

LATTICE QCD AND THREE-BODY EXOTIC SYSTEMS IN THE STATIC LIMIT*

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The static potentials for quark–antiquark–gluon and 3-gluon systems are computed with lattice QCD methods. For the quark–antiquark–gluon hybrid meson the static potential is obtained for different values of the angle between the quark–gluon and antiquark–gluon segments. The simulations support the formation of an adjoint string for small angles, while for large angles, the adjoint string is replaced by two fundamental strings connecting the gluon and the quarks. For the 3-gluon glueball, we discuss the corresponding Wilson loops and show that the gluons are connected by fundamental strings when the gluons are far apart.

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1. Introduction

We explore, in lattice QCD, the static potential of three-body systems with gluon(s) using Wilson loops, namely the quark–antiquark–gluon hybrid and the three gluons glueball. The interest in this systems is increasing because of the future experiments BESIII at IHEP in Beijin, GLUEX at JLab and PANDA at GSI in Darmstadt, dedicated to study the mass range of charmonium, with a focus in its plausible excitations and in hybrid and glueball production. The three-gluon glueballs are also relevant to the odderon. Thus several models are being developed and the static potential should, at least, provide one important component of the full interaction. The first

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lattice studies of the gluon interactions were performed by Michael [1, 2] and Bali extended them to other $SU(3)$ representations [3]. Okiharu and colleagues [4, 5] studied tetraquarks and pentaquarks on the lattice. Bicudo, Cardoso and Oliveira studied the hybrid quark–antiquark–gluon static potential, [6, 7].

We compare different models of confinement, namely the Casimir Scaling, type I and type II superconductor models. In the type I model, adjoint strings are formed while in the type II model, the fundamental strings repulse and adjoint strings are not formed. This comes from the analogy with the two types of condensed matter superconductors (see Fig. 1).

In the case of the hybrid meson type II models mean a fundamental string linking the quark and the antiquark to the gluon, while type I models require an adjoint string. For the three-gluon glueball, the type II means three fundamental string linking the gluons in a triangle shape, while for the type I the three adjoint strings fuse at a common point giving rise to a starfish geometry (see Fig. 1).

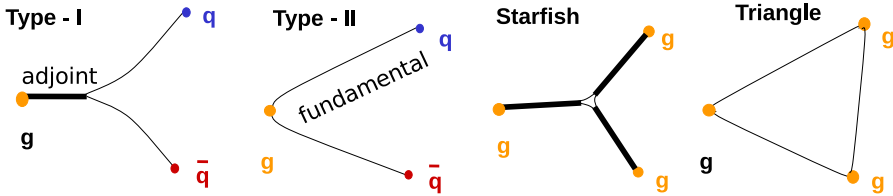


Fig. 1. The two models of confinement: type I and type II.

2. Wilson loops

To measure the static potentials in lattice QCD, we construct an operator — the Wilson loop — which corresponds to our system. The mean value of this operator could be expanded in Euclidean time as

$$\langle W(t) \rangle = \sum_n C_n e^{-V_n t} \quad (1)$$

with V_0 being the static potential of the system.

For the case of a meson this operator is simply given by a closed loop of lattice links

$$W_{q\bar{q}}(R, T) = \text{Tr} \left[U_\mu(0, 0) \dots U_\mu(R-1, 0) U_4(R, 0) \dots U_4(R, T-1) \right. \\ \left. \times U_\mu(R-1, T)^\dagger \dots U_\mu(0, T)^\dagger U_4(0, T-1)^\dagger \dots U_4(0, 0)^\dagger \right]. \quad (2)$$

In the cases of hybrid mesons and glueballs, we have lines corresponding not only to quarks and antiquarks, but also to gluons. Since the gluons are in the adjoint representation of SU(3) the gluonic lines correspond to adjoint paths in the lattice $\tilde{U}^{ab} = \frac{1}{2} \text{Tr}[\lambda^a U \lambda^b U^\dagger]$. This operators could be simplified by using the Fierz relation:

$$\sum_a \lambda_{ij}^a \lambda_{kl}^a = 2\delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl}. \tag{3}$$

For the results in this work, we used 141 $24^3 \times 48$ SU(3) lattice configurations with $\beta = 6.2$, generated with MILC code [9].

2.1. Quark-antiquark-gluon Wilson loop

In the case of the quark-antiquark-gluon the Wilson loop is given by [6, 7] (Fig. 2):

$$\begin{aligned} W_{q\bar{q}g} = & \text{Tr} \left[U_4(x, 0) \dots U_4(0, t-1) \lambda^a U_4^\dagger(0, t-1) \dots U_4^\dagger(0, 0) \lambda^b \right] \\ & \times \text{Tr} \left[\lambda^b L_1(0) U_4(\mathbf{x}_1, 0) \dots U_4(\mathbf{x}_1, t-1) L_1^\dagger(t) \right. \\ & \left. \times \lambda^a L_2(t) U_4^\dagger(\mathbf{x}_2, t-1) \dots U_4^\dagger(\mathbf{x}_2, 0) L_2^\dagger(0) \right], \end{aligned} \tag{4}$$

where $L_i(t)$ is a spatial path that links the origin (where the gluon is) to the position \mathbf{x}_i in the time slice t . Using (3), this operator becomes $W_{q\bar{q}g} \propto W_1 W_2 - \frac{1}{3} W_3$ where W_1, W_2 and W_3 are the three fundamental (*i.e.* mesonic) Wilson loops.

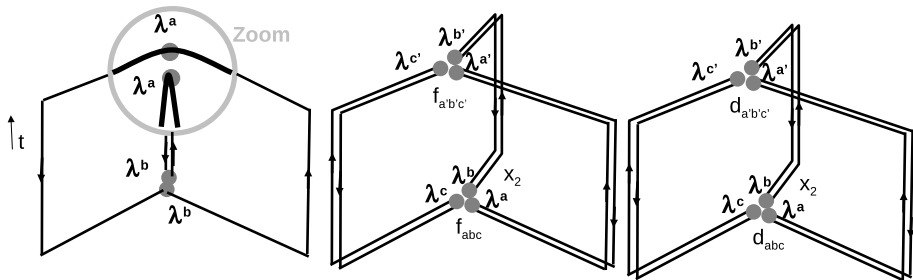


Fig. 2. Left: Quark-antiquark-gluon Wilson loop. Center and right: Antisymmetric and symmetric three-gluons Wilson loop.

2.2. Three gluons Wilson loop

For the three gluons there are two possible colour wavefunctions, corresponding to opposite charge conjugation properties. One is antisymmetric and the other symmetric for the permutation of two gluons. For this case, the Wilson loop operators have the form [8]

$$W_{3g}^{A/S} = T_{abc} T_{a'b'c'} \tilde{P}_1^{aa'} \tilde{P}_2^{bb'} \tilde{P}_3^{cc'}, \quad (5)$$

where $T_{abc} = f_{abc}$ for the antisymmetric case and $T_{abc} = d_{abc}$ for the symmetric, and

$$P_i = L_i(0) U_4(\mathbf{x}_i, 0) \dots U_4(\mathbf{x}_i, t-1) L_i^\dagger(t). \quad (6)$$

The two three-gluon Wilson loops, as a function of fundamental paths are given by

$$W_{3g}^A \propto \text{Re} \left\{ \text{Tr} \left[P_1 P_2^\dagger \right] \text{Tr} \left[P_2 P_3^\dagger \right] \text{Tr} \left[P_3 P_1^\dagger \right] \right\} - \text{Re} \text{Tr} \left[P_1 P_2^\dagger P_3 P_1^\dagger P_2 P_3^\dagger \right] \quad (7)$$

and

$$\begin{aligned} W_{3g}^S \propto & \text{Re} \text{Tr} \left[P_1 P_2^\dagger P_3 P_1^\dagger P_2 P_3^\dagger \right] + \text{Re} \left\{ \text{Tr} \left[P_1^\dagger P_2 \right] \text{Tr} \left[P_2^\dagger P_3 \right] \text{Tr} \left[P_3^\dagger P_1 \right] \right\} \\ & - \frac{2}{3} \text{Tr} \left[P_1 P_2^\dagger \right] \text{Tr} \left[P_1^\dagger P_2 \right] - \frac{2}{3} \text{Tr} \left[P_2 P_3^\dagger \right] \text{Tr} \left[P_2^\dagger P_3 \right] \\ & - \frac{2}{3} \text{Tr} \left[P_3 P_1^\dagger \right] \text{Tr} \left[P_3^\dagger P_1 \right] + \frac{4}{3}. \end{aligned} \quad (8)$$

3. Static potential results

3.1. Hybrid meson

For the hybrid meson, we study the static potential of the system as a function of the quark–gluon distance (r_1), the antiquark–gluon distance (r_2), and the angle θ between the quark–gluon and antiquark–gluon segments. By using off-axis directions on the lattice we can, not only compute the potential for $\theta = 0^\circ$, 90° and 180° , but also for $\theta = 45^\circ$, 60° , 120° and 135° .

First, we study the results for the hybrid meson for the special case $r_1 = r_2 = r$, for the different values of θ . As can be seen in Fig. 3, for large r the potentials are given by $V(r) \sim \sigma' r$. We can also see that for all angles we have $\sigma' \simeq 2\sigma$, except for $\theta = 0^\circ$, in which case we have $\sigma' = \frac{9}{4}\sigma$, which is the value predicted by Casimir Scaling. This means that for $\theta = 0^\circ$ an adjoint string is formed while for the other angles we essentially have two independent fundamental strings.

Also in Fig. 3, we see the results for the potential with $r_1 = r_2 = r$ as a function of θ for various values of r . This results are fitted to a Coulomb like potential, which in this case changes only with θ in the type II superconductor model. This model fits well the results for large θ and r .

With this results, we can see the existence of two regimes — one with adjoint string formation (for $\theta = 0$) and the other with fundamental strings. To better understand the transition between the two, we are studying a geometry with a shape of a U , given in Fig. 3.

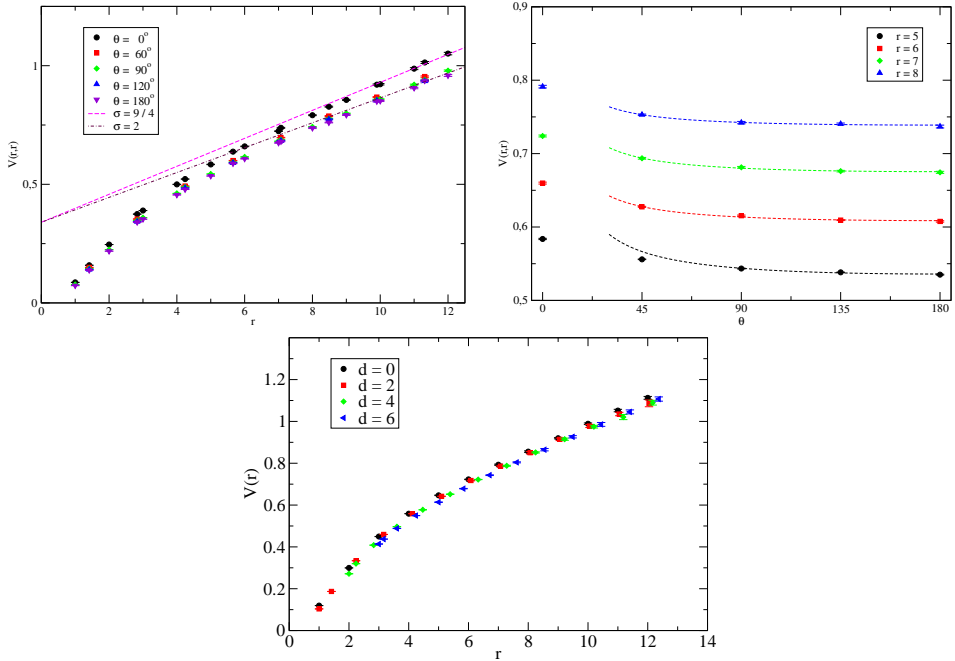


Fig. 3. Upper left: Results for the $V(r, r)$ on the hybrid meson for various angles and comparison with two different string tensions. Upper right: Results for $V(r, r)$ as a function of θ for different values of r . Bottom: Results for the U geometry as function of $r_1 = r_2 = r$ for various values of quark–antiquark distance.

3.2. Three gluons

For the case of the three-gluon glueball, we study two geometries. One, in which the three gluons are in three perpendicular axis and at the same distance of the origin, forming an equilateral triangle, and another in which one of the three gluons is at the origin and the other two are in the axis and at the same distance of the other gluon, drawing a rect isosceles triangle.

For this geometries we study the difference between the two potentials and the two potentials separately. Both results are shown in Fig. 4.

For $V_{\text{symm}} - V_{\text{anti}}$ we see that the difference is systematically positive and rising linearly with the perimeter of the triangle, with a fitted string tension $\sigma_{\text{diff}} = 0.04\sigma$.

By fitting V_{anti} and V_{symm} to a potential of the form $V = C_0 - \alpha \sum_{i < j} \frac{1}{r_{ij}} + \sigma' p$, where p is the perimeter of the triangle, we get $\sigma' = \sigma$, showing that the geometry of the strings is the triangle geometry and not the starfish geometry, which would have given $\sigma' = \frac{9}{4\sqrt{3}}\sigma$ for the equilateral triangle, and $\sigma' = \frac{9(1+\sqrt{3})}{8(1+\sqrt{2})}\sigma$ for the rect isosceles triangle.

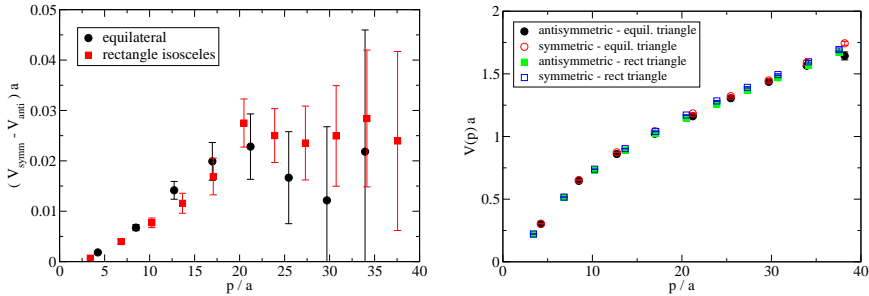


Fig. 4. Left: Results for the difference of the three gluon static potential in the two colour arrangements. Right: Results for the three gluon potential as a function of the perimeter of the triangle formed by the three quarks.

4. Conclusions

From both the results we see that the confinement is essentially described by a type II superconductor model, except when the fundamental strings overlap, in which case adjoint strings are formed. This result is important for constituent quark and gluon models. In the three-gluon glueball this type II superconductor behaviour manifests itself by the triangular string geometry. However, this picture does not account for the formation of adjoint strings and does not explain the little systematic difference between the static potential of the two colour wavefunctions.

In order to understand better the formation of the adjoint string and the validity of the type II superconductor model, we are computing the static distributions of the chromoelectric field in both systems.

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