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UNQUENCHING AND REQUENCHING THE QUARK MODEL*

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Theoretical issues with incorporating virtual quark effects in the constituent quark model are discussed. A formalism for integrating out virtual meson channels is described.

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1. Introduction

The issue of coupled channel effects in the hadron spectrum is coming under increasing scrutiny. This is in part due to continued experimental effort that is pushing higher into the excitation spectrum of light and charmonium mesons. Certainly one expects open and virtual meson–meson channels to affect the mass spectrum at the level of tens (or more) MeV. Similarly, form factors, transition amplitudes, leptonic widths, *etc.* should all be affected at some level. Perhaps the most dramatic manifestation of coupled channel effects would be the genesis of (non-nuclear) hadron–hadron bound states. Many candidates exist, including the $f_0(980)$ (as $K\bar{K}$), the $f_0(1710)$ ($K^*\bar{K}^*$), the $D_{s0}(2317)$ (DK), $X(3872)$ ($D^*\bar{D}$), and $\Lambda(1405)$ (NK).

On the theoretical side, virtual loop effects should disrupt naive valence expectations. For example, the near degeneracy of the ω and ρ mesons is hard to understand in light of the different continua that couple to these particles. The earliest observation of this problem that I am aware of is called the 'Oakes–Yang problem', after Oakes and Yang [1] who noted that the Gell-Mann–Okubo mass formula should be ruined by virtual threshold effects.

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Three different aspects of this problem are reviewed here: *(i)* the efficacy with which coupled channel effects can be absorbed into a low-energy quark model parameters, *(ii)* a set of theorems concerning mixing and mass shifts due to virtual meson loops, *(iii)* a similarity transformation is used to derive an effective Hamiltonian that accounts for virtual loop effects.

2. Unquenching the quark model

The constituent quark model is a nonrelativistic potential model, and therefore does not consider quark pair creation that gives rise to strong decay and coupled channel effects. It is a simple matter to correct this by including the appropriate operator in a second quantised version of the quark model (although one pays the price in turning a quantum mechanical formalism into a field-theoretic formalism). The difficult part is determining the structure of the pair creation operator: this must involve gluonic flux tube breaking and is, therefore, a nontrivial aspect of strongly coupled QCD. At present, there is little evidence upon which to build a model operator; even observing string breaking in lattice gauge theory is extremely difficult [2]. One is forced to model; the most popular model is the '$^3P_0$' model, which postulates quark pair creation with vacuum quantum numbers. An alternative is assuming a quark interaction of the form $\int \psi^\dagger\psi(x)K(x-y)\psi^\dagger\psi(y)$, which often gives similar results to the $^3P_0$ model [3].

A contemporary example of mixing induced by the $^3P_0$ operator is provided by a computation of the mixing amplitude of a $D^*\bar{D}$ molecular X with a $c\bar{c}\,\chi'_{c1}$. Evaluating this matrix element yields surprisingly large mixing amplitudes, even for quite diffuse molecular X states [3]. This has important implications for X phenomenology.

3. Renormalising the quark model

It is convenient to define a simple field theoretical model to examine the possibility of absorbing virtual meson into renormalised model parameters. The model Hamiltonian is

$$\begin{aligned}\hat{H} \;=\; & -\int d^3x\hat{\psi}^\dagger_f\tau_3\left(m_f-\frac{\nabla^2}{2m_f}\right)\hat{\psi}_f+g\int d^3x\hat{\psi}^\dagger_f\tau_1\hat{\psi}_f \\ & +\frac{1}{2}\int d^3xd^3y\hat{\psi}^\dagger_{f'}(x)\hat{\psi}_{f'}(x)V(x-y)\hat{\psi}^\dagger_f(y)\hat{\psi}_f(y)\,. \end{aligned} \tag{1}$$

The potential V is taken to have a simple quadratic form.

The theory is truncated by projecting onto the lowest two Fock space sectors. The resulting Bethe–Heitler equation is solved with the addition of a weakly coupled probe channel to locate the poles in the T-matrix. A typical computation is shown in Fig. 1. Heavy meson poles below threshold are quite narrow and easily seen by the probe channel. Identifying resonance locations above threshold is more difficult and hence Argand diagrams were often employed.

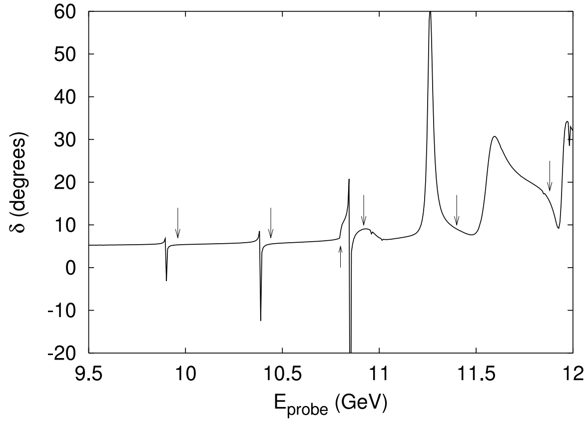

Fig. 1. Probe phase shift for $g = 300$ MeV. The arrows indicate bare pole locations. Threshold is shown at 10.8 GeV.

One finds that up to five of the lowest levels can be accurately reproduced by adjusting the 'quark' mass and the potential spring constant [4].

4. Some unquenching theorems

It is possible to derive general theorems concerning perturbative mass shifts and mixing amplitudes of mesons due to meson loops. These theorems hold assuming that the quark pair creation operator factorises into spin and spatial components and that the virtual mesons fall into degenerate orbital multiplets. Under these conditions [5]

(1) The loop mass shifts are identical for all states within a given N, L multiplet.

(2) These states have the same total open-flavour decay widths.

(3) Loop-induced valence configuration mixing vanishes provided that $L_i \neq L_f$ or $S_i \neq S_f$.

These results have been extended by Close and Thomas [6] to the case of hybrid and canonical meson mixing.

These results have important implications: if the conditions are approximately true it implies that meson mass shifts can be completely absorbed into a constant term in the quark potential. Furthermore, if the virtual channels have masses split by subleading operators, then the mesons that couple to them will follow the same patterns of mass shifts. This implies that the quark model is robust under coupling to virtual meson channels.

5. Requenching the quark model with an effective Hamiltonian

It is evident that dealing with sums over infinitely many continuum meson–meson channels is a technically difficult problem. An attractive alternative is to attempt to integrate out the effects of the transition operator in the model field theory, yielding an effective Hamiltonian in the $q\bar{q}$ sector. Here I propose to do this with a similarity transformation up to the order of $1/m^2$.

To be specific, I will consider a nonrelativistic constituent quark model with the Fock sector mixing driven by the $^3P_0$ model vertex. The result is a nonrelativistic field theory defined in the particle representation as

$$\begin{aligned} H \;=\; & \sum_k \epsilon_k \left(b_k^\dagger b_k + d_k^\dagger d_k \right) + \sum_{pqk} v_q^{(M)} b_{k+q}^\dagger d_{p-q}^\dagger d_p b_k \\ & + \frac{1}{2} \sum_{pqk} V_q^{(B)} b_{k+q}^\dagger b_{p-q}^\dagger b_p b_k + \sum_k \left(b_k^\dagger \gamma_k d_{-k}^\dagger + \text{h.c.} \right) . \end{aligned} \tag{2}$$

The quark pair creation vertex is taken to be a one-body operator with a general kernel denoted γ_k. The $^3P_0$ model is obtained for $\gamma_k = \gamma\sigma \cdot \vec{k}$. The potentials are $V_q^{(M)}$ = Fourier transform of $-\frac{4}{3}\frac{\alpha_s}{r} + br$ and $V^{(B)} = \frac{1}{2}V^{(M)}$.

The next step is to remove the interaction that is off-diagonal in particle number with a unitary transformation, $\psi = e^{-iS}\psi'$ and $H' = e^{iS}He^{-iS}$. We proceed by determining S and H' as a series in $1/m$; thus $S = S_1 + S_2 + S_3 + \ldots$ where S_i is of the order of $1/m^i$. Similarly $H = H_0 + T + V + \mathit{\Gamma}$ where H_0 is of the order of m, T is of the order of $1/m$, and V and $\mathit{\Gamma}$ are of the order of m^0. Here $\mathit{\Gamma}$ represents all interactions that change particle number. The S_i are determined by requiring that particle number changing operators are eliminated at the order of $1/m^i$. Thus, for example,

$$[iS_1, H_0] = -\mathit{\Gamma} \tag{3}$$

determines S_1. At this order one obtains expressions for the effective interaction given by

$$V' + \mathit{\Gamma}' = T + [iS_1, V + \mathit{\Gamma}] + \tfrac{1}{2}[iS_1, [iS_1, H_0]] \tag{4}$$

and S_2 is determined by $[iS_2, H_0] = -\mathit{\Gamma}'$.

Repeating the procedure yields

$$
\begin{aligned}
V'' + \Gamma'' &= [iS_2, V] + [iS_2, \Gamma] + [iS_1, T] + \tfrac{1}{2}\,[iS_1, [iS_2, H_0]] \\
&\quad + \tfrac{1}{2}\,[iS_2[iS_1, H_0]] + \tfrac{1}{2}\,[iS_1, [iS_1, V]] + \tfrac{1}{2}\,[iS_1, [iS_1, \Gamma]] \\
&\quad + \tfrac{1}{6}\,[iS_1, [iS_1, [iS_1, H_0]]] \\
&= [iS_2, V] + \tfrac{1}{2}\,[iS_2, \Gamma] + [iS_1, T] + \tfrac{2}{3}\,[iS_1, T'] \qquad (5)
\end{aligned}
$$

and a final result

$$
\begin{aligned}
H_{\text{eff}} = H'' &= \sum_k \left(m^* + \gamma^2 m^* + \frac{k^2}{2m^*} \right) \left(b_k^\dagger b_k + d_k^\dagger d_k \right) \\
&\quad + \sum_{pkq} V_q^{(M)} b_{k+q}^\dagger d_{p-q}^\dagger d_p b_k \\
&\quad \times \left[1 - \frac{1}{8m^2} \gamma^2 \left(p^2 + (p-q)^2 + k^2 + (k-q)^2 \right) \right] \\
&\quad + \frac{1}{2} \sum_{pkq} V_q^{(B)} b_{k+q}^\dagger b_{p-q}^\dagger b_p b_k \\
&\quad + \frac{1}{8m^2} \sum_{pkq} V_q^{(M)} \left[b_{k+q}^\dagger b_{p-q}^\dagger \gamma_{p-q} \gamma_p b_p b_k + \text{h.c.} \right]
\end{aligned}
$$

with $m = m^*(1+\gamma^2)$.

One sees that the interquark interaction in mesons is modified by a spin-independent interaction at the order of $1/m^2$. It is likely that this interaction will be difficult to disentangle from many other sources of spin-independent corrections to the static potential. Alternatively, the interaction in baryons receives a correction of spin-orbit type. This can have important implications for the famous ‘spin-orbit’ problem in baryons; certainly it is clear that there need not be a simple relationship between mesonic and baryonic spin-orbit forces, as is usually assumed in constituent quark models.

6. Conclusions

Amongst the important conclusions are:

(i) virtual meson loop effects can be largely subsumed into model parameters if attention is restricted to low lying states;

(ii) in an SU(6) limit, a large class of quark creation operators yield identical real and imaginary shifts in pole locations. This observation underpins the stability of the constituent quark model to unquenching effects;

(iii) mixing with the continuum can induce an effective short range interaction. This interaction is in general spin-dependent and can have a substantial effect on phenomenology. Sorting out spin-dependence due to gluon exchange, instantons, relativistic dynamics, and continuum mixing remains a serious challenge for model builders;

(iv) there is a lore that passing the lowest continuum threshold in a given channel is associated with a deterioration of the quality of potential models of hadrons. But we have seen that hadron masses are shifted throughout the spectrum (including below threshold) due to a given channel. Rather, it is the proximity of a continuum channel which can cause local distortions of the spectrum. Of course, the problem in hadronic physics is that continuum channels tend to get dense above threshold.

A number of additional issues complicate the application of these ideas to hadronic physics. For example, the QCD transition operator is determined by nonperturbative gluodynamics and is certainly not as simple as the ${}^3P_0$ model. In addition, as stated above, many continuum channels are present. Summing over these is nontrivial — indeed the sum may not converge. Furthermore, one expects that when the continuum virtuality is much greater than $\Lambda_{\rm QCD}$ quark–hadron duality will be applicable and the sum over hadronic channels should evolve into perturbative quark loop corrections to the quark model potential. Correctly incorporating this into constituent quark models requires marrying QCD renormalisation with effective models and is not a simple task. Finally, pion and multipion loops can be expected to dominate the virtual continuum component of hadronic states (where allowed) due to the light pion mass. This raises the issue of correctly incorporating chiral dynamics into unquenched quark models.

The results presented here imply that constituent quark models must become progressively less accurate high in the spectrum. However, this effect is likely to be overwhelmed by more serious problems: the nonrelativistic constituent quark model must eventually fail as gluonic degrees of freedom are activated and because chiral symmetry breaking (which is the pedestal upon which the constituent quark model rests) becomes irrelevant for highly excited states. It is clear that an exploration of the excited hadron spectrum is required to understand the interesting physics behind the unquenched quark model, gluonic degrees of freedom, and chiral symmetry breaking.

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