

EXCITED BARYONS AS EXPERIMENTAL PROBES OF THE QUARK MASS*

FELIPE J. LLANES-ESTRADA

Universidad Complutense de Madrid, 28040 Madrid, Spain

PEDRO BICUDO, MARCO CARDOSO

CFTP, Dep. Física, Instituto Superior Técnico, Lisboa, Portugal

TIM VAN CAUTEREN

Department of Subatomic and Radiation Physics, Ghent University
Ghent, Belgium

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We observe that excited hadrons provide an opportunity to probe from experiment the power-law running of the quark mass in the mid infrared, while the condition $m(k) < k$ remains valid. A relatively clean analysis is possible for the maximum spin excitations of the Δ baryons accessible at current experimental facilities such as ELSA and Jlab.

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Quark masses are fundamental parameters of the Standard Model that appear in the Lagrangian density of Quantum Chromodynamics, whose non-interacting part reads

$$\mathcal{L}_{\text{QCD}}^0 = \sum_i m_i \bar{\psi} \psi + i \sum_i \bar{\psi} \not{\partial} \psi. \quad (1)$$

These masses can also be read off an expansion of the quark propagator for large momenta in perturbation theory

$$S(k) = \frac{i}{\not{k} - m} \quad (2)$$

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and information about them can be gathered in several ways. For light quarks, only very coarse information is available from sum rules and lattice gauge theory, although quark mass ratios are somewhat better known from chiral perturbation theory [1].

As the momentum of the propagating quarks becomes smaller towards the QCD scale Λ_{QCD} , the mass in the denominator runs like a power-law

$$S(k) = \frac{iZ(k^2)}{k - m(k^2)} \quad \text{with} \quad m(k^2) \propto k^\alpha \quad (3)$$

as has been shown by Dyson–Schwinger studies [2] in Landau gauge and lattice gauge theory [3]. For even lower k the running mass function $m(k)$ presumably stabilizes and gives sense to the concept of constituent quarks, but to date there is no precise dynamical meaning to the quark masses obtained from quark model fits to data in terms of the QCD parameters.

What we have recently proposed, and elaborate in this article, is that the exponent of the power-law running in the mid-infrared is accessible to experiment by studying excited hadrons. The key idea is to exploit the recently understood parity doubling in the high hadron spectrum, by which excited hadrons decouple from the pion and instead become degenerate with a partner of equal parity, the mass difference being of order the quark mass that wants to be extracted, $|M_+ - M_-| \propto m(\langle k \rangle)$. The scale at which this mass is probed is the average momentum of the quark in the excited hadron. The proportionality constant is difficult to calculate, so the absolute normalization of the quark mass is not readily accessible, but by comparing increasingly excited hadrons with larger and larger $\langle k \rangle$, one can extract its running.

In Hamiltonian dynamics, the running $m(k)$ defined from the propagator appears in the Dirac spinors $U_{\kappa\lambda}$. For light quarks one can expand spinors in the ultrarelativistic (large-momentum) limit in the inverse ratio $m(k)/k$

$$U_{\kappa\lambda} = \frac{1}{\sqrt{2E(k)}} \left[\frac{\sqrt{E(k) + m(k)}\chi_\lambda}{\sqrt{E(k) - m(k)}\vec{\sigma} \cdot \hat{k}\chi_\lambda} \right] \quad (4)$$

$$\xrightarrow{k \rightarrow \infty} \frac{1}{\sqrt{2}} \left[\frac{\chi_\lambda}{\vec{\sigma} \cdot \hat{k}\chi_\lambda} \right] + \frac{1}{2\sqrt{2}} \frac{m(k)}{k} \left[\frac{\chi_\lambda}{-\vec{\sigma} \cdot \hat{k}\chi_\lambda} \right]$$

with $E(k) = \sqrt{k^2 + m(k)^2}$, having kept the leading chiral invariant term, and a next order chiral symmetry breaking $m(k)/k$ term. Non-chiral, spin-dependent potentials in the quark–quark interaction originate from the second term in the expansion Eq. (4).

This translates into an expansion of H^{QCD} in the weak sense (that is, not of the Hamiltonian operator itself, but its restriction to the Hilbert space of highly excited resonances, where $\langle k \rangle$ is large) is:

$$\langle n | H^{\text{QCD}} | n' \rangle \simeq \langle n | H_{\chi}^{\text{QCD}} | n' \rangle + \langle n | \frac{m(k)}{k} H_{\chi}^{\text{QCD}'} | n' \rangle + \dots \quad (5)$$

The parity degeneracy [4] for highly excited hadrons follows from invariance under transformations generated by the chiral charge [5] $Q_5^a = \int d\mathbf{x} \psi^\dagger(x) \gamma_5 (\tau^a/2) \psi(x)$ and $[Q_5^a, H] = 0$. For low-lying hadrons, Chiral Symmetry is spontaneously broken by the ground state, $Q_5^a |0\rangle \neq 0$ providing a large quark mass in the propagator, $m(k)$, pseudo-Goldstone bosons (π , K , η), and the loss of parity degeneracy in ground state baryons. Substituting the spinors, and in terms of Bogoliubov-rotated $q\bar{q}$ normal modes B , D ,

$$\begin{aligned} Q_5^a = & \int \frac{d^3k}{(2\pi)^3} \sum_{\lambda\lambda' f f' c} \left(\frac{\tau^a}{2} \right)_{ff'} \frac{k}{\sqrt{k^2 + m^2(k)}} \\ & \times \left((\boldsymbol{\sigma} \cdot \hat{\mathbf{k}})_{\lambda\lambda'} \left(B_{k\lambda f c}^\dagger B_{k\lambda' f' c} + D_{-k\lambda' f' c}^\dagger D_{-k\lambda f c} \right) \right. \\ & \left. + \frac{m(k)}{k} (i\sigma_2)_{\lambda\lambda'} \left(B_{k\lambda f c}^\dagger D_{-k\lambda' f' c}^\dagger + B_{k\lambda' f' c} D_{-k\lambda f c} \right) \right). \end{aligned} \quad (6)$$

In the presence of spontaneous χ SB, $m(k) \neq 0$, and the last term realizes chiral symmetry non-linearly in the spectrum as it creates/destroys a pion. But when $\langle k \rangle$ is large, it is the $\boldsymbol{\sigma} \cdot \hat{\mathbf{k}}$ -term that dominates, and chiral symmetry is realized linearly (with only quark counting operators flipping parity and spin). This happens for a baryon resonance high enough in the spectrum, whose constituents have a momentum distribution peaked higher than the IR enhancement of $m(k)$, as in Fig. 1.

To exploit the smallness of the $|M^{P=+} - M^{P=-}|$ mass difference, we employ increasing angular momentum and establish how the j -scaling of the splitting is related to the k -scaling of $m(k)$. The M^\pm in our proposed study are the masses of the ground state quartets of the Δ spectrum, with parity \pm and in the limit of large total angular momentum $j \gg 3/2$. The two approximately degenerate masses M^+ and M^- are in the same leading linear Regge trajectory, phenomenologically fixing their j -scaling

$$j = \alpha_0 + \alpha M^{\pm 2} \xrightarrow{j \rightarrow \infty} \alpha M^{\pm 2}. \quad (7)$$

The parity of the ground state alternates between “+” and “−” as the angular momentum steps up by one. Large j is equivalent to large quark

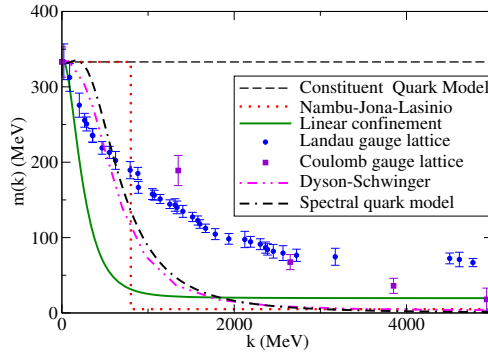


Fig. 1. IR enhancement of the light quark mass, generated when spontaneous chiral symmetry breaking occurs. Shown are quark masses in the main approaches to QCD, all multiplied by an arbitrary factor to match them at zero momentum. (References in the text.)

orbital angular momentum l since the spin is finite, and also to a large average linear momentum $\langle k \rangle$. This is illustrated in Fig. 2 where we show how the overlap of the wavefunction with the running $m(k)$ blinds high-lying states to χ SB. From the relativistic version of the virial theorem [7] between kinetic and total energies,

$$\langle k \rangle \rightarrow c_2 M^\pm \rightarrow \frac{c_2}{\sqrt{\alpha}} \sqrt{j}, \quad (8)$$

where c_2 is a constant; for a linear potential model, and 3 quarks in a baryon, $c_2 = 1/6$.

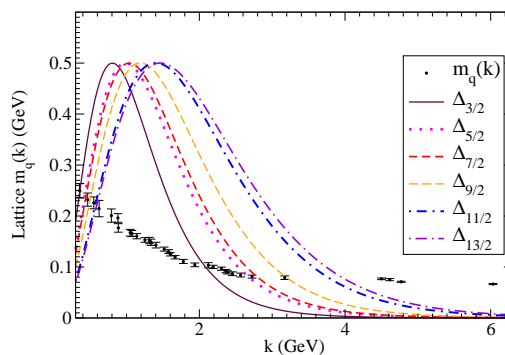


Fig. 2. Typical momentum distributions of increasingly excited $\Delta_{3/2}, \dots, \Delta_{13/2}$ resonances overlap less and less with the dynamically generated IR quark mass so that $\langle n | (m(k)/k) H_\chi^{\text{QCD}'} | n \rangle$ becomes small. (Illustrative variational wavefunctions for a linear potential with string tension $\sigma = 0.135 \text{ GeV}^2$, not normalized for visibility.) This computation has been reported in [6].

The first term in Eq. (5) cancels out in the difference $|M^+ - M^-| \ll M^\pm$ (while both M^\pm are dominated by the chiral invariant term, their difference stems from the dynamically generated quark mass) thus exposing the second term in Eq. (5), proportional to $\langle m(k)/k \rangle$, *viz.*

$$|M^+ - M^-| \rightarrow \left\langle \frac{m(k)}{k} H_\chi^{\text{QCD}'} \right\rangle \rightarrow c_3 \frac{m(\langle k \rangle)}{\langle k \rangle} \langle H_\chi^{\text{QCD}'} \rangle \quad (9)$$

(the factorization is allowed by the mean-value theorem at the price of a constant that we do not attempt to determine here). This equation is analogous to the renowned Gell-Mann–Oakes–Renner relation $M_\pi^2 = -m_q \langle \bar{\psi}\psi \rangle / f_\pi^2$ but active when chiral symmetry is realized linearly, as in the high-baryon excitations we examine.

The $\boldsymbol{\sigma} \cdot \hat{\mathbf{k}}$ from the spinors induce spin-dependent potentials in $H_\chi^{\text{QCD}'}$. We need its large j -scaling, equivalent to large $\langle k \rangle$ and M^\pm , and separately consider its angular and radial dependences $\langle H_\chi^{\text{QCD}'} \rangle \propto \langle H_\chi^{\text{QCD}'} \rangle_{\text{ang}} \times \langle H_\chi^{\text{QCD}'} \rangle_{\text{rad}}$. The angular matrix element generally includes a spin-orbit term, that leads its j -counting

$$\langle H_\chi^{\text{QCD}'} \rangle_{\text{ang}} \rightarrow j, \quad (10)$$

while the centrifugal barrier dominates the radial part,

$$l(l+1) \langle H_\chi^{\text{QCD}'} \rangle_{\text{rad}} \rightarrow j^2 \langle H_\chi^{\text{QCD}'} \rangle_{\text{rad}} \rightarrow c_4 M^\pm$$

and thus $\langle H_\chi^{\text{QCD}'} \rangle_{\text{rad}} \propto j^{-2} M^\pm$. From Eqs. (7), (10)

$$\langle H_\chi^{\text{QCD}'} \rangle \rightarrow c_5 M^\pm j^{-1} \rightarrow \frac{c_5}{\sqrt{\alpha}} \sqrt{\frac{1}{j}}. \quad (11)$$

(As a corollary, note that a spin-independent potential scales like $1/j^{3/2}$.)

The result of the j -scaling analysis reads then

$$|M^+ - M^-| \rightarrow \frac{c_3 m(\langle k \rangle)}{\langle k \rangle} \times c_5 M^\pm \times j^{-1} = \frac{c_3 c_5}{c_2} m(\langle k \rangle) j^{-1} \quad (12)$$

that links the IR enhancement of the quark mass to baryon spectroscopy in a usable way. An experimental extraction proceeds by just fitting the exponent of the j -scaling for the splitting $|M^+ - M^-| \propto j^{-i}$. Then, in view of Eq. (8), one obtains both

$$m(\Lambda \times \sqrt{j}) \propto j^{-i+1} \quad \text{and} \quad m(k) \propto k^{-2i+2}. \quad (13)$$

The same exponent i in this last equation can be obtained from the fit to the $|M^+ - M^-|$ with increasing j .

In conclusion, the scaling with j of the mass-splittings in the Δ spectrum can be related to the k -running of the quark mass as defined from the quark propagator. This remarkable result holds as long as the expansion in $m(\langle k \rangle)/\langle k \rangle$ makes sense, that is, for highly excited resonances.

Other consequences of Insensitivity to Chiral Symmetry Breaking high in the spectrum, such as the decoupling of excited resonances from pions (for example, the widths $\Gamma_{\Delta^* \rightarrow N\pi}$) can likewise be exploited to probe the running quark mass.

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