

INFLUENCE OF VECTOR MESONS ON THE $f_0(600)$ DECAY WIDTH IN A LINEAR SIGMA MODEL WITH GLOBAL CHIRAL INVARIANCE*

DENIS PARGANLIJA^a, FRANCESCO GIACOSA^a, DIRK H. RISCHKE^{a,b}

^aInstitute for Theoretical Physics, Goethe University
Max-von-Laue-Str. 1, 60438 Frankfurt am Main, Germany

^bFrankfurt Institute for Advanced Studies, Goethe University
Ruth-Moufang-Str. 1, 60438 Frankfurt am Main, Germany

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We consider a globally invariant chiral Lagrangian that contains vector and axial-vector mesons. We compute the $f_0(600) \rightarrow \pi\pi$ decay width and $\pi\pi$ scattering lengths and compare with the corresponding results in which the (axial-)vector degrees of freedom decouple. We show that the role of vector mesons has a great impact on both quantities.

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1. Introduction

One way to investigate Quantum Chromodynamics (QCD) at low energies is by using effective models — such as the Linear Sigma Model — that possess the same global symmetries as QCD, most notably the chiral $SU(N_f)_r \times SU(N_f)_l$ symmetry (with N_f flavours). In these effective models hadronic degrees of freedom, rather than quark and gluon degrees of freedom, are present; spontaneous breaking of the chiral invariance leads to the emergence of nearly massless pseudoscalar Goldstone bosons (*e.g.*, pions for $N_f = 2$). Their chiral partners, the scalar states, remain massless.

If the Linear Sigma Model is used, there are at least two possibilities to choose the hadronic degrees of freedom in the corresponding Lagrangian: with (pseudo-)scalar degrees of freedom only, or with the addition of (axial-)vector degrees of freedom. In this paper we present a linear sigma model *with* vector and axial-vector mesons for $N_f = 2$ and discuss two important quantities: the $\pi\pi$ scattering lengths and the decay width of $f_0(600)$ (in our notation: σ) into two pions. In particular, we aim to investigate the role

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of (axial-)vector mesons for these quantities. To this end we investigate the limit in which the (axial-)vector mesons decouple and compare the results with the results of the full model.

The paper is organized as follows: in Section 2 the model is introduced, in Section 3 the results with and without vector mesons are presented and in Section 4 we present our conclusions.

2. The model

The model is described by a $SU(2)_r \times SU(2)_l$ chirally symmetric Lagrangian [1–3]:

$$\begin{aligned}
 \mathcal{L} = & \text{Tr}[(D^\mu \Phi)^\dagger (D^\mu \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 \\
 & - \frac{1}{4} \text{Tr}[(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \frac{m_1^2}{2} \text{Tr}[(L^\mu)^2 + (R^\mu)^2] + \text{Tr}[H(\Phi + \Phi^\dagger)] \\
 & + c(\det \Phi + \det \Phi^\dagger) - 2ig_2(\text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
 & + \frac{h_1}{2} \text{Tr}(\Phi^\dagger \Phi) \text{Tr}[(L^\mu)^2 + (R^\mu)^2] + h_2 \text{Tr}[|L^\mu \Phi|^2 + |\Phi R^\mu|^2] \\
 & + 2h_3 \text{Tr}(\Phi R_\mu \Phi^\dagger L^\mu) + \dots, \tag{1}
 \end{aligned}$$

where $\Phi = (\sigma + i\eta_N)t^0 + (\vec{a}_0 + i\vec{\pi}) \cdot \vec{t}$ (scalar and pseudoscalar degrees of freedom), $L^\mu = (\omega^\mu + f_1^\mu)t^0 + (\vec{\rho}^\mu + \vec{a}_1^\mu) \cdot \vec{t}$ and $R^\mu = (\omega^\mu - f_1^\mu)t^0 + (\vec{\rho}^\mu - \vec{a}_1^\mu) \cdot \vec{t}$ (vector and axial-vector degrees of freedom; our model is currently constructed for $N_f = 2$ — thus, our eta meson η_N contains only non-strange degrees of freedom); $t^0, \vec{t}$ are the generators of $U(2)$; $D^\mu \Phi = \partial^\mu \Phi - ig_1(L^\mu \Phi - \Phi R^\mu) - ieA^\mu[t^3, \Phi]$ (A^μ denotes the photon field), $L^{\mu\nu} = \partial^\mu L^\nu - ieA^\mu[t^3, L^\nu] - (\partial^\nu L^\mu - ieA^\nu[t^3, L^\mu])$, $R^{\mu\nu} = \partial^\mu R^\nu - ieA^\mu[t^3, R^\nu] - (\partial^\nu R^\mu - ieA^\nu[t^3, R^\mu])$. The dots refer to further globally invariant terms which are irrelevant in the following. The explicit breaking of the global symmetry is described by the term $\text{Tr}[H(\Phi + \Phi^\dagger)] \equiv h\sigma$ ($h = \text{const.}$). The chiral anomaly is described by the term $c(\det \Phi + \det \Phi^\dagger)$ [4]. Note that the model possesses global chiral invariance; the reasons to use the global invariance (instead of the local one) may be found, *e.g.*, in Ref. [5] as well as Refs. [1, 6]. The reason to consider operators up to the fourth order only has been presented in Ref. [7].

When $m_0^2 < 0$, spontaneous symmetry breaking $SU(2)_r \times SU(2)_l \rightarrow SU(2)_V$ takes place. The sigma field is shifted by its (constant) vacuum expectation value ϕ : $\sigma \rightarrow \sigma + \phi$. As a result of this shift non-diagonal mixing terms of the form $-g_1 \phi \vec{a}_1^\mu \cdot \vec{\pi}$ and $-g_1 \phi f_1^\mu \eta_N$ arise. In order to remove them, the fields f_1 and $\vec{a}_1$ are also shifted as $f_1^\mu \rightarrow f_1^\mu + w \partial^\mu \eta$, $\vec{a}_1^\mu \rightarrow \vec{a}_1^\mu + w \partial^\mu \vec{\pi}$ with $w = g_1 \phi / m_{a_1}^2$ and the pseudoscalar fields are renormalized: $\vec{\pi} \rightarrow Z \vec{\pi}$, $\eta_N \rightarrow Z \eta_N$ where $Z = m_{a_1} / (m_{a_1}^2 - g_1^2 \phi^2)^{\frac{1}{2}}$ [3, 8].

The identification of mesons with particles listed by the PDG [9] is straightforward in the pseudoscalar and (axial-)vector sectors: The fields $\vec{\pi}$ and η_N correspond to the pion and the SU(2) counterpart of the η meson, $\eta_N \equiv \sqrt{1/2}(\bar{u}u + \bar{d}d)$, (with a mass m_{η_N} of about 700 MeV, obtained by “unmixing” the physical η and η' mesons). The fields ω^μ , $\vec{\rho}^\mu$ represent the $\omega(782)$ and the $\rho(770)$ vector mesons; the fields f_1^μ , $\vec{a}_1^\mu$ represent the $f_1(1285)$ and $a_1(1260)$ axial-vector mesons. Unfortunately, the identification of the σ and $\vec{a}_0$ fields is controversial with the possibilities being the pairs $\{f_0(600), a_0(980)\}$ and $\{f_0(1370), a_0(1450)\}$. For the purpose of this paper we chose the assignment $\{f_0(600), a_0(980)\}$, which allows us to study the decay $f_0(600) \rightarrow \pi\pi$ and the $\pi\pi$ scattering lengths.

Eleven parameters are present in the Lagrangian: $\lambda_1, \lambda_2, c, h_0, h_1, h_2, h_3, m_0^2, g_1, g_2, m_1$. By using the masses $m_\pi, m_{\eta_N}, m_{a_0}, m_\rho$ and m_{a_1} , the pion decay constant f_π (via the Eq. $\phi = Z f_\pi$) and the experimentally well-known decay widths $\Gamma_{\rho \rightarrow \pi\pi} = 149.4 \pm 1.0$ MeV and $\Gamma_{f_1 \rightarrow a_0\pi} = 8.748 \pm 2.097$ MeV [9], one can eliminate eight parameters. All quantities of interest can be then expressed in terms of three independent combinations of parameters, which for convenience are chosen to be m_σ, Z and h_1 . Explicit expressions and a detailed description of the procedure can be found in Ref. [10].

For the present purpose, it is useful to write explicitly the equation for the ρ mass

$$m_\rho^2 = m_1^2 + \frac{\phi^2}{2} [h_1 + h_2(Z) + h_3(Z)], \quad (2)$$

where the decomposition of m_ρ^2 in terms of two terms is evident: the term $(\phi^2/2) [h_1 + h_2(Z) + h_3(Z)]$ represents the contribution of the chiral condensate (and vanishes in the limit $\phi \rightarrow 0$), while m_1^2 is independent of the chiral condensate and can be related to the gluon condensate. Note that we made explicit that $h_2 \equiv h_2(Z)$ and $h_3 \equiv h_3(Z)$ are functions of Z . In the following we will consider m_1 as varying between 0 and $m_\rho/2$: in fact, a negative m_1 would imply that the vacuum is not stable in the limit $\phi \rightarrow 0$; $m_1^2 > m_\rho^2$ would imply that the contribution of the chiral condensate is negative. This is counter-intuitive and at odds with various microscopic approaches such as the NJL model.

3. Results and discussions

Results with (axial-)vector mesons: The scattering lengths $a_0^0 \equiv a_0^0(m_\sigma, Z, h_1)$ and $a_0^2 \equiv a_0^2(m_\sigma, Z, h_1)$ and $\Gamma_{a_1 \rightarrow \pi\gamma} \equiv \Gamma_{a_1 \rightarrow \pi\gamma}[Z]$ are functions of m_σ, Z and h_1 (explicit expressions can be found in Ref. [10]). We use the experimental values $a_0^0 = 0.218 \pm 0.020$ and $a_0^2 = -0.0457 \pm 0.0125$ from the NA48/2 Collaboration [11] and $\Gamma_{a_1 \rightarrow \pi\gamma} = 0.640 \pm 0.246$ MeV [9]. A standard fit procedure allows us to determine the parameters m_σ, Z , and h_1 and their

errors: $Z = 1.67 \pm 0.20$, $h_1 = -68 \pm 338$ and $m_\sigma = (332 \pm 456)$ MeV. Obviously, the errors of h_1 and m_σ are too large. A drastic improvement can be obtained via a physical consideration: as we discussed after Eq. (2) we impose that m_1^2 is varying between 0 and m_ρ^2 . As a consequence for $Z = 1.67$, the parameter h_1 needs to lie in the interval $h_1 \in [-32, -83] \Leftrightarrow h_1 = -68^{+36}_{-15}$ (the upper boundary of h_1 corresponds to $m_1 = 0$). The value of m_σ can be constrained in a similar way using the scattering length a_0^0 : $m_\sigma \in [279, 500]$ MeV $\Leftrightarrow m_\sigma = 332^{+168}_{-53}$ MeV.

With these parameter values it is possible to calculate the σ decay width, $\Gamma_{\sigma \rightarrow \pi\pi} \equiv \Gamma_{\sigma \rightarrow \pi\pi}(m_\sigma, Z, h_1)$, which is shown in the following Fig. 1.

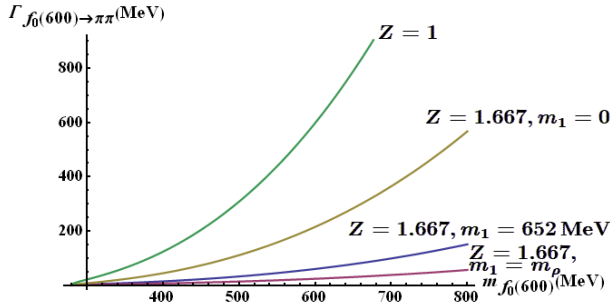


Fig. 1. Two-pion decay width of $f_0(600)$ with ($Z = 1.667$) and without ($Z = 1$) vector and axial-vector mesons.

The three lower curves in the plot depict the σ decay width in the model that contains vector and axial-vector degrees of freedom. It is clear that the decay width becomes smaller when the vector and axial-vector degrees of freedom are introduced. In that case, the value of the σ decay width changes very slowly with m_σ so that even for $m_\sigma \cong 650$ MeV the width is not larger than approximately 300 MeV — a value that is too small when compared, *e.g.*, with the results from Refs. [12, 13]. Note that the value of $m_\sigma \cong 650$ MeV is actually too large to agree with the scattering length a_0^0 (as apparent from Fig. 2) if the (axial-)vector fields are considered — then, the σ mass should not be larger than 450 MeV which in turn yields a σ decay width smaller than about 100 MeV.

Results without (axial-)vector mesons: This limit is obtained for $g_1 \rightarrow 0$, *i.e.* $Z \rightarrow 1$, and $h_1 = h_2 = h_3 = 0$. In this case the decay width $\Gamma_{\sigma \rightarrow \pi\pi} \equiv \Gamma_{\sigma \rightarrow \pi\pi}(m_\sigma, Z = 1, h_1 = 0)$ and the scattering length $a_0^0 \equiv a_0^0(m_\sigma, Z = 1, h_1 = 0)$ are functions of m_σ only and are plotted in Fig. 1 and Fig. 2. The scattering length a_0^0 allows for $m_\sigma \in [500, 620]$ MeV (see Fig. 2). Thus, the value of the $f_0(600)$ decay width in this case is *much larger* than in the previous cases: given that $m_\sigma \in [500, 620]$ MeV, one obtains $\Gamma_\sigma \in [350, 600]$ MeV (see Fig. 1). These values are in agreement with

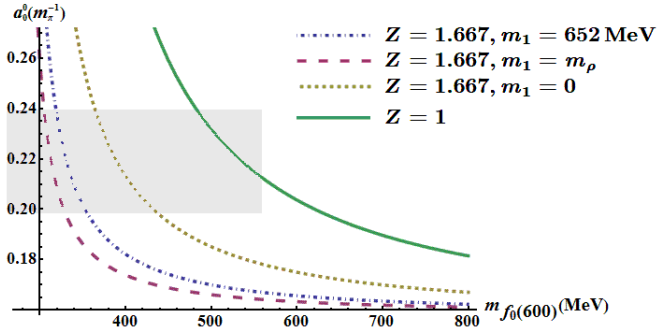


Fig. 2. Scattering length a_0^0 with ($Z = 1.667$) and without ($Z = 1$) vector and axial-vector mesons. The shaded area corresponds to NA48/2 results [11].

the result obtained by Leutwyler *et al.* ($\Gamma_\sigma/2 = 272_{-12.5}^{+9}$ MeV) [12] and the result $\Gamma_\sigma/2 = (255 \pm 16)$ MeV obtained by Peláez *et al.* [13]. However, this limit also implies that $\Gamma_{\rho \rightarrow \pi\pi} = \Gamma_{a_1 \rightarrow \rho\pi} = 0$, which is clearly at odds with experiment [9].

4. Conclusions

The σ model without (axial-)vector degrees of freedom can properly describe $\pi\pi$ scattering and $f_0(600)$ phenomenology. This is also one of the reasons of its success in low-energy hadron physics. However, the inclusion of vector and axial-vector fields drastically worsens the agreement with data: a simultaneous description of both $\pi\pi$ scattering lengths and $f_0(600) \rightarrow \pi\pi$ is no longer possible. This is due to the fact that a notable decrease in the decay width of the $f_0(600)$ resonance, $\Gamma_{\sigma \rightarrow \pi\pi} \cong 100$ MeV, takes place and due to the important role of the ρ meson in $\pi\pi$ scattering.

In light of these results one could be lead to neglect (axial-)vector degrees of freedoms. This is, however, not feasible: vector and axial-vector mesons are well-known quark–antiquark states which should necessarily be part of a realistic low-energy model. Neglecting them would imply that they decouple from the pseudoscalar Goldstone bosons, which is obviously at odds with the experimentally measured decay widths $\Gamma_{\rho \rightarrow \pi\pi}$ and $\Gamma_{a_1 \rightarrow \rho\pi}$. A way out of the problem is to change the scenario for scalar mesons by identifying the scalar fields of the model with $\{f_0(1370), a_0(1450)\}$ (as done — for instance — in Refs. [14, 15]). Still, a light scalar resonance with mass of about 400 MeV is needed to describe $\pi\pi$ scattering: the resonance $f_0(600)$ can be interpreted as an additional tetraquark state. In this way mixing of tetraquark and quarkonia takes place [15]. A detailed study of this possibility and the generalization to $N_f = 3$ (involving also the scalar glueball) are possible extensions of the present approach.

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