THE PHYSICS OF GLUEBALLS*

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I present a model for glueballs with two and three constituent gluons. I show that, even if spin-1 gluons seem to reproduce properly the lattice QCD spectrum for $C = +$ states, the extension for $C = -$ cannot match with the lattice results. Resorting to the helicity formalism, we show how transverse gluons fit in better agreement the lattice QCD spectrum. We then conclude that even if gluons gain an effective mass, they remain transverse particles.

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1. Introduction

Quantum Chromodynamics (QCD) allows the self-coupling of the gauge bosons, the gluons. Therefore, states with no valence quarks, the glueballs, are a beautiful consequence and prediction of QCD. Recently, a comprehensive review was devoted to the physics of glueballs [1]. Here I present a short summary of this review with a special emphasis on constituent models. The interested reader will find technical details about other techniques and more references in the review [1].

The observation of glueballs, however, remains difficult, probably because the lightest one, the scalar 0^{++}, should mix with mesons [2]. Some experimental glueball candidates are currently known, such as the $f_0(1370)$, $f_0(1500)$, $f_0(1710)$, ... but no definitive conclusions can be drawn concerning the nature of these states [1–3].

On the other hand, pure gauge QCD has been investigated by lattice QCD for many years, leading to a well established glueball spectrum below 4 GeV [4]. Our aim is to reproduce this hierarchy with the most simple models with constituent gluons. Since two gluons can only bind into positive-C, we have to consider three-gluon glueballs to account for the existence of low-lying negative-C states.

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I report the main results of lattice QCD and QCD spectral sum rules in, respectively, Sections 2 and 3. Section 3 is devoted to constituent models.

2. Lattice

The pure gauge spectrum was investigated on a lattice by Morningstar and Peardon [4]. They found 15 glueballs states with various quantum numbers below 4 GeV, see Fig. 1. The scalar glueball is resolved to be the lightest glueball. It is worth also mentioning that no $1^{\pm +}$ state was found, at least below 4 GeV. When including dynamical quarks in the gluonic operators one expects a decreases of the glueball masses. However, no definitive conclusion can be drawn [1].

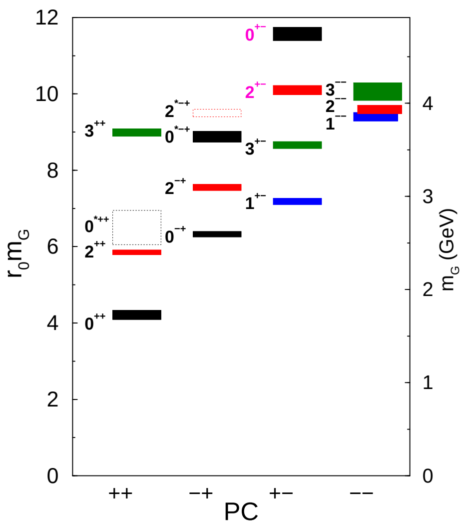

Fig. 1. Lattice spectrum.

3. QCD spectral sum rules

The QCD spectral sum rules is a powerful technique to describe hadronic resonances. In the glueball sector, a comprehensive study of the low-lying gluonic correlators was performed by Forkel [5]. He showed how instanton induced forces are needed to satisfied low-energy theorems and stability under the sum rules. Instanton forces are resolved to be attractive in the scalar and repulsive in the pseudoscalar channels, leading to the following masses

$$m_{\rm S} \approx 1.25 \text{ GeV}, \qquad m_{\rm P} \approx 2.2 \text{ GeV}. \tag{1}$$

Forkel's spectrum is somehow lower than pure gauge lattice masses. From which we can expect a decrease of lattice masses in the unquenched case.

4. Constituent models

4.1. Two-gluon glueballs

In Ref. [6], the authors provide a relevant model of two-gluon glueballs. Assuming Casimir scaling for the string tension of the flux tube, the Hamiltonian, endowed with One-Gluon Exchange (OGE) potentials, reads

$$H_{gg} = 2\sqrt{\boldsymbol{p}^2 + m^2} + \tfrac{9}{4}\sigma r + V_{\text{oge}}(r; \alpha_{\text{S}}, \mu; \boldsymbol{S}, \boldsymbol{L})\,. \tag{2}$$

Although they use a bare mass $m = 0$ in the kinetic term, their gluons have longitudinal components and are spin-1 particles. Therefore, many states are degenerate and the authors resorted to spin-dependent potentials coming from the OGE to lift these degeneracies. The corrections are of the order of μ^{-2}, where $\mu = \left\langle \boldsymbol{p}^2 \right\rangle$ is an effective constituent mass. The parameters were fitted on the low-lying states and the final spectrum is displayed in Fig. 2 (left).

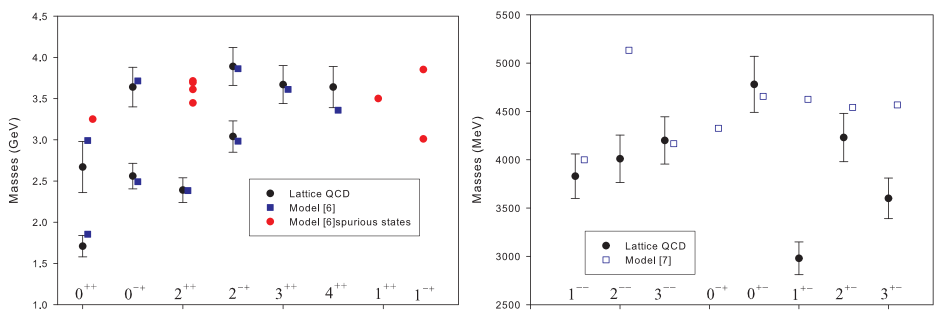

Fig. 2. Left: Spectrum of Hamiltonian (2.1) with longitudinal gluons. Right: Spectrum of Hamiltonian (3.1) with longitudinal gluons.

All states (squares) fall into lattice error bars. However, we noticed some spurious states (circles) not found by any lattice study. For instance, $J = 1$ states are forbidden by Yang's theorem and should not be present in the spectrum of two-gluon bound states. The appearance of such states is induced by the longitudinal component of gluons and should disappear when considering transverse gluons.

4.2. Three-gluon glueballs

Let us forget about the spurious states for the moment and let us generalize the model of the previous section for three-gluon glueballs. We used a generalisation of the flux tube for the confinement. In heavy baryons, the confinement has a Y-shape, but in our case, we replaced it by a center-of-mass junction. The Hamiltonian is supplemented by the potential coming from the OGE and reads:

$$H_{ggg} = \sum_i^3 \sqrt{\boldsymbol{p}_i^2} + \frac{9}{4} f\sigma |\boldsymbol{r}_i - \boldsymbol{R}_{\mathrm{cm}}| + \sum_{i<j} V_{\mathrm{oge}}(r_{ij}; \alpha_{\mathrm{S}}, \mu; \boldsymbol{L}_{ij}, \boldsymbol{S}_{ij}) \,. \qquad (3)$$

We refer the reader to Ref. [7] for further details concerning the Hamiltonian. This semirelativistic Hamiltonian is easily diagonalized in a Gaussian basis [8, 9].

We impose the symmetric colour function $d_{abc} A^a_\mu A^b_\nu A^c_\rho$, which ensures a negative C-parity. Then the spin symmetry determines the symmetry of the space wave-function. Since the coupling of three spin-1 is given by

$$\mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1} = \mathbf{3}_s \oplus \mathbf{2}_m \oplus \mathbf{1}_s \oplus \mathbf{0}_a \,, \qquad (4)$$

the lowest state with 2^{--} has a mixed symmetry and cannot lie in the same mass range as 1^{--} and 3^{--}, as it was already noticed in Ref. [10]. Moreover, a positive parity requires an odd angular momentum. Then, all $(0,1,2,3)^{+-}$ are degenerate with a large component $L = 1$ in the wave function. But the lattice QCD exhibits a gap around 2 GeV between 0^{+-} and 1^{+-} which are respectively the highest and lowest states with a negative conjugation charge. This gap cannot be reproduced within this model and the spectrum of the Hamiltonian (3.1), shown in Fig. 2 (right), is nearly in complete disagreement with lattice QCD. The symmetry arguments are Hamiltonian-independent and we can, therefore, conclude that models with longitudinal gluons are not appropriate to reproduce the lattice pure gauge spectrum.

4.3. Transverse gluons

In order to solve the problems encountered (spurious states, hierarchy in the PC $= +-$ sector), we implemented a formalism developed by Jacob and Wick [11]. This formalism allows us to handle transverse particles. Indeed, in the previous models, longitudinal components implied extra states. When applying it to two-gluon glueballs, we remarked that the Bose symmetry (and the parity) implies selection rules. Three families were identified [12]: $(2k)^{++}, (2k+3)^{++}, (2k+2)^{-+}$ with $k \in N$. One easily checks that no spurious $J = 1$ states appear. Moreover, with this special construction, we can project out the wave function on the usual spectroscopic basis. For the low-lying states, we have the following helicity structures in term of spectroscopic states:

$$\left|S_+;0^{++}\right\rangle = \left|(++)+(--)\right\rangle = \sqrt{\frac{2}{3}}\left|{}^1S_0\right\rangle + \sqrt{\frac{1}{3}}\left|{}^5D_0\right\rangle\,, \tag{5a}$$

$$\left|S_-;0^{-+}\right\rangle = \left|(++)-(--)\right\rangle = \left|{}^3P_0\right\rangle\,, \tag{5b}$$

$$\left|D_+;2^{++}\right\rangle = \left|(+-)+(-+)\right\rangle = \sqrt{\frac{2}{5}}\left|{}^5S_2\right\rangle + \sqrt{\frac{4}{7}}\left|{}^5D_2\right\rangle + \sqrt{\frac{1}{7}}\left|{}^5G_2\right\rangle\,. \tag{5c}$$

With this decomposition, it is now easily to compute matrix elements. Moreover, one does not need to use complicated spin-dependent potentials (since the orbital wave function are different).

We checked the wave functions using a simple Hamiltonian:

$$H_{gg} = 2\sqrt{\boldsymbol{p}^2} + \frac{9}{4}\sigma r - 3\frac{\alpha_{\mathrm{S}}}{r}\,. \tag{6}$$

The resulting spectrum, displayed in Fig. 3 is in good agreement with the lattice QCD data without the inclusion of spin-dependent potentials. But instanton-induced interactions were needed for $J = 0$ states. Indeed, with this simple Hamiltonian scalar and pseudoscalar states are degenerate. But, as shown by Forkel using QCD spectral sum rules [5], one has to add an attractive (repulsive) force in the 0^{++} (0^{-+}) glueball correlator.

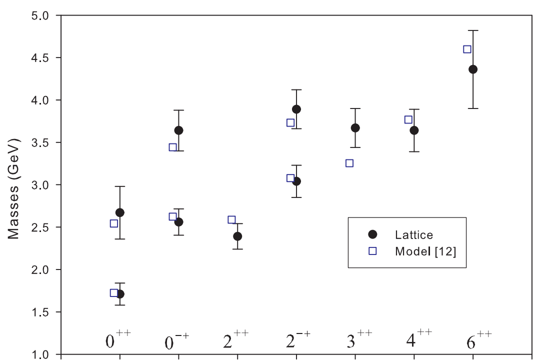

Fig. 3. Spectrum of Hamiltonian (4.1) with transverse gluons.

In addition, all states are present with no spurious state. We then conclude that, in order to reproduce the glueball hierarchy observed in lattice QCD, one has to enforce that the lightest states with a positive C-parity are bound states of two transverse gluons.

The next step is to implement this formalism for three-gluon glueballs. This work is in progress. However, we have some indications that the lowest-lying three-gluon glueballs with transverse gluons are spin 1 and 3 [13]. Symmetry arguments are also in favor of a four-gluon interpretation for 0^{+-}.

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