Vol. 2 (2009) *Acta Physica Polonica B Proceedings Supplement* No 2

THE FOUR-GLUON VERTEX AND THE RUNNING COUPLING IN LANDAU GAUGE YANG–MILLS THEORY*

Christian Kellermann

Institut für Kernphysik, Technische Universität Darmstadt
Schlossgartenstrasse 9, 64289 Darmstadt, Germany

Christian S. Fischer

Institut für Kernphysik, Technische Universität Darmstadt
Schlossgartenstrasse 9, 64289 Darmstadt, Germany
and
GSI Helmholtzzentrum für Schwerionenforschung GmbH
Planckstr. 1, 64291 Darmstadt, Germany

(Received October 2, 2009)

We summarise results for the running coupling from the four-gluon vertex in Landau gauge, SU(N_c) Yang–Mills theory as given by a combination of dressing functions of the vertex and the gluon propagator. These functions have been determined numerically from the corresponding set of Dyson–Schwinger equations. In the infrared we obtain a nontrivial infrared fixed point which is three orders of magnitude smaller than the corresponding one in the coupling of the ghost-gluon vertex.

PACS numbers: 12.38.Aw, 14.70.Dj, 12.38.Lg, 11.15.Tk

1. Introduction

In recent years the running coupling of Yang–Mills theory has been investigated in a number of approaches; for a review see [1]. These include lattice QCD [2–7], analytic perturbation theory [8, 9], the functional renormalization group [10–12], Dyson–Schwinger equations (DSE) [13–17] and phenomenological extractions from experiment [18, 19]. The goal of these investigations is an extension of our knowledge of the coupling from the large momentum region towards small momenta of the order of $\Lambda_{\rm QCD}$ and below. Perturbation theory alone, plagued by the problem of the Landau

* Presented at the International Meeting "Excited QCD", Zakopane, Poland, February 8–14, 2009.

pole, is clearly insufficient for this task. However, even improving the perturbation series with analytic constraints already leads to a well-defined coupling that has a fixed-point in the infrared [9]. Furthermore, such fixed-points are also found in the frameworks of the Functional Renormalisation Group and Dyson–Schwinger equations (DSEs). In these approaches non-perturbative running couplings can be defined in terms of (gauge dependent) dressing functions of the propagators and primitively divergent vertices of the theory.

There are many possibilities to define renormalisation group invariant couplings from these vertices; for a detailed discussion of the gauge and scheme dependence see [20]. Here, we consider a definition of the coupling from the four-gluon vertex given by [17]

$$\alpha^{4g}\left(p^2\right) = \frac{g^2}{4\pi}\left[\Gamma^{4g}\left(p^2\right)\right] Z^2\left(p^2\right) , \tag{1}$$

where $g^2/4\pi$ is the coupling at the renormalization point μ^2 and $Z(p^2)$ denotes the dressing function of the gluon propagator. The function $\Gamma^{4g}(p^2)$ describes the nonperturbative dressing of the tree-level tensor structure of the four-gluon vertex in the presence of only one external scale p^2. Here we used the asymmetric momentum point $(p, p, p, -3p)$; other choices are possible. Both, $Z(p^2)$ and $\Gamma^{4g}(p^2)$ are determined numerically from their Dyson–Schwinger equations. Below we summarise our calculational scheme and results for $\alpha^{4g}(p^2)$ as obtained in [21].

As discussed above, the infrared behaviour of the coupling Eq. (1) is of particular interest. In general, there are two different types of infrared solutions of Yang–Mills theory, denoted "scaling" and "decoupling", discussed in detail in [22] and references therein. Here we summarise the results of [21], which have been obtained for infrared scaling, corresponding results for decoupling are subject of future work.

2. The four-gluon vertex and its Dyson–Schwinger equation

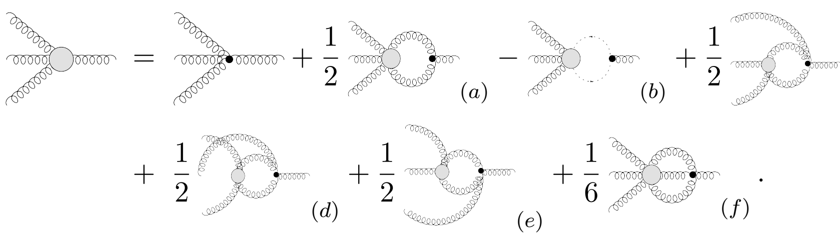
The four-gluon vertex is a highly complicated object with four Lorentz- and four color indices. This complexity forces a two step procedure: one first works with a restricted subset of possible combinations of Lorentz- and color tensors. This reduced complexity allows for a first study of the most important properties of the vertex and its Dyson–Schwinger equation. On the basis of these results one can then attack the full problem in a second step. We will outline the most important parts of the first step (a detailed treatment of can be found in [21]) leaving the second step for future investigations.

2.1. Nonperturbative structure of the four-gluon vertex

A suitable subset of fifteen tensor-structures of the four-gluon vertex has been suggested in [23]. Implementing Bose-symmetry further reduces the number of allowed structures to three, one being the tree-level structure. Since these are quite lengthy we will not present them here, but refer to [21] instead.

2.2. The DSE for the four-gluon vertex

In compact notation the DSE of the four-gluon vertex reads [23]:

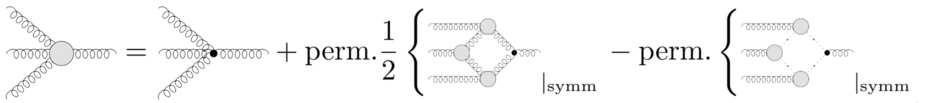


(2)

Here the vertex-blobs denote connected vertex functions which can be further decomposed into irreducible ones (see [23]). Clearly the full DSE is much too complicated to be calculated straightforwardly. Instead we employ a truncation scheme, which results in a much simpler approximate equation, which contains the dominant IR and UV contributions and reproduces asymptotic freedom. This scheme is

- The fully dressed ghost and gluon propagators in the internal loops are taken from their own coupled system of DSEs as given in [22, 24];
- The vertices are decomposed into irreducible ones by means of a dressed skeleton expansion and n-point functions with $n \geq 4$ are neglected;
- The leading diagrams then are identified and the others are dropped. This procedure is described in detail in [21];
- Selfconsistency effects of the four-gluon vertex will be neglected;
- The dressed ghost-gluon vertex will be replaced by the bare vertex [25–27];
- We employ an ansatz for the three-gluon vertex, which is constructed such that it has the infrared behaviour as given in [17] and guarantees the correct UV-behaviour of the four-gluon DSE as known from perturbation theory.

Details can be found in [21]. The resulting approximate equation reads



(3)

Here "perm." denotes permutations of the three external dressed legs of the ghost-box and the gluon-box diagram. The subscript "symm" indicates that we average over all possible locations of the bare vertex in the diagrams thus restoring Bose symmetry on the diagrammatic level.

3. Numerical results

From Eq. (3) we determined the dressing functions of the four-gluon vertex numerically. Using the result for the projection on the tree-level tensor-structure together with Eq. (1) we obtained the momentum dependence of the corresponding running coupling shown in Fig. (1). Also shown in the plot is the corresponding numerical result for the coupling from the ghost-gluon vertex as reported in Ref. [24]. The renormalisation conditions used in both calculations are adapted such that the couplings agree in the ultraviolet momentum regime. The nonperturbative scale inherent in both couplings stems from the gluon propagator and, therefore, is the same for both calculations. Consequently both couplings agree in the ultraviolet momentum regime as expected from perturbation theory.

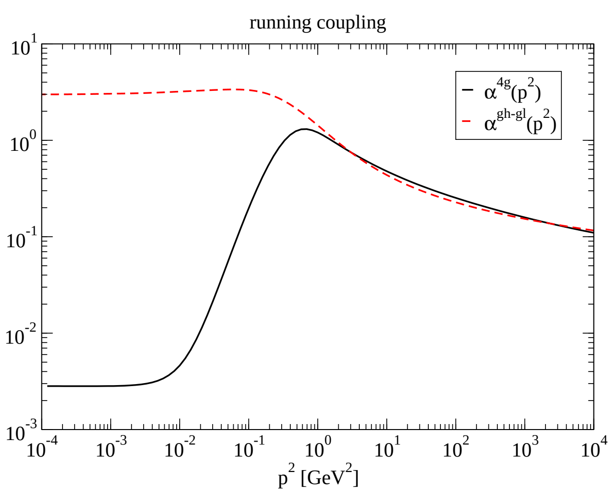


Fig. 1. The running coupling from the four-gluon vertex compared to the coupling from the ghost-gluon vertex from Ref. [24].

At small momenta, however, the couplings start to deviate. Both couplings develop an infrared fixed point which can be analysed analytically, see [15, 21] for details. Here we find

$$\alpha_{4g}\left(p^2 \to 0\right) \approx \frac{0.0083}{N_c}\,. \tag{4}$$

This value agrees well with our numerical result. Note that the infrared fixed-point of the coupling from the four-gluon vertex is three orders of magnitudes smaller than the one from the ghost-gluon vertex. Preliminary results also indicate that a corresponding coupling from the three-gluon vertex is of similar magnitude as our value for the four-gluon coupling [28, 29]. Within the scaling scenario of infrared Yang–Mills theory adopted here this means that the small momentum properties are dominated by the behaviour of the Faddeev–Popov determinant in agreement with the analysis of Zwanziger in Ref. [30].

This work was supported by the Helmholtz-University Young Investigator Grant number VH-NG-332 and by the Helmholtz International Center for FAIR within the framework of the LOEWE program (Landesoffensive zur Entwicklung Wissenschaftlich-Ökonomischer Exzellenz) launched by the State of Hesse.

REFERENCES

[1] G.M. Prosperi, M. Raciti, C. Simolo, *Prog. Part. Nucl. Phys.* **58**, 387 (2007) `[arXiv:hep-ph/0607209]`.

[2] [ALPHA Collaboration] M. Della Morte, R. Frezzotti, J. Heitger, J. Rolf, R. Sommer, U. Wolff, *Nucl. Phys.* **B713**, 378 (2005) `[arXiv:hep-lat/0411025]`.

[3] O. Kaczmarek, F. Zantow, *Phys. Rev.* **D71**, 114510 (2005) `[arXiv:hep-lat/0503017]`.

[4] A. Cucchieri, T. Mendes, *Braz. J. Phys.* **37**, 484 (2007) `[arXiv:hep-ph/0605224]`.

[5] A. Sternbeck, K. Maltman, L. von Smekal, A.G. Williams, E.M. Ilgenfritz, M. Muller-Preussker, *PoS* **LAT2007**, 256 (2007) `[arXiv:0710.2965[hep-lat]]`.

[6] C. Allton, M. Teper, A. Trivini, *J. High Energy Phys.* **0807**, 021 (2008) `[arXiv:0803.1092[hep-lat]]`.

[7] L. von Smekal, K. Maltman, A. Sternbeck, `arXiv:0903.1696[hep-ph]`.

[8] D.V. Shirkov, I.L. Solovtsov, *Phys. Rev. Lett.* **79**, 1209 (1997) `[arXiv:hep-ph/9704333]`.

[9] D.V. Shirkov, I.L. Solovtsov, *Theor. Math. Phys.* **150**, 132 (2007) [arXiv:hep-ph/0611229].

[10] H. Gies, *Phys. Rev.* **D66**, 025006 (2002) [arXiv:hep-th/0202207].

[11] J.M. Pawlowski, D.F. Litim, S. Nedelko, L. von Smekal, *Phys. Rev. Lett.* **93**, 152002 (2004) [arXiv:hep-th/0312324].

[12] J. Braun, H. Gies, *Phys. Lett.* **B645**, 53 (2007) [arXiv:hep-ph/0512085].

[13] L. von Smekal, R. Alkofer, A. Hauck, *Phys. Rev. Lett.* **79**, 3591 (1997) [arXiv:hep-ph/9705242].

[14] R. Alkofer, L. von Smekal, *Phys. Rep.* **353**, 281 (2001) [arXiv:hep-ph/0007355].

[15] C. Lerche, L. von Smekal, *Phys. Rev.* **D65**, 125006 (2002) [arXiv:hep-ph/0202194].

[16] C.S. Fischer, *J. Phys. G* **32**, R253 (2006) [arXiv:hep-ph/0605173].

[17] R. Alkofer, C.S. Fischer, F.J. Llanes-Estrada, *Phys. Lett.* **B611**, 279 (2005) [arXiv:hep-th/0412330]; R. Alkofer, C.S. Fischer, F.J. Llanes-Estrada, K. Schwenzer, *Ann. Phys.* **324**, 106 (2009) [arXiv:0804.3042[hep-ph]].

[18] S.J. Brodsky, S. Menke, C. Merino, J. Rathsman, *Phys. Rev.* **D67**, 055008 (2003) [arXiv:hep-ph/0212078].

[19] M. Baldicchi, A.V. Nesterenko, G.M. Prosperi, C. Simolo, *Phys. Rev.* **D77**, 034013 (2008) [arXiv:0705.1695[hep-ph]].

[20] W. Celmaster, R.J. Gonsalves, *Phys. Rev.* **D20**, 1420 (1979).

[21] C. Kellermann, C.S. Fischer, *Phys. Rev.* **D78**, 025015 (2008) [arXiv:0801.2697[hep-ph]].

[22] C.S. Fischer, A. Maas, J.M. Pawlowski, arXiv:0810.1987[hep-ph].

[23] L. Driesen, M. Stingl, *Eur. Phys. J.* **A4**, 401 (1999) [arXiv:hep-th/9808155].

[24] C.S. Fischer, R. Alkofer, *Phys. Lett.* **B536**, 177 (2002) [arXiv:hep-ph/0202202].

[25] W. Schleifenbaum, A. Maas, J. Wambach, R. Alkofer, *Phys. Rev.* **D72**, 014017 (2005) [arXiv:hep-ph/0411052].

[26] A. Sternbeck, E.M. Ilgenfritz, M. Muller-Preussker, A. Schiller, I.L. Bogolubsky, *PoS* **LAT2006**, 076 (2006) [arXiv:hep-lat/0610053].

[27] A. Cucchieri, A. Maas, T. Mendes, *Phys. Rev.* **D74**, 014503 (2006) [arXiv:hep-lat/0605011]; arXiv:0803.1798[hep-lat].

[28] R. Alkofer, M.Q. Huber, K. Schwenzer, arXiv:0812.4045[hep-ph].

[29] K. Schwenzer, private communications.

[30] D. Zwanziger, *Phys. Rev.* **D69**, 016002 (2004) [arXiv:hep-ph/0303028].