Vol. 2 (2009) *Acta Physica Polonica B Proceedings Supplement* No 2

BABY STEPS BEYOND RAINBOW–LADDER*

Richard Williams[a], Christian S. Fischer[a,b]

[a]Institute for Nuclear Physics, Darmstadt University of Technology
Schlossgartenstrasse 9, 64289 Darmstadt, Germany

[b]GSI Helmholtzzentrum für Schwerionenforschung GmbH
Planckstr. 1, 64291 Darmstadt, Germany

(Received October 2, 2009)

We discuss the impact of including corrections beyond single gluon exchange in light mesons within the nonperturbative framework of Dyson–Schwinger equations (DSE) and Bethe–Salpeter equations (BSE). We do this by considering unquenching effects in the form of hadronic resonance contributions, notably pion exchange, and by the inclusion of the dominant gluon self-interactions to the quark–gluon vertex. Thus we make steps towards an *ab initio* description of light mesons by functional methods.

PACS numbers: 11.10.St, 11.30.Rd, 12.38.Lg, 14.65.Bt

1. Introduction

With the absence of quarks and gluons in the physical spectrum, we are forced to probe their interaction at low energies by the study of colourless composites of these particles. This information is encoded in the Green's functions of our QFT, in particular the four-quark scattering matrix and the quark–gluon vertex. Faced with the richness of the hadronic spectrum, it is not unreasonable to draw a comparison with the complex nonperturbative particulars of these Green's functions. In this paper we will consider the simplest bound states — the light mesons — as our probes and investigate how their properties depend on information present in the quark–gluon vertex.

2. Dyson–Schwinger equations

To solve the DSEs, shown for the inverse quark propagator and quark–gluon vertex by (1) and (2) of Fig. 1, we need to introduce a truncation at some point in the infinite tower. This need for a truncation also applies to

* Presented by R. Williams at the International Meeting "Excited QCD", Zakopane, Poland, February 8–14, 2009.

the BSE since therein we must specify the (2PI) four-point scattering kernel. This is a delicate process since we must be careful to preserve various symmetries of the theory. The most important of these relates to chiral symmetry, expressed via the axial-vector Ward–Takahashi identity (axWTI). This underpins the observed mass spectrum of the light mesons, and ensures that pions are indeed the (pseudo)-Goldstone bosons of the theory. This identification is a necessary feature of any serious model and must be exhibited by any truncation of the BSEs and DSEs.

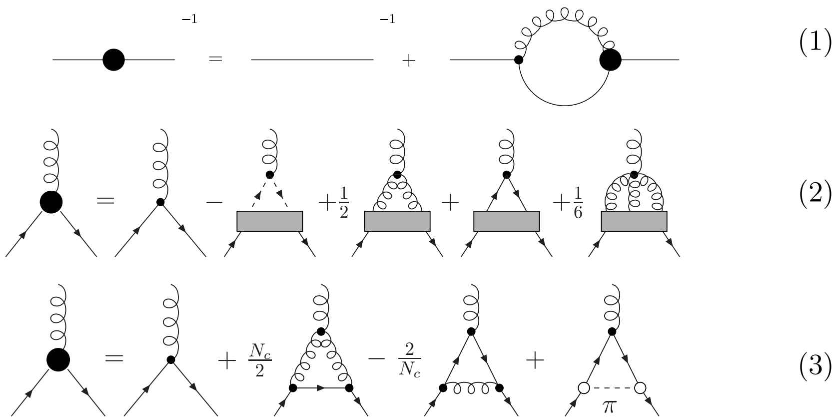


Fig. 1. DSEs for: (1) fully dressed quark propagator; (2) full quark–gluon vertex; (3) truncated quark–gluon vertex. Internal propagators are dressed, with gluons shown by wiggly lines, quarks by straight lines and dashed lines mesons. White-filled circles show meson amplitudes whilst black-filled represent vertex dressings.

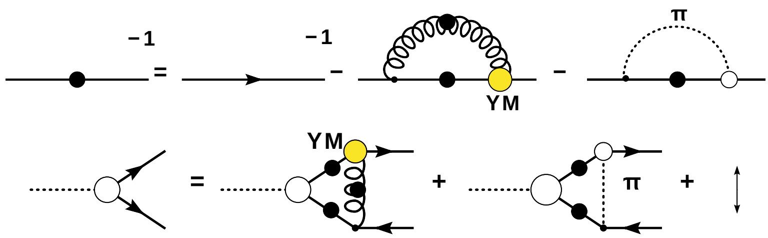
The simplest truncation that satisfies this criterion is that of Rainbow–Ladder (R–L) whereby the full quark–gluon vertex is replaced by a bare vertex, see *e.g.* [1]. The axWTI preserving kernel in the BSE then corresponds to a single gluon exchange, re-summed to all orders thus providing the 'ladder'. These R–L models are designed to reproduce predominantly s-wave mesons due to the simple vector–vector structure of the interaction. To compensate for this simplicity one constructs a phenomenological model for the gluon, effectively subsuming additional vertex corrections from the Yang–Mills sector and unquenching effects.

To separate the phenomenological from the *ab initio*, we must investigate the quark–gluon vertex and determine the impact of corrections beyond tree-level to our quarks and mesons. Such studies have been made *e.g.* [2–8]. Following the analysis of [9, 10] we approximate the full DSE (2) with the (nonperturbative) one-loop structure of (3). Here the first 'non-Abelian' loop-diagram in (3) subsumes the first two diagrams in the full DSE to first order in a skeleton expansion of the four-point functions. We neglect the two-loop diagram in the full DSE (2), which is justified for small and large momenta [6,9]. The remaining 'Abelian' contributions are split into the non-resonant second loop-diagram in (3) and a third diagram containing effects due to hadron back-reactions.

In the next section we consider these resonance contributions, using a R–L truncation for the non-resonant parts. We follow this by exploring a new truncation in which leading non-resonant parts from the non-Abelian vertex are included self-consistently in the quark DSE and meson BSE.

3. Including unquenching effects

The prescription for including pion degrees of freedom in the DSEs and BSEs in a manner consistent with the axWTI have been proposed and investigated in [10–12], with the resultant system of equations depicted by



The first loop diagram of both equations relates to the usual rainbow-ladder, where the infrared suppressed gluon is enhanced by the vertex dressing, indicated by YM. The second diagram contributing to the quark DSE and meson BSE represents the back-reaction of the pion onto the quark. This requires input of the quark–pion vertex, which we parameterise by a chiral approximation of the leading pion Bethe–Salpeter amplitude [12]

$$\Gamma_\pi^j(p;P) = \tau^j \gamma_5 B_\chi\left(p^2\right)/f_\pi\,. \tag{4}$$

Here $B_\chi(p^2)$ is the scalar dressing function of the quark propagator in the chiral limit. The effects of neglecting the three sub-leading amplitudes have been quantified for a real-value approximation in Ref. [10], and found to be on the level of a few percent. The advantage of the approximation in (4) is that we can then directly calculate the quark propagator in the complex plane.

We need to specify the gluon propagator and a quark–gluon vertex that subsumes the non-resonant parts of the interaction into an effective rainbow-ladder model. Since here we wish to employ a gluon propagator as calculated from its DSE, we employ the Soft Divergent model (SD) for the quark–gluon vertex as described in [9, 12].

We calculated a range of meson observables and observe that the effect of the pion back-reaction has only a small impact on the pion mass itself. The impact of including pion-cloud effects on the leptonic decay constant is more substantial, with effects of the order of 10 MeV:

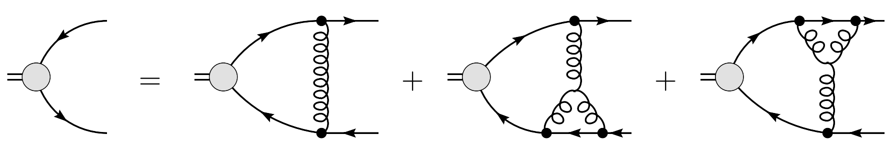
Model	M_π	f_π	M_ρ	f_ρ	M_σ	M_{a_1}	M_{b_1}
Quenched	125	102	795	159	638	941	879
Unquenched	138	93.8	703	162	485	873	806
PDG [17]	138	92.4	776	156	400–1200	1230	1230

For the remaining heavier mesons, the common trend is that the inclusion of the pion cloud decreases the masses by 100–200 MeV. Most notable of these are for the rho, where we see that unquenching from the pion-cloud yields a bound-state that is ~ 90 MeV lighter. This will prove important in what follows when we consider gluon self-interaction contributions.

It is clear, however, that in order to reproduce the rich spectrum of light mesons we need to include spin dependent contributions from the Yang–Mills part of the quark–gluon vertex. We now consider this in the absence of pion-cloud effects in the next section.

4. Including gluon self-interactions

Looking back to our proposed truncation of the quark–gluon vertex, we identified the second loop diagram of (3) as the dominant contribution. Ignoring all other contributions the resulting equation, coupled with the quark DSE, may be solved numerically provided we know the gluon propagator and three-gluon vertex dressing. Since we work in Euclidean space we need the quark propagator for complex values of its momenta. This means our quark–gluon vertex must also be solved for complex momenta. Through judicious choice of momenta in both the quark and vertex DSE, this can be performed without unconstrained analytic continuation of the gluon propagator and three-gluon vertex. The axWTI preserving truncation for the BSE [2, 13], consistent with our choice of the quark–gluon vertex is [14]



Since this work is a preliminary study of a new and sophisticated truncation [15] we choose to simplify things further by employing a simple momentum dependent ansatz for the gluon [16]

$$\alpha Z\left(q^2\right) = \left(\pi D/\omega^2\right) q^4 e^{-q^2/\omega^2}\,, \tag{5}$$

with two parameters D and ω which provide for the scale and strength of the effective gluon interaction. Naturally, this ansatz provides only a first step towards a full calculation including input from the DSEs for the three-gluon vertex and the gluon propagator. Nevertheless, we believe that the ansatz (5) is sufficient to provide for reliable qualitative results as concerns the effects due to the non-Abelian diagram onto meson properties.

The results of our calculation follow, wherein we compare our truncation including gluon self-interaction effects in all twelve tensor structures of the quark–gluon vertex to that of R–L with only vector–vector interactions.

Model	ω	D	m_π	f_π	m_ρ	f_ρ	m_σ	m_{a_1}	m_{b_1}
R–L	0.50	16	138	94	758	154	645	926	912
BTR			138	111	881	176	884	1055	972
R–L	0.45	25	136	92	746	149	675	917	858
BTR			141	112	873	173	796	1006	902
PDG [17]			138	92.4	776	156	400–1200	1230	1230

The model parameters ω and D were tuned such that for the latter we obtain reasonable pion observables, and the quark mass is fixed at 5 MeV.

What we find is that the mass of the ρ-meson is enhanced by ~ 120 MeV as compared to pure R–L. This is intriguing since it has long been suspected that corrections beyond R–L approximately cancel in the ρ [2, 3, 18]. This is supported by known estimates of mass shifts from the resonant and non-resonant Abelian diagrams in (3), calculated at $\sim$ 90 MeV (see previous section) and 30 MeV [5], respectively.

5. Conclusions

We presented a study of light mesons using two truncations of the Bethe–Salpeter equations beyond R–L. We considered unquenching effects associated with the pion cloud, and the dominant non-Abelian corrections to the quark–gluon vertex stemming from gluon self-interactions. For the latter truncation, we obtain masses for the rho meson of $\sim$ 900 MeV, consistent with quenched lattice simulations. The subsequent inclusion of unquenching effects and other non-resonant contributions to the quark–gluon interaction brings the rho mass back to its physical value thus supporting a long suspected cancellation mechanism. An investigation based on Yang–Mills DSE results together with unquenching effects is currently underway.

This work was supported by the Helmholtz-University Young Investigator Grant number VH-NG-332 and by the Helmholtz International Center for FAIR within the framework of the LOEWE program (Landesoffensive zur Entwicklung Wissenschaftlich-Ökonomischer Exzellenz) launched by the State of Hesse.

REFERENCES

[1] P. Maris, C.D. Roberts, *Phys. Rev.* **C56**, 3369 (1997).

[2] A. Bender, C.D. Roberts, L. Von Smekal, *Phys. Lett.* **B380**, 7 (1996).

[3] A. Bender, W. Detmold, C.D. Roberts, A.W. Thomas, *Phys. Rev.* **C65**, 065203 (2002).

[4] M.S. Bhagwat *et al. Phys. Rev.* **C70**, 035205 (2004).

[5] P. Watson, W. Cassing, P.C. Tandy, *Few-Body Syst.* **35**, 129 (2004); P. Watson, W. Cassing, *Few-Body Syst.* **35**, 99 (2004).

[6] M.S. Bhagwat, P.C. Tandy, *Phys. Rev.* **D70**, 094039 (2004).

[7] H.H. Matevosyan, A.W. Thomas, P.C. Tandy, *Phys. Rev.* **C75**, 045201 (2007).

[8] L. Chang, C.D. Roberts, *Phys. Rev. Lett.* **103**, 081601 (2009) [arXiv:0903.5461 [nucl-th]].

[9] R. Alkofer, C.S. Fischer, F.J. Llanes-Estrada, K. Schwenzer, *Ann. Phys.* **324**, 106 (2009).

[10] C.S. Fischer, D. Nickel, J. Wambach, *Phys. Rev.* **D76**, 094009 (2007).

[11] C.S. Fischer, D. Nickel, R. Williams, *Eur. Phys. J.* **C60**, 1434 (2008).

[12] C.S. Fischer, R. Williams, *Phys. Rev.* **D78**, 074006 (2008).

[13] H.J. Munczek, *Phys. Rev.* **D52**, 4736 (1995).

[14] P. Maris, P.C. Tandy, *Nucl. Phys. Proc. Suppl.* **161**, 136 (2006).

[15] C.S. Fischer, R. Williams, *Phys. Rev. Lett.* **103**, 122001 (2009) [arXiv:0905.2291 [hep-ph]].

[16] R. Alkofer, P. Watson, H. Weigel, *Phys. Rev.* **D65**, 094026 (2002).

[17] C.R. Allton *et al.*, *Phys. Lett.* **B628**, 125 (2005).

[18] G. Eichmann *et al.*, *Phys. Rev.* **C77**, 042202 (2008).