = 0$, the values $\phi_P = (44.8 \pm 4.3)°$ and $Z_{\eta'}^2 = 0.29 \pm 0.21$ with $\chi^2/$d.o.f. $= 2.3/2$ and $\phi_P = (45.0 \pm 4.3)°$ and $Z_{\eta'}^2 = 0.30 \pm 0.20$ with $\chi^2/$d.o.f. $= 1.9/2$ are obtained for $x = 0.81$ and $x = 1$, respectively. For $\phi_V = +3.2°$, one gets $\phi_P = (44.5 \pm 4.3)°$ and $Z_{\eta'}^2 = 0.28 \pm 0.21$ with $\chi^2/$d.o.f. $= 3.0/2$ and $\phi_P = (44.6 \pm 4.3)°$ and $Z_{\eta'}^2 = 0.30 \pm 0.21$ with $\chi^2/$d.o.f. $= 2.6/2$, respectively. These fits seem to favour a substantial gluonic component in η' which is, however, compatible with zero at 2σ due to the large uncertainty. In all cases, the mixing angle and most of the other parameters are consistent with those assuming no gluonium but with larger uncertainties due to fewer constraints. The parameter r' weighting the relative gluonic production amplitude is consistent with zero and has a large uncertainty. These results are in agreement with the values $\phi_P = (45 \pm 4)°$ and $Z_{\eta'}^2 = 0.30 \pm 0.21$ — or $\phi_{\eta'G} = (33 \pm 13)°$ — found in Ref. [3].

5. Conclusions

In this work we have performed an updated phenomenological analysis of an accurate and exhaustive set of $V \to P\gamma$ and $J/\psi \to VP$ decays with the purpose of determining the quark and gluon content of the η and η' mesons.

Our conclusions from the $V \to P\gamma$ analysis are the following. First, accepting the absence of gluonium for the η meson, the current experimental data on $VP\gamma$ transitions indicate within our model a negligible gluonic content for the η' meson, $Z_{\eta'}^2 = 0.04 \pm 0.09$. Second, this gluonic content of the η' wave function amounts to $|\phi_{\eta'G}| = (12 \pm 13)°$ and the η–η' mixing angle is found to be $\phi_P = (41.4 \pm 1.3)°$. Third, imposing the absence of gluonium for both mesons one finds $\phi_P = (41.5 \pm 1.2)°$, in agreement with the former result. Finally, we would like to stress that more refined experimental data, particularly for the $\phi \to \eta'\gamma$ channel, will contribute decisively to clarify this issue.

Our conclusions from the $J/\psi \to VP$ analysis follow. First, assuming the absence of gluonium, the η–η' mixing angle is found to be $\phi_P = (40.5 \pm 2.4)°$, in agreement with recent experimental measurements [4] and phenomenological estimates [3]. Second, if gluonium is allowed in the η' wave function, the values obtained are $\phi_P = (44.5 \pm 4.3)°$ and $Z_{\eta'}^2 = 0.28 \pm 0.21$ — or $|\phi_{\eta'G}| = (32 \pm 13)°$, which suggest within our model a substantial gluonic component in η'. Third, the inclusion of vector mixing angle effects, not included in previous analyses, turns out to be irrelevant. Finally, it is worth noticing that the recent reported values of $B(J/\psi \to \rho\pi)$ by the BABAR [20] and BES [21] Collaboration are crucial to obtain a consistent description of data.

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