Vol. 2 (2009) *Acta Physica Polonica B Proceedings Supplement* No 2

ENERGY LOSSES IN A HOT PLASMA*

Stéphane Peigné

SUBATECH, UMR 6457, Université de Nantes
Ecole des Mines de Nantes, IN2P3/CNRS
4 rue Alfred Kastler, 44307 Nantes Cedex 3, France

(Received October 30, 2009)

The energy loss of a fast charged particle crossing a hot plasma is reviewed.

PACS numbers: 12.38.Mh, 25.75.–q, 52.27.Ny

1. Introduction

The phenomenon of hadron suppression observed at large p_{T} at the Relativistic Heavy Ion Collider, called *jet-quenching*, is considered as one of the few spectacular effects possibly signalling the production of a Quark–Gluon Plasma (QGP) in heavy-ion collisions. Jet-quenching is qualitatively explained by the energy loss ΔE suffered by the energetic parent parton when crossing the plasma (and before hadronization). This motivates a review study of energy losses of ultrarelativistic particles in hot plasmas.

The physics of parton energy loss is quite rich and depends on several effects, among which transverse momentum broadening in multiple scattering, the Landau–Pomeranchuk–Migdal (LPM) suppression of radiation due to coherence effects, and the "dead-cone" suppression of radiation for massive particles. Parton energy loss has been studied theoretically by many groups, with various assumptions and using different methods. It is worth stressing that the results obtained so far are always model-dependent. For instance, the medium-induced radiative energy loss of a hard parton travelling the distance L in a QGP has been calculated to be $\Delta E_{\mathrm{rad}}^{\mathrm{ind}} = c_1 \alpha_{\mathrm{s}} \hat{q} L^2$, with a determined numerical coefficient c_1. This might look as an exact, model-independent expression, but it is not. Indeed, it depends on certain model parameters, namely the QGP Debye mass $\mu \sim gT$ and parton mean free path $\lambda \sim 1/(\alpha_{\mathrm{s}} T)$, through the parameter:

$$\hat{q} \equiv \frac{\mu^2}{\lambda} \sim \alpha_{\mathrm{s}}^2 T^3 \,. \qquad (1)$$

* Presented at the International Meeting "Excited QCD", Zakopane, Poland, February 8–14, 2009.

Usually, $\hat{q}$ is interpreted as a transport coefficient — the transverse momentum squared acquired by the parton per unit length — which can be calculated separately. But this procedure hides the fact that in the (model-independent) expression of $\Delta E_{\rm rad}^{\rm ind}$ in terms of *observable* quantities and in particular of the plasma temperature T, $\Delta E_{\rm rad}^{\rm ind} = c_2 \alpha_s^3 T^3 L^2$, the constant c_2 is actually not known[1].

In this context, using simple heuristic arguments to find the parametric dependence of ΔE may be as good as using complicated formalisms. This is what we tried in Ref. [1]. In the following we mention the main results for the energy loss of fast charges in relativistic plasmas[2].

2. Collisional energy loss

Let us start with collisional energy loss, and consider the QED case of a muon (of energy E and mass M) crossing an e^+e^- plasma, assuming $E \gg M \gg T$. The muon can lose energy via two kinds of *collisional* processes, namely Coulomb and Compton scattering off thermal particles, see Fig. 1.

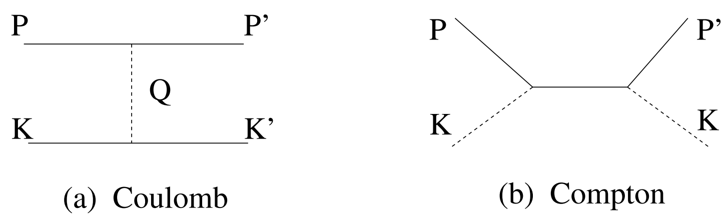

Fig. 1. The basic processes for collisional energy loss.

The differential Coulomb scattering cross-section reads

$$\frac{d\sigma_{\rm Coul}}{dt} \sim \frac{\alpha^2}{(t-\mu^2)^2}\,, \tag{2}$$

where $\mu \sim eT$ is the QED Debye mass. Denoting $\lambda \sim 1/(\alpha T)$ the muon mean free path, the rate of energy loss per unit length can be expressed as

$$\left.\frac{dE}{dx}\right|_{\rm Coul} = \frac{1}{\lambda}\,\langle \Delta E \rangle_{\rm 1scat.} \sim n\sigma\,\frac{1}{\sigma}\int dt\,\frac{d\sigma}{dt}\,Q_0\,, \tag{3}$$

where we used $\lambda \sim 1/(n\sigma)$, with $n \sim T^3$ the density of thermal particles. Using now $t = -2KQ = -2(K_0 Q_0 - \vec{K}\cdot\vec{Q})$, we estimate $Q_0 \sim |t|/K_0 \sim |t|/T$, and (3) then gives:

---

1 If the theoretical models used so far are realistic enough, one might hope that $c_1 \simeq c_2$.

2 See [1] for details and references to original works.

$$\left.\frac{dE}{dx}\right|_{\rm Coul} \sim n \int dt\, \frac{\alpha^2}{t^2}\, \frac{|t|}{T} \sim \alpha^2 T^2 \int \frac{dt}{t} \sim \alpha^2 T^2 \ln \frac{ET}{\mu^2}\,. \tag{4}$$

The Coulomb contribution arises from the broad logarithmic region $\mu^2 \ll |t| \ll |t|_{\max} \sim ET$, implying $Q_0 \ll E$. To leading logarithmic accuracy, the final muon is thus the *leading* particle.

Compton scattering also gives a logarithmic contribution to dE/dx, which, however, arises from scattering in the u-channel. (Only t-channel scattering is represented in Fig. 1(b).) Assuming $M^2 \ll |u| \ll s \sim ET$ we have

$$\frac{d\sigma_{\rm Comp}}{dt} \sim \frac{\alpha^2}{su} \Rightarrow \left.\frac{dE}{dx}\right|_{\rm Comp} \sim n \int dt \frac{\alpha^2}{su}\, \frac{|t|}{T} \sim \alpha^2 T^2 \int \frac{du}{u} \sim \alpha^2 T^2 \ln \frac{ET}{M^2}\,. \tag{5}$$

The Compton contribution arises from a kinematical domain, where $|t| \simeq |t|_{\max} \simeq s \Leftrightarrow Q_0 \simeq E$, *i.e.*, from *full stopping* of the muon. This property of Compton scattering is well-known and used to produce energetic photons (Compton backscattering of laser beams). In spite of different nature, the contributions to dE/dx from Coulomb and Compton scattering are of the same order.

What is the collisional energy loss of an electron (with $E \gg T \gg m \sim m_{\rm thermal}$) crossing an e^+e^- plasma? It is tempting to use (4) and (5), and replace $M \to m_{\rm thermal} \sim \mu$ to find the result. But this recipe is incorrect, because in the Compton contribution, the fully stopped electron cannot be distinguished from thermal electrons. To define the *observable* energy loss of a light particle, we must demand the final particle to be leading, hence only the Coulomb contribution (4) survives.

This discussion also holds in the QCD case of a parton crossing a QGP. To leading logarithmic accuracy, the collisional loss of a *tagged* (and thus possibly non-leading) heavy quark receives an additional contribution compared to the case of a light parton.

3. Radiative energy loss

Here we discuss the radiative energy loss $\Delta E_{\rm rad}$ induced by elastic rescatterings in the plasma. Heuristically, $\Delta E_{\rm rad}$ can be obtained from the essential quantum features of the problem. In QED photon radiation is coherent over a distance corresponding to the photon formation length[3]

$$\ell_{\rm f}(\omega, \theta) \sim \frac{1}{\omega \theta^2}\,, \tag{6}$$

[3] A similar discussion holds for gluon radiation in QCD.

where ω is the photon energy and θ its emission angle. Roughly speaking, the emission of a second photon can occur only if the first is already emitted. If $\ell_{\rm f} \gg \lambda$, we expect a suppression of the radiation spectrum compared to the regime where the spectrum is additive in the number of elastic scatterings. This is the so-called LPM effect. For a large medium, $L \gg \ell_{\rm f}$, the spectrum induced by crossing the distance L is thus given by

$$\omega \frac{dI}{d\omega d^2\vec{\theta}}\bigg|_L \sim \frac{L}{\ell_{\rm f}}\, \omega \frac{dI}{d\omega d^2\vec{\theta}}\bigg|_{\rm BH,\, eff} , \tag{7}$$

where the last factor is the Bethe–Heitler (BH) spectrum induced by a single *effective* scattering over the length $\ell_{\rm f}$. In particular, the typical momentum transfer $q_\perp$ over $\ell_{\rm f}$ is given by the random walk estimate $q_\perp^2 \sim (\ell_{\rm f}/\lambda)\mu^2$.

Recall that for a given momentum transfer $\vec{q}_\perp$ to the radiating charge, the BH spectrum is of the form

$$\omega \frac{dI}{d\omega d^2\vec{\theta}}\bigg|_{\rm BH} \sim \alpha\, \frac{\theta_s^2}{\theta^2(\vec{\theta} - \vec{\theta}_s)^2} , \tag{8}$$

$$\vec{\theta}_s = \frac{\vec{q}_\perp}{E}\ ({\rm QED})\, , \qquad \vec{\theta}_s = \frac{\vec{q}_\perp}{\omega}\ ({\rm QCD})\, . \tag{9}$$

In the following we will neglect logarithms. In a single scattering the BH radiative energy loss thus reads

$$\Delta E_{1,{\rm BH}} = \int d\omega\, d^2\vec{\theta}\, \omega \frac{dI}{d\omega\, d^2\vec{\theta}}\bigg|_{\rm BH} \sim \alpha E\, , \tag{10}$$

arising from typical emission angles $\theta^2 \sim \theta_s^2$.

We need to distinguish two physical situations: *(i)* the "asymptotic" case of a particle produced in the remote past and entering the plasma and *(ii)* the case of a particle initially produced in the plasma. In the latter case, the relevant quantity is not the *total* radiative energy loss — since a newly created particle radiates even in vacuum — but the *medium-induced* radiative loss, obtained by subtracting the vacuum contribution. In the former case, total and medium-induced radiative losses coincide — an asymptotic particle travelling in vacuum does not radiate.

3.1. Asymptotic particle

Let us determine heuristically the parametric behaviour of $\Delta E_{\rm rad}(L)$ in the two limits $L \ll \lambda$ and $L \to \infty$. When $L \ll \lambda$, $\Delta E_{\rm rad}$ is expressed as the probability L/λ to have one elastic scattering, times the integrated BH spectrum (induced by the transfer $q_\perp \sim \mu$),

$$\Delta E_{\rm rad}(L \ll \lambda) \sim \frac{L}{\lambda}\, \Delta E_{1,\rm BH} \sim \alpha E\, \frac{L}{\lambda} \sim \alpha^2 ET\, L\,. \tag{11}$$

The latter result arises from typical formation lengths

$$\ell_{\rm f} \sim \frac{1}{\omega\theta^2} \sim \frac{1}{E\theta_{\rm s}^2} \sim \frac{E}{\mu^2} \gg \lambda \gg L\,, \tag{12}$$

where the first inequality is due to $E \gg T$. When $L \to \infty$, the typical formation length can be estimated by replacing $\mu^2 \to \mu_{\rm eff}^2$ in the above estimate,

$$\ell_{\rm f} \sim \frac{E}{\mu_{\rm eff}^2} \sim \frac{E}{\mu^2 \ell_{\rm f}/\lambda} \Rightarrow \ell_{\rm f} \sim \sqrt{\frac{\lambda E}{\mu^2}} \equiv L^*\,. \tag{13}$$

When $L \gg L^*$ we obtain

$$\Delta E_{\rm rad}(L \gg L^*) \sim \frac{L}{\ell_{\rm f}}\, \Delta E_{1,\rm BH} \sim \frac{L}{L^*}\, \alpha E \sim \alpha\sqrt{\hat{q}E}\, L \sim \alpha^2\sqrt{ET^3}L\ . \tag{14}$$

The parametric behaviours (11) and (14) are represented in Fig. 2 (upper curve). The transition region $\lambda \ll L \ll L^*$ has a smooth logarithmic dependence in L [1]. These results for $\Delta E_{\rm rad}$ are valid both in QED and QCD (for light particles though). QED and QCD start to qualitatively differ when radiation spectra are considered, and when the radiating particle is massive [1].

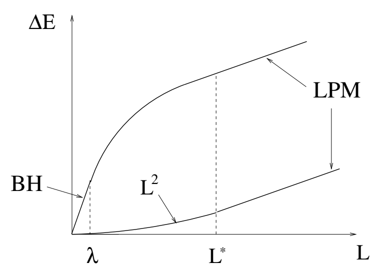

Fig. 2. Radiative energy loss of a light particle crossing the distance L in the plasma. Upper curve: Total radiative loss of a particle produced in the remote past. Lower curve: Medium-induced radiative loss of a particle initially produced in the plasma. The results hold in QED and QCD for light particles.

3.2. Particle produced in the plasma

For a particle created in the plasma, the *medium-induced* radiative loss is obtained by subtracting from the total loss the vacuum radiation due to the sudden acceleration of the particle. Since the radiation associated with a formation length $\ell_{\rm f} > L$ cancels in this difference, the medium-induced loss can be obtained heuristically as for an asymptotic particle, but with the additional constraint $\ell_{\rm f} \leq L$,

$$\Delta E_{\rm rad}^{\rm ind}(L) \sim \int d\omega\, d^2\vec{\theta}\,\omega \left.\frac{dI}{d\omega\, d^2\vec{\theta}}\right|_L \Theta(\ell_{\rm f} \leq L)\,. \tag{15}$$

When $L \ll \lambda$, the induced loss is estimated as

$$\Delta E_{\rm rad}^{\rm ind}(L \ll \lambda) \sim \int d\omega\, d\theta^2 \Theta\left(\frac{1}{\omega\theta^2} \leq L\right) \frac{L}{\lambda}\,\alpha\,\frac{\theta_s^2}{\theta^2(\vec{\theta}-\vec{\theta}_s)^2} \sim \alpha\hat{q}L^2\,. \tag{16}$$

The quadratic law $\Delta E_{\rm rad}^{\rm ind} \sim L^2$ is actually valid up to $L \sim L^*$ [1]. In the case of an asymptotic particle, we saw that $\Delta E_{\rm rad}$ arises from $\ell_{\rm f} \gg L$. Thus, the constraint $\ell_{\rm f} \leq L$ strongly suppresses $\Delta E_{\rm rad}^{\rm ind}$, see Fig. 2. When $L \gg L^*$, $\Delta E_{\rm rad}$ arises from $\ell_{\rm f} \sim L^*$, satisfying the condition $\ell_{\rm f} \leq L$. Hence the behaviour of $\Delta E_{\rm rad}^{\rm ind}(L \gg L^*)$ is given by (14). The overall behaviour of the medium-induced loss is shown in Fig. 2. This behaviour holds for light particles in QED and QCD. In particular, contrary to what is sometimes claimed, the quadratic behaviour at small L is not specific to QCD.

The same line of arguments can be used to derive the medium-induced radiative loss of a heavy QED or QCD charge of mass M. Up to logarithms, the results for $\Delta E_{\rm rad}^{\rm ind}$ are the same in QED and QCD. The quadratic law (16) appears to be universal, and valid up to a saturation scale $L_{\rm cr}$, above which a linear regime (with a slope reduced compared to (14)) sets in. The length $L_{\rm cr}$ decreases when M increases and is given by

$$L_{\rm cr} = {\rm Min}(L^*, \tilde{L}^*, \frac{E}{\mu M})\,, \qquad \tilde{L}^* \equiv \left(\frac{\lambda E^2}{\mu^2 M^2}\right)^{1/3}\,. \tag{17}$$

For a particle created in the plasma, the rate of (medium-induced) radiative loss is given by the following simple heuristic formula, which is valid in QED and QCD, and for light and massive particles [1]:

$$\left.\frac{dE}{dx}\right|_{\rm induced} \sim \alpha\,\hat{q}\,{\rm Min}(L, L_{\rm cr})\,. \tag{18}$$

REFERENCES

[1] S. Peigné, A.V. Smilga, `arXiv:0810.5702[hep-ph]`.