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ABOUT THE ORIGIN OF THE MASS OF THE NUCLEON IN A LINEAR SIGMA MODEL*

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Beyond the contribution of the chiral condensate to the nucleon mass, in the so-called mirror assignment a chirally invariant mass term $\sim m_0$ is possible. In the present work it is discussed on light of recent results which is the role of both terms in generating the nucleon mass. Also, the origin of m_0 in terms of tetraquark and gluon condensates is briefly discussed.

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1. Introduction

Understanding the mass of the nucleon is one of the most important issues in modern physics. Neglecting a small contribution of the nonzero current quark masses, there are essentially two ways to generate the mass of the nucleon in the context of a linear sigma model, which will be discussed in the following.

In the classical linear sigma model the mass of the nucleon is generated exclusively through spontaneous breaking of the chiral symmetry, leading to the appearance of a chiral condensate $\phi \sim f_\pi$, where $f_\pi = 92.4$ MeV is the pion decay constant. The chiral condensate ϕ can be directly related to the fundamental quark condensate $\langle \bar{q}q \rangle$ as $\phi \simeq \Lambda_{\rm QCD}^5 \langle \bar{q}q \rangle$, where $\Lambda_{\rm QCD}$ is the QCD Yang–Mills scale. Indeed, in the framework of QCD sum rules the relation between the mass of the nucleon m_N and $\langle \bar{q}q \rangle$ is expressed via the Ioffe's formula [1]: $m_N \sim ((-4\pi^2)/\Lambda_B^2) \langle \bar{q}q \rangle$, where Λ_B is an energy scale of the order of $\sim 1\,$GeV.

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However, not only the quark condensate, but also other condensates, such as the gluon and the tetraquark ones, can contribute to the mass of the nucleon m_N [2] and it is not yet settled which is their quantitative role. A possibility to study this problem in the context of a linear sigma model goes via the so-called mirror assignment, discussed extensively in Ref. [3] and further used in Ref. [4] to study the properties of symmetric and non-symmetric nuclear matter. In this assignment, the nucleon N and its chiral partner N^* (to be identified here with $N(1535)$) form a doublet of the chiral group. As discussed in the next section, it is possible to introduce a chirally invariant mass term parametrized by m_0. The latter, in turn, means that the masses of the nucleon and its chiral partner do not vanish in the chirally restored phase where $\phi \to 0$ but acquire the same non-vanishing mass m_0.

2. Parity-doubling model

The model: The $\mathrm{SU}(2)_r \times \mathrm{SU}(2)_l$ invariant linear sigma model in the baryonic sector reads:

$$\begin{aligned}\mathcal{L}_{\mathrm{bar}} \;=\; & \overline{\Psi}_{1\mathrm{L}} i\gamma_\mu D^\mu_{1\mathrm{L}} \Psi_{1\mathrm{L}} + \overline{\Psi}_{1\mathrm{R}} i\gamma_\mu D^\mu_{1\mathrm{R}} \Psi_{1\mathrm{R}} \\ & + \overline{\Psi}_{2\mathrm{L}} i\gamma_\mu D^\mu_{2\mathrm{R}} \Psi_{2\mathrm{L}} + \overline{\Psi}_{2\mathrm{R}} i\gamma_\mu D^\mu_{2\mathrm{L}} \Psi_{2\mathrm{R}} \\ & - \widehat{g}_1 \left(\overline{\Psi}_{1\mathrm{L}} \Phi \Psi_{1\mathrm{R}} + \overline{\Psi}_{1\mathrm{R}} \Phi^\dagger \Psi_{1\mathrm{L}} \right) - \widehat{g}_2 \left(\overline{\Psi}_{2\mathrm{L}} \Phi^\dagger \Psi_{2\mathrm{R}} + \overline{\Psi}_{2\mathrm{R}} \Phi \Psi_{2\mathrm{L}} \right), \quad (1)\end{aligned}$$

where $D^\mu_{1\mathrm{R}} = \partial^\mu - ic_1 R^\mu$, $D^\mu_{1\mathrm{L}} = \partial^\mu - ic_1 L^\mu$ and $D^\mu_{2\mathrm{R}} = \partial^\mu - ic_2 R^\mu$, $D^\mu_{2\mathrm{L}} = \partial^\mu - ic_2 L^\mu$ are the covariant derivatives for the nucleonic fields, parametrized by the coupling constants c_1 and c_2. The scalar and pseudoscalar meson fields are included in the matrix $\Phi = (\sigma + i\eta)t^0 + (\overrightarrow{a}_0 + i\overrightarrow{\pi}) \cdot \overrightarrow{t}$ and the (axial-)vector fields are represented by the matrices $R^\mu = (\omega^\mu - f_1^\mu)t^0 + (\overrightarrow{\rho}^\mu - \overrightarrow{a_1}^\mu) \cdot \overrightarrow{t}$ and $L^\mu = (\omega^\mu + f_1^\mu)t^0 + (\overrightarrow{\rho}^\mu + \overrightarrow{a_1}^\mu) \cdot \overrightarrow{t}$ ($\overrightarrow{t} = \frac{1}{2}\overrightarrow{\tau}$, where $\overrightarrow{\tau}$ are the Pauli matrices and $t^0 = \frac{1}{2}1_2$). The fields Ψ_1 and Ψ_2 are the baryon doublets, where Ψ_1 has positive parity and Ψ_2 negative parity.

In the naive assignment Ψ_1 and Ψ_2 transform in the same way under chiral transformation:

$$\begin{aligned}\Psi_{1\mathrm{R}} &\longrightarrow U_\mathrm{R} \Psi_{1\mathrm{R}}\,, \qquad \Psi_{1\mathrm{L}} \longrightarrow U_\mathrm{L} \Psi_{1\mathrm{L}}\,, \\ \Psi_{2\mathrm{R}} &\longrightarrow U_\mathrm{R} \Psi_{2\mathrm{R}}\,, \qquad \Psi_{2\mathrm{L}} \longrightarrow U_\mathrm{L} \Psi_{2\mathrm{L}}\,, \qquad (2)\end{aligned}$$

while in the mirror assignment Ψ_1 and Ψ_2 transform as:

$$\begin{aligned}\Psi_{1\mathrm{R}} &\longrightarrow U_\mathrm{R} \Psi_{1\mathrm{R}}\,, \qquad \Psi_{1\mathrm{L}} \longrightarrow U_\mathrm{L} \Psi_{1\mathrm{L}}\,, \\ \Psi_{2\mathrm{R}} &\longrightarrow U_\mathrm{L} \Psi_{2\mathrm{R}}\,, \qquad \Psi_{2\mathrm{L}} \longrightarrow U_\mathrm{R} \Psi_{2\mathrm{L}}\,, \qquad (3)\end{aligned}$$

i.e. Ψ_2 transforms in a "mirror way" under chiral transformations. These latter field transformations allow to write down an additional chirally invariant mass term for the fermions, parametrized by m_0, which plays a crucial role in the generation of the nucleon's mass:

$$\mathcal{L}_{\rm mass} = -m_0 \left(\overline{\Psi}_{\rm 1L}\Psi_{\rm 2R} - \overline{\Psi}_{\rm 1R}\Psi_{\rm 2L} - \overline{\Psi}_{\rm 2L}\Psi_{\rm 1R} + \overline{\Psi}_{\rm 2R}\Psi_{\rm 1L}\right) \,. \tag{4}$$

Mass generation through Spontaneous Symmetry Breaking (SSB): When the baryon fields transform as in Eq. (2), the baryonic Lagrangian is expressed solely by Eq. (1). In fact, the mass term of Eq. (4) is not chirally invariant in this case. The fields Ψ_1 and Ψ_2 are identified with the nucleon N and with the resonances $N^* \equiv N(1535)$, respectively ($\Psi_1 = N$ and $\Psi_2 = N^*$).

A nonzero vacuum expectation value $\phi = Zf_\pi$ of the scalar–isoscalar field σ is generated by a Mexican hat potential in the mesonic sector, where $Z \approx 1.67$ arises when also (axial-)vector degrees of freedom are included [5]. As a consequence, the shift $\sigma \to \sigma + \phi$ leads to the generation of baryonic masses:

$$\mathcal{L}_{\rm bar,\sigma} = -\frac{\widehat{g}_1}{2}\,\overline{\Psi}_1(\sigma+\phi)\,\Psi_1 - \frac{\widehat{g}_2}{2}\,\overline{\Psi}_2(\sigma+\phi)\Psi_2\,. \tag{5}$$

It is evident that the N and N^* masses read

$$m_N = \frac{\widehat{g}_1}{2}\phi\,, \qquad m_{N^*} = \frac{\widehat{g}_2}{2}\phi\,. \tag{6}$$

Using the experimental values $m_N = 939\,\text{MeV}$ and $m_{N^*} = 1535\,\text{MeV}$ [7] together with $\phi = Zf_\pi = 154.3\,\text{MeV}$, one determines $\widehat{g}_1 = 12.2$ and $\widehat{g}_2 = 20$. Both masses are a simple linear function of ϕ, see Eq. (6), which vanish in the limit $\phi \to 0$.

Mass generation in the mirror assignment: When the baryon fields transform according to the mirror assignment of Eq. (3), the Lagrangian term of Eq. (4) is also chirally invariant. Therefore, the full Lagrangian in this case reads

$$\mathcal{L}_{\rm mirror} = \mathcal{L}_\sigma + \mathcal{L}_{\rm mass}\,. \tag{7}$$

The term parametrized by m_0 generates a mixing between the fields Ψ_1 and Ψ_2. The physical fields N and N^*, referring to the nucleon and its chiral partner, arise by diagonalizing the corresponding mass matrix in the Lagrangian:

$$\begin{pmatrix} N \\ N^* \end{pmatrix} = \frac{1}{\sqrt{2\cosh\delta}} \begin{pmatrix} e^{\delta/2} & \gamma_5 e^{-\delta/2} \\ \gamma_5 e^{-\delta/2} & -e^{\delta/2} \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}. \tag{8}$$

The masses of the nucleon and its partner are then obtained as:

$$m_{N,N^*} = \sqrt{m_0^2 + \left(\frac{\widehat{g}_1+\widehat{g}_2}{4}\right)^2 \phi^2} \pm \frac{(\widehat{g}_1-\widehat{g}_2)\phi}{4}\,. \tag{9}$$

Note that in this case the chiral condensate generates a mass splitting between m_N and m_{N^*}, whereas the masses degenerate to m_0 ($m_N = m_{N^*} = m_0$) when $\phi \to 0$, *i.e.* when chiral symmetry is restored. The parameter δ in Eq. (8) is related to the masses and the parameter m_0 by the expression:

$$\cosh\delta = \frac{m_N + m_{N^*}}{2m_0} \,. \tag{10}$$

If $\delta \to \infty$ (*i.e.* $m_0 \to 0$) the mixing of the field disappears and $\Psi_1 = N$ and $\Psi_2 = N^*$. In this case the masses are generated as discussed in the naive case discussed previously.

As one can observe in Eq. (9), the nucleon mass *cannot* be linearly expressed as $m_N = m_0 + c\phi$, thus m_0 should not be interpreted as a linear contribution to the nucleon mass. Such a linearization is only possible in the case when m_0 dominates or the chiral condensate dominates. Present results [6] show, however, that this is not the case but that both quantities are sizable. In fact, the parameters m_0, $\widehat{g}_1$ and $\widehat{g}_2$ and their errors have been determined in Ref. [6] as $m_0 = 460 \pm 136$ MeV, $\widehat{g}_1 = 11.0 \pm 1.5$, $\widehat{g}_2 = 18.8 \pm 2.4$. Note, m_0 is larger than what originally found in Ref. [3] and points to a sizable interplay of other condensates to the nucleon mass.

For a correct discussion of the influence of different terms contributing to the nucleon mass, we consider the following cases:

(a) The parameter m_0 is kept fixed to the value of 460 MeV and the chiral condensate ϕ is left as a free parameter. The corresponding masses are plotted in the right panel of Fig. 1. When varying ϕ from 0 to the physical value Zf_π the nucleon mass goes from $m_0 = 460$ MeV to 939 MeV, thus showing that in limit $\phi \to 0$ the mass does not vanish.

(b) The chiral condensate ϕ is kept fixed to its physical value Zf_π but the parameter m_0 is left free. The corresponding baryon masses are plotted as function of m_0 in Fig. 1, left panel. Due to the expression

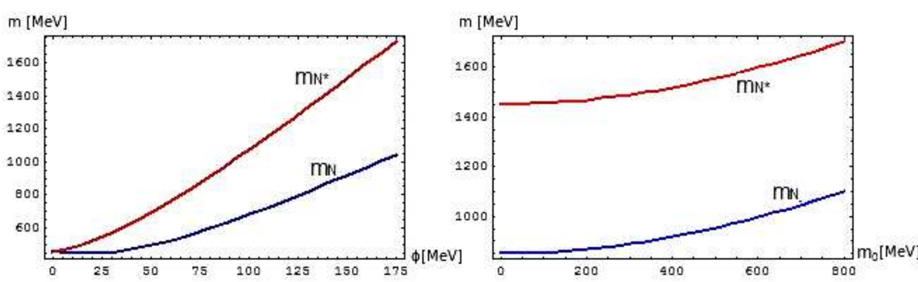

Fig. 1. The masses of the nucleon and its chiral partner as a function of the quark condensate at fixed $m_0 = 460$ MeV (left figure) and as a function m_0 at fixed $\phi = 154.3$ MeV (right figure).

of Eq. (9), the nucleon m_N is actually a slowly increasing function of m_0. In the limit $m_0 \to 0$ the nucleon mass, as originated from the chiral condensate only, reads $m_N = \widehat{g}_1\phi/2 \simeq 850$ MeV and the mass of the partner reads $m_{N^*} = \widehat{g}_2\phi/2 \simeq 1450$ MeV. From this point of view the obtained non-zero value of $m_0 = 460$ MeV induces an increase of only 100 MeV to the nucleon mass and its partner.

As a last point we discuss the origin of m_0 in terms of tetraquark and gluon condensates. The term of Eq. (4) is the only one in the baryon sector which is not dilatation invariant. In order to render it such, as required by QCD, by preserving chiral symmetry we can couple it to the chirally invariant dilaton field G [8] and tetraquark field $\chi \equiv [\overline{u}, \overline{d}][u, d]$:

$$\mathcal{L}_{\text{mass}} = (a\chi + b\,G)\left(\overline{\Psi}_{1\text{L}}\Psi_{2\text{R}} - \overline{\Psi}_{1\text{R}}\Psi_{2\text{L}} - \overline{\Psi}_{2\text{L}}\Psi_{1\text{R}} + \overline{\Psi}_{2\text{R}}\Psi_{1\text{L}}\right)\,, \tag{11}$$

where a and b are dimensionless coupling constants. Note, the excitation of the scalar field G is identified with the scalar glueball [9] while the field χ with the resonance $f_0(600)$ (see Ref. [10], where also the tetraquark condensate is discussed, and references therein). When shifting both fields around their vacuum expectation values $G \to G_0 + G$ and $\chi \to \chi_0 + \chi$ we recover the term of Eq. (4) by identifying

$$m_0 = a\chi_0 + bG_0\,, \tag{12}$$

where χ_0 and G_0 are the tetraquark and gluon condensates, respectively. We thus have re-expressed the parameter m_0 as the sum of the contributions of the tetraquark and gluon condensates.

3. Summary

Different ways of mass generation of the nucleon in the framework of a chirally invariant sigma model have been discussed: *(i)* the standard SSB mechanism, in which the scalar field σ develops a vacuum expectation value $\phi \neq 0$ and produces a baryon mass *(ii)* an additional term (proportional to the parameter m_0) which is chirally invariant when the baryon fields transformed according to the mirror assignment. In this case the nucleon mass does not vanish in the limit $\phi \to 0$.

We have shown that when both terms of *(i)* and *(ii)* are sizable, as predicted in Ref. [6], the nucleon and the partner masses cannot be simply expressed as two separate contributions. An interplay of both terms takes place. As a last point we discussed the emergence of m_0 in terms of the gluon and tetraquark condensates. The study of this scenario at nonvanishing density and its relation to the postulated quarkyonic phase [11] constitute an outlook of the present work.

REFERENCES

[1] B.L. Ioffe, *Nucl. Phys.* **B188**, 317 (1981); Erratum **B191**, 591 (1981).

[2] T.D. Cohen, R.J. Furnstahl, D.K. Griegel, X.m. Jin, *Prog. Part. Nucl. Phys.* **35**, 221 (1995).

[3] C. DeTar, T. Kunihiro, *Phys. Rev.* **D39**, 2805 (1989); D. Jido, M. Oka, A. Hosaka, *Prog. Theor. Phys.* **106**, 873 (2001); D. Jido, Y. Nemoto, M. Oka, A. Hosaka, *Nucl. Phys.* **A671**, 471 (2000).

[4] D. Zschiesche, L. Tolos, J. Schaffner-Bielich, R.D. Pisarski, *Phys. Rev.* **C75**, 055202 (2007); V. Dexheimer, G. Pagliara, L. Tolos, J. Schaffner-Bielich, S. Schramm, *Eur. Phys. J.* **A38**, 105 (2008).

[5] D. Parganlija, F. Giacosa, D.H. Rischke, *AIP Conf. Proc.* **1030**, 160 (2008) [`arXiv:0804.3949[hep-ph]`]; D. Parganlija, F. Giacosa, D.H. Rischke, `arXiv:0812.2183 [hep-ph]`; S. Strüber, D.H. Rischke, *Phys. Rev.* **D77**, 085004 (2008).

[6] S. Gallas, F. Giacosa, D.H. Rischke, `arXiv:0907.5084[hep-ph]`.

[7] C. Amsler *et al., Phys. Lett.* **B667**, 1 (2008).

[8] A. Salomone, J. Schechter, T. Tudron, *Phys. Rev.* **D23**, 1143 (1981).

[9] C. Amsler, F.E. Close, *Phys. Rev.* **D53**, 295 (1996); F.E. Close, A. Kirk, *Eur. Phys. J.* **C21**, 531 (2001); F. Giacosa, T. Gutsche, V.E. Lyubovitskij, A. Faessler, *Phys. Rev.* **D72**, 094006 (2005).

[10] A. Heinz, S. Struber, F. Giacosa, D.H. Rischke, *Phys. Rev.* **D79**, 037502 (2009); F. Giacosa, *Phys. Rev.* **D75**, 054007 (2007); A.H. Fariborz, R. Jora, J. Schechter, *Phys. Rev.* **D72**, 034001 (2005).

[11] L. McLerran, R.D. Pisarski, *Nucl. Phys.* **A796**, 83 (2007); L. McLerran, K. Redlich, C. Sasaki, `arXiv:0812.3585 [hep-ph]`.