

GLUEBALLS AND STATISTICAL MECHANICS OF THE GLUON PLASMA*

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Describing the gluon plasma as an ideal gas of transverse gluons with a temperature-dependent mass $m(T)$ allows to reproduce its equation of state, computed in pure glue lattice QCD. Dealing with $m(T)$ demands to build a thermodynamically consistent framework. We propose a general way to obtain such a formalism and apply it to the gluon plasma. Then we argue that the peculiar behavior of $m(T)$ near the critical temperature T_c is due to color interactions between the gluons. Using an effective glueball model with a lattice QCD-derived screened color interaction, we show that those interactions are strong enough to bind the lightest glueballs up to $1.13 T_c$. Moreover, the thermodynamical properties of the gluon plasma can be understood by describing it as a mixture of an ideal gas of free gluons and glueballs, with a temperature-dependent glueball abundance.

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1. The gluon plasma

It is expected that, at high enough temperatures or densities, a phase transition from hadronic matter to quark–gluon plasma will occur. Experimental evidences for such a new state of matter seem to have been found at RHIC, while on the theoretical side there exists a great amount of works devoted to the study of the quark–gluon plasma and involving various approaches like perturbative methods, potential models, AdS/QCD duality, lattice QCD, *etc.*

In the present work, which is a summary of Ref. [1], we focus on the gluon plasma, that is pure glue QCD above the critical temperature. The equation of state of an SU(3) gluon plasma has been obtained in lattice

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QCD [2]; results are shown in Fig. 1. Two important features of the energy and entropy are observed: A sharp increase just after T_c and an apparent saturation below the Stefan–Boltzmann constant in the range $T/T_c \approx 2$ –5. There have been many attempts to understand those lattice results by using effective approaches, especially models taking explicitly into account bound states above T_c , or quasiparticle models describing the gluon plasma as an ideal gas of gluons, see [3, 4] for example. Let us now show how to build a quasiparticle approach reproducing lattice QCD.

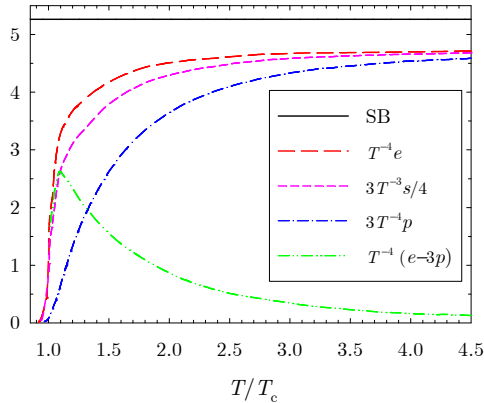


Fig. 1. Energy density, entropy density, pressure, and trace anomaly of the gluon plasma *versus* T/T_c , as measured in pure glue lattice QCD [2] (dashed lines). The horizontal line is the Stefan–Boltzmann limit for a gas of massless transverse gluons.

2. Ideal boson gas

The formal expressions of the energy density, entropy density and pressure of an ideal Bose gas, respectively denoted e_0 , s_0 , and p_0 , are standard in statistical physics, see [5] for example. They involve the Bose–Einstein probability density, $[e^{\beta\epsilon} - 1]^{-1}$, with ϵ the energy of a boson. Assuming that the gluon plasma is an ideal gas of massless gluons, *i.e.* $\epsilon = k$ with k the gluon’s momentum, one gets the Stefan–Boltzmann limit, which does not correspond to the lattice data as shown in Fig. 1. A nonzero constant gluon mass or a momentum cutoff cannot reproduce the data either. One has thus to introduce a temperature-dependent gluon mass, $m(T)$, and to use the dispersion relation $\epsilon = \sqrt{k^2 + m^2(T)}$.

When ϵ depends on the temperature, the first and second laws of thermodynamics are not trivially satisfied. We have shown in Ref. [1] that the usual expressions e_0 , s_0 , and p_0 can still be used provided that β is no longer equal to $1/T$, but rather computed through the relation $T = 1/f(\beta)$, where $f(\beta)$ satisfies the first-order nonlinear differential equation:

$$f(\beta) = \beta \left[1 - \frac{\overline{\partial_\beta \epsilon(T = 1/f(\beta))}}{\partial_\beta e_0(T = 1/f(\beta))} \right], \quad (1)$$

$\overline{\partial_\beta \epsilon}$ being the phase-space average of $\partial_\beta \epsilon$. In standard cases where $\partial_\beta \epsilon = 0$, $f(\beta) = \beta$ is recovered. This general procedure preserves the formal expressions of all the relevant observables of the problem: The only modification is the definition of β , which is not a physical parameter in itself. For computational applications however, this formalism is rather complicated since it demands an *a priori* knowledge of the solution of Eq. (1). It is not the case here since $m(T)$ is a free parameter of the model; therefore it is of interest to find alternative solutions.

One can choose to keep the standard link $\beta = 1/T$ as well as the formal expression of one thermodynamical quantity. Then, the definitions of the two other have to be modified. For example, if the energy density is given by $e = e_0$, the thermodynamical consistency imposes

$$s = s_0 + B, \quad p = p_0 + \frac{B}{\beta}, \quad \text{with} \quad B = \int_{\beta_\star^{(1)}}^{\beta} \nu \overline{\partial_\beta \epsilon}|_{\beta=\nu} d\nu. \quad (2)$$

That solution has been previously used in Ref. [6]. Similarly, either the entropy can be kept invariant, as done in Ref. [7], or the pressure [8].

The thermal gluon mass $m(T)$ can be fitted on one thermodynamical quantity obtained in lattice QCD by using one of the three possible alternative solutions. The observed qualitative behavior is always of the form of $m(T) \sim T + (T - T_c)^{-0.4}$. It can be analytically shown that the linear behavior of $m(T)$ at large T is needed to reach the apparent saturation below the Stefan–Boltzman limit, while the singularity near T_c allows to reproduce the sharp increase of e and s at $T \sim T_c$. An asymptotic linear part, in a temperature range which is not too large, is actually expected from perturbative QCD, where it can be shown that $m(T) \propto \sqrt{\alpha_s(T)} T$ [9]. Near T_c however, the ideal gas picture is questionable. In that temperature range indeed, lattice QCD shows that the color interactions are still large, although non confining [10]. The dispersion relation $\sqrt{k^2 + m^2(T)}$ should then be replaced by $\sqrt{k^2 + \bar{m}^2(T)} + V(r, T)$, with V an interaction potential between the gluons. In a regime where the interactions become dominant, $m(T \sim T_c)$ thus mainly accounts in an effective way for an average potential “felt” by a gluon in the plasma.

3. Mixed glueball–gluon model

A more consistent description of the gluon plasma in terms of an ideal gas could be the following: Color interactions above T_c are strong enough to generate glueballs (bound states of gluons), and the gluon plasma is rather a mixture of an ideal glueball gas and an ideal gas of transverse gluons.

The expected glueballs to be produced in the gluon plasma should be two-gluon ones in a color singlet channel since they are the lightest and presumably the most strongly bound ones. The dynamics of the gluon pair also comes into play: The minimal allowed value of the square orbital angular momentum is $\langle \vec{L}^2 \rangle = 2$, corresponding to the $0^{\pm+}$ glueballs as shown in Ref. [11]. The mass spectrum of the lightest glueballs can be computed from the spinless Salpeter Hamiltonian $2\sqrt{\vec{p}^2 + T^2} + (9/4)V_{qq}(r, T)$. T is approximately the mass of a free gluon in the plasma, that is $m(T)$ in the asymptotic regime, and $V_{qq}(r, T)$ is the static internal energy of a quark–antiquark pair, which has been computed on the lattice [10]. The lowest-lying glueball mass we find from the latter Hamiltonian is around 1.8 GeV at $T \sim T_c$, that is a value compatible with the one at $T = 0$, and then quickly increases to reach a plateau at about 2.8 GeV. We, moreover, find that the ground state is bound up to $T = 1.14 T_c$ and then dissociates in the medium above this temperature. Glueballs are nevertheless expected to be present at higher temperatures as resonances in the continuum, see Ref. [12] for example, in which glueball resonances are found up to $1.9 T_c$.

Since glueballs can be present in the deconfined medium, we can recompute the thermodynamical properties of the gluon plasma by assuming that it is a mixture of an ideal gas of transverse gluons and an ideal gas of glueballs. The free parameter is now the glueball abundance $n(T)$, that depends on the temperature and can be fitted on the lattice energy for example.

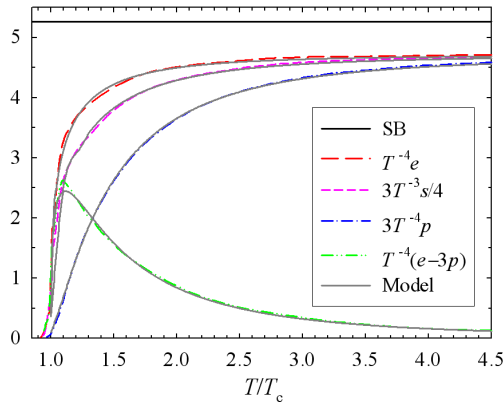


Fig. 2. The same as Fig. 1, but the results obtained with the mixed glueball–gluon model are plotted for comparison.

It has been done in details in Ref. [1]; let us then just mention the result, which is that $n(T)$ is of the form $n(T) \approx e^{-3.4(T/T_c-1)^{0.5}}$. The glueball abundance is very large near the critical temperature and logically becomes negligible after the dissociation temperature. We have checked that the quantitative behavior of $n(T)$ is not very sensitive to the glueball mass; the key result is rather that free gluons alone are unable to fit the available data. Finally, our mixed glueball–gluon model successfully compares to the complete lattice QCD data, as shown in Fig. 2.

4. Conclusion

The equation of state of the gluon plasma, coming from pure gauge lattice QCD computations, can be accurately reproduced by modeling the gluon plasma as a gas of transverse gluons with a temperature-dependent mass. In that case, standard formulas in statistical mechanics have to be modified in order to enforce the thermodynamical consistency. We have shown that all the existing modified formalisms can be derived in a simple unified way, and that a new possible framework exists in which the standard form of each thermodynamical quantity is preserved while β is no longer equal to $1/T$.

Then it can be shown that, independently of the considered formulation, reproducing the lattice data leads to constraints on the thermal gluon mass. It must be strongly decreasing just after the critical temperature and grow linearly asymptotically. The linear growth can be expected from perturbative QCD while the singular behavior of the thermal gluon mass near T_c accounts for residual color interactions, which are still strong in the early stages after deconfinement. It appears that they are actually strong enough to bind the lightest glueballs up to $T \sim 1.13 T_c$.

Therefore, we have proposed to describe the gluon plasma as a mixed glueball–gluon gas, with a temperature-dependent glueball abundance. This model is able to reproduce accurately the lattice data while offering a more consistent physical picture and drawing a bridge between the quasiparticle approach and other models focusing on the existence of bound states after deconfinement. The very large glueball abundance near T_c suggests that the relevant degrees of freedom of pure glue QCD near the phase transition are glueball ones rather than gluonic ones. The nontrivial contribution of glueballs near T_c has also been pointed out in the recent work [13]. Finally, it follows from our study that the gluon plasma, and thus presumably the quark–gluon plasma, might be a glueball-rich medium in the range $T/T_c \simeq 1.0$ – 1.5 . An experimental observation of glueball decays in the early stages after deconfinement, especially in the $\gamma\gamma$ channel, might thus be possible in future experiments.

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