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EXPLORING EXCITED HADRONS IN LATTICE QCD*

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Progress in extracting the spectrum of excited hadron resonances in lattice QCD Monte Carlo calculations is reviewed and the key issues and challenges in such computations are outlined. The importance of multi-hadron states as simulations with lighter pion masses are done is discussed, and the need for all-to-all quark propagators is emphasized.

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Experiments show that many excited-state hadrons exist, and there are significant experimental efforts to map out the QCD resonance spectrum, such as Hall B and the proposed Hall D at Jefferson Lab, ELSA associated with the University of Bonn, COMPASS at CERN, PANDA at GSI, and BESIII in Beijing. Hence, there is a great need for *ab initio* determinations of such states in lattice QCD.

Higher-lying excited hadrons are a new frontier in lattice QCD, and explorations of new frontiers are usually fraught with dangers. Excited states are more difficult to extract in Monte Carlo calculations; correlation matrices are needed and operators with very good overlaps onto the states of interest are crucial. To study a particular state of interest, all states lying below that state must first be extracted, and as the pion gets lighter in lattice QCD simulations, more and more multi-hadron states will lie below the excited resonances. To reliably extract these multi-hadron states, multi-hadron operators made from constituent hadron operators with well-defined relative momenta will most likely be needed, and the computation of temporal correlation functions involving such operators will require the use of all-to-all quark propagators. The evaluation of disconnected diagrams will

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ultimately be required. Perhaps most worrisome, most excited hadrons are unstable (resonances), so the results obtained for finite-box stationary-state energies must be interpreted carefully.

Reliably capturing the masses of excited states requires the computation of correlation matrices $C_{ij}(t) = \langle 0|T\Phi_i(t)\Phi_j^\dagger(0)|0\rangle$ associated with a large set of N different operators $\Phi_i(t)$. It has been shown in Ref. [1] that the N *principal effective masses* $W_\alpha(t)$, defined by

$$W_\alpha(t) = \ln\left(\frac{\lambda_\alpha(t,t_0)}{\lambda_\alpha(t+1,t_0)}\right),$$

where $\lambda_\alpha(t,t_0)$ are the eigenvalues of $C(t_0)^{-1/2}\ C(t)\ C(t_0)^{-1/2}$ and $t_0 < t/2$ is usually chosen, tend to the eigenenergies of the lowest N states with which the N operators overlap as t becomes large. The eigenvectors associated with $\lambda_\alpha(t,t_0)$ can be viewed as variationally optimized operators. When combined with appropriate fitting and analysis methods, such variational techniques are a particularly powerful tool for investigating excitation spectra.

The use of operators whose correlation functions $C(t)$ attain their asymptotic form as quickly as possible is crucial for reliably extracting excited hadron masses. An important ingredient in constructing such hadron operators is the use of smeared fields. Operators constructed from smeared fields have dramatically reduced mixings with the high frequency modes of the theory. Both link-smearing and quark-field smearing should be applied. Since excited hadrons are expected to be large objects, the use of spatially extended operators is another key ingredient in the operator design and implementation. A more detailed discussion of these issues can be found in Ref. [2].

Hadron states are identified by their momentum $\boldsymbol{p}$, intrinsic spin J, projection λ of this spin onto some axis, parity $P = \pm 1$, and quark flavor content (isospin, strangeness, *etc.*). Some mesons also include G-parity as an identifying quantum number. If one is interested only in the masses of these states, one can restrict attention to the $\boldsymbol{p} = \boldsymbol{0}$ sector, so operators must be invariant under all spatial translations allowed on a cubic lattice. The little group of all symmetry transformations on a cubic lattice which leave $\boldsymbol{p} = \boldsymbol{0}$ invariant is the octahedral point group $\mathrm{O_h}$, so operators may be classified using the irreducible representations (irreps) of $\mathrm{O_h}$. The continuum-limit spins J of our states must be deduced by examining degeneracy patterns across the different $\mathrm{O_h}$ irreps.

The key ingredients in our approach to extracting the hadron spectrum of QCD from Markov-chain Monte Carlo calculations of temporal correlations using a space-time lattice are *(i)* gauge field smearing using the stout-link procedure [3], *(ii)* a new quark-field smearing scheme known as Laplacian

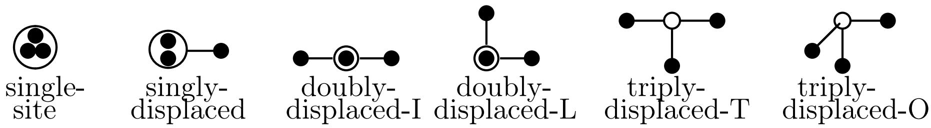

Fig. 1. The spatial arrangements of the extended three-quark baryon operators. Smeared quark-fields are shown by solid circles, line segments indicate gauge-covariant displacements, and each hollow circle indicates the location of a Levi-Civita color coupling. For simplicity, all displacements have the same length in an operator. Results presented here used displacement lengths of $3a_{\rm s}$ (~ 0.3 fm).

Heaviside (Laph), *(iii)* the use of covariantly-displaced, smeared quark fields as our basic building blocks, *(iv)* the use of lattice symmetry operations and group-theoretical projections to assemble the basic building blocks into hadron operators, and *(v)* the use of stochastic estimators with diluted noise in the low-lying Laph subspaces to evaluate the propagators of the basic building blocks.

We use a variety of spatially-extended hadron operators. The use of different spatial configurations (see Fig. 1 for the baryon configurations and Fig. 2 for the meson configurations) yield operators which effectively build up the necessary orbital and radial structures of the hadron excitations. The design of these operators is such that a large number of them can be evaluated very efficiently, and components in their construction can be used for both meson, baryon, and multi-hadron computations.

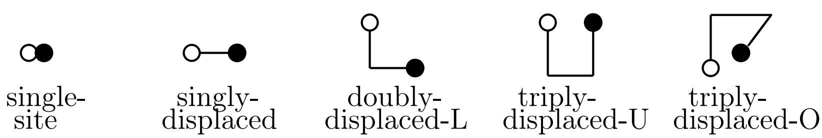

Fig. 2. The spatial arrangements of the quark–antiquark meson operators. In the illustrations, the smeared quarks fields are depicted by solid circles, each hollow circle indicates a smeared "barred" antiquark field, and the solid line segments indicate covariant displacements.

We recently presented results for the nucleon resonances on $N_f = 2$ configurations on a $24^3 \times 64$ lattice using a stout-smeared clover fermion action and a Symanzik-improved anisotropic gauge action [4] with lattice spacing $a_{\rm s} \sim 0.1$ fm and $a_{\rm s}/a_t \sim 3$. Results, shown in Fig. 3, were obtained using 430 gauge configurations with a quark mass yielding a pion mass $m_\pi = 416\,\mathrm{MeV}$ and using 363 gauge configurations for $m_\pi = 578\,\mathrm{MeV}$. The low-lying odd-parity band shows the exact number of states in each channel as expected from experiment. The two figures show the splittings in the band increasing as the quark mass is decreased. At these heavy pion masses, the first excited state in the G_{1g} channel is significantly higher than the experimentally measured Roper resonance. It remains to be seen whether or not this level will drop down as the pion mass is further decreased. The lowest four energies were reported in each of the six irreducible representations of the octahedral group at each pion mass. Clear evidence was found for a $5/2^-$ state in the pattern of negative-parity excited states. This agrees with the pattern of physical states and spin 5/2 has been realized for the first time on the lattice. Most of the levels in these plots lie very close to two-particle thresholds, shown by empty boxes. The use of two-hadron operators will be needed to go to lighter pion masses.

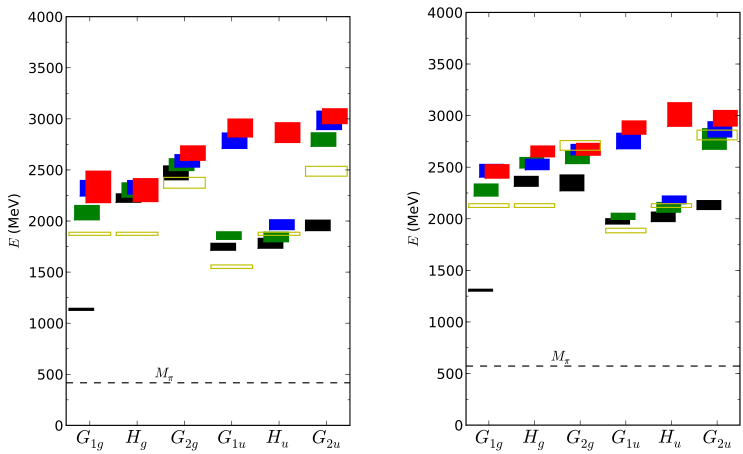

Fig. 3. The energies obtained for each symmetry channel of isospin 1/2 baryons are shown based on the $24^3 \times 64$ $N_f = 2$ lattice QCD data for $m_\pi = 416\,\mathrm{MeV}$ (left panel) and $m_\pi = 578\,\mathrm{MeV}$ (right panel). The scale shows energies in MeV and errors are indicated by the vertical size of the boxes. The overall error in the scale setting is not included. Empty boxes show thresholds for multi-hadron states.

To study a particular eigenstate of interest, all eigenstates lying below that state must first be extracted, and as the pion gets lighter in lattice QCD simulations, more and more multi-hadron states will lie below the excited resonances. Evaluating correlators with multi-hadron sources whose constituents have well defined relative momenta requires quark propagators from all lattice sites on a given time slice to all sites on another time slice. Computing all such elements of the propagators exactly is not possible (except on very small lattices). Hence, some way of stochastically estimating them is needed.

Random noise vectors η whose statistical expectations satisfy $E(\eta_i) = 0$ and $E(\eta_i \eta_j^*) = \delta_{ij}$ are useful for stochastically estimating the inverse of a large matrix M as follows. Assume that for each of N_R noise vectors, we can solve the following linear system of equations: $MX^{(r)} = \eta^{(r)}$ for $X^{(r)}$.

Then $X^{(r)} = M^{-1}\eta^{(r)}$, and

$$E(X_i\eta_j^*) = E\left(\sum_k M_{ik}^{-1}\eta_k\eta_j^*\right) = \sum_k M_{ik}^{-1}E(\eta_k\eta_j^*) = M_{ij}^{-1}\,. \tag{1}$$

The expectation value on the left-hand can be estimated using the Monte Carlo method. Unfortunately, such estimates usually have variances which are much too large to be useful.

Progress is only possible if stochastic estimates of the quark propagators with reduced variances can be made. Techniques of *diluting* the noise vectors have been developed which accomplish such a variance reduction [5]. A given dilution scheme introduces a complete set of $N \times N$ projection matrices $P^{(a)}$. Define $\eta_k^{[a]} = P_{kk'}^{(a)}\eta_{k'}$ and define $X^{[a]}$ as the solution of $M_{ik}X_k^{[a]} = \eta_i^{[a]}$, then we have

$$M_{ij}^{-1} = \sum_a M_{ik}^{-1}E(\eta_k^{[a]}\eta_j^{[a]*}) = \sum_a E\left(X_i^{[a]}\eta_j^{[a]*}\right)\,. \tag{2}$$

The variance of such an estimate is much smaller than that without noise dilution. Of course, the effectiveness of the variance reduction depends on the dilution projectors chosen.

A comparison of different dilution schemes is shown in Fig. 4. The black (red) dashed-dotted line shows the decrease in error expected by simply increasing the number of noise vectors, starting from the time (time + even/odd-space) dilution point. The advantage in using increased dilutions compared to an increased number of noise vectors with only time dilution is evident by the symbols lying below the dashed-dotted lines. Note that time + spin + color + even/odd-space dilution yields an error comparable with the gauge-noise limit using only a single noise vector! A new smearing scheme [6] which utilizes the low-lying eigenvectors of the covariant Laplacian (Laplacian Heaviside smearing) has recently been introduced, and introducing noise vectors and dilution projectors in the subspace spanned by the smearing produces estimates with even smaller variances. A detailed study of this new method should be published soon. These encouraging results demonstrate that the inclusion of good multi-hadron operators will certainly be possible using stochastic all-to-all quark propagators with diluted-source variance reduction.

This talk discussed the key issues and challenges in exploring excited hadrons in lattice QCD. The importance of multi-hadron operators and the need for all-to-all quark propagators were emphasized. Given the major experimental efforts to map out the QCD resonance spectrum, there is a great need for *ab initio* determinations of such states in lattice QCD. The exploration of excited hadrons in lattice QCD is well underway.

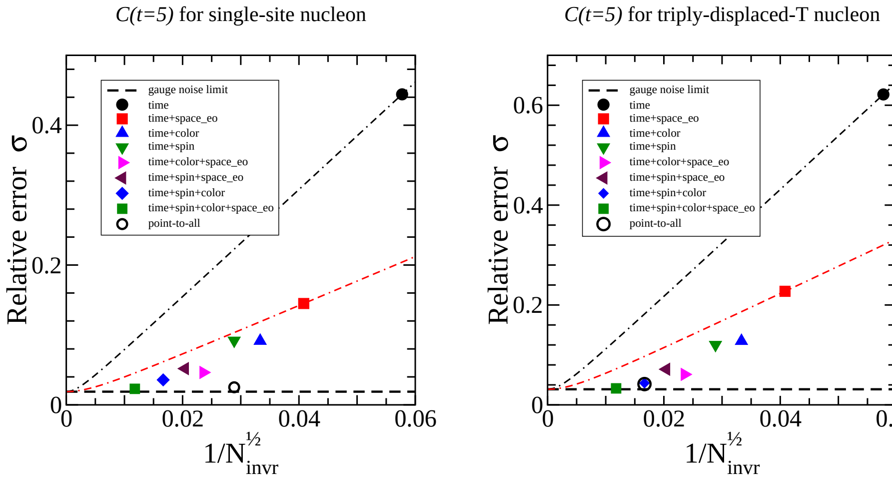

Fig. 4. (Left) the relative errors in the correlation function of a single-site nucleon operator for temporal separation $t = 5a_t$ evaluated using stochastically-estimated quark propagators with different dilution schemes against $1/N_{\rm inv}^{1/2}$, where $N_{\rm inv}$ is the number of Dirac matrix inversions required. The open circle shows the point-to-all error, and the horizontal dashed line shows the gauge-noise limit. The black (red) dashed-dotted line shows the decrease in error expected by simply increasing the number of noise vectors, starting from the time (time + even/odd-space) dilution point. (Right) the same as the left plot, except for a triply-displaced-T nucleon operator. These results used 100 quenched configurations on an anisotropic $12^3 \times 48$ lattice with a Wilson fermion and gauge action.

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