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HOLOGRAPHIC QCD WITH MATTER*

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In this brief note, we will discuss the quark density dependence of the deconfinement phase transition using the probe approximation, in which only the low density regime is valid. When considering the first correction of the quark density, the deconfinement temperature decreases as the quark density increases like the lattice QCD result.

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Recently, related to the RHIC and LHC experiments, understanding strongly interacting QCD is requesting much attention. Although a powerful method for this subject, the lattice QCD, is being developed, when it comes to the dense matter problem, lattice calculation has difficulty and not much result is produced so far. The AdS/CFT correspondence [1] in the string theory, can shed many aspect of hadron theory [2–10] as well as in strongly interacting quark gluon plasma [11–16]. The theory can easily accommodate the dense matter problem at least for deconfined phase [4–6, 17]. However, for the the hadron phase, the status is not very clear since even the phase diagram is qualitatively different from that of the real QCD [4, 18].

A study [19] based on a Hawking–Page type transition in the AdS/QCD models calculated the deconfinement temperature. In this analysis, the effect of the quark density was not considered, since they are suppressed by $1/N_c$ compared to the gravitational part: the gravitational coupling scales as $\kappa \approx g_{\mathrm{s}} \approx 1/N_c$, and the contribution from the quark scales only as N_c. In this brief note, we will talk about how the quark density modify the Hawking–Page transition using the probe approximation.

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When ignoring the effect from the quark density, the Euclidean gravitational action given by

$$S_{\mathrm{grav}} = -\frac{1}{2\kappa^2}\int d^5x\sqrt{g}\left(\mathrm{R}+\frac{12}{L^2}\right)\,, \tag{1}$$

where the gravitational constant G_5 is given by $G_5 = \kappa^2/(8\pi)$ and L is the radius of the AdS_5. There are two relevant background solutions for the Einstein equation. The first one is cut-off thermal AdS(tAdS), where the cut-off was introduced to explain the quark confinement, with the line element

$$ds^2 = \frac{L^2}{z^2}\left(d\tau^2 + dz^2 + d\vec{x}_3^2\right)\,, \tag{2}$$

where the radial coordinate runs from the boundary of tAdS space $z=0$ to the cut-off z_{IR}, which corresponds to an infrared cut-off in energies proportional to $1/z_{\mathrm{IR}}$ from the point of view of the boundary dual theory. The second one is AdS black hole (AdSBH) with the line element

$$ds^2 = \frac{L^2}{z^2}\left(f(z)d\tau^2 + \frac{dz^2}{f(z)} + d\vec{x}_3^2\right)\,, \tag{3}$$

where $f(z) = 1-(z/z_{\mathrm{h}})^4$ and z_{h} is the horizon of the black hole. Note that the above two metrics are solutions of the Einstein equation. The Hawking temperature of the black hole solution is $T = 1/(\pi z_{\mathrm{h}})$ which is given by regularizing the metric near the horizon. In the tAdS case, the periodicity in the Euclidean time-direction is fixed by comparing two geometries at an UV cut-off ε where the periodicity of the time-direction in both cases is locally the same. Then, the time periodicity of tAdS is given by

$$\beta = \pi z_{\mathrm{h}}\sqrt{f(\varepsilon)}. \tag{4}$$

Now we calculate the action density V, which is defined by the action divided by the common volume factor of R^3. The regularized action density of the tAdS is given by

$$V_1(\varepsilon) = \frac{4L^3}{\kappa^2}\int_0^{\beta'} d\tau\int_\varepsilon^{z_{\mathrm{IR}}}\frac{dz}{z^5}\,, \tag{5}$$

and that of the AdSBH is given by

$$V_2(\varepsilon) = \frac{4L^3}{\kappa^2}\int_0^{\pi z_{\mathrm{h}}} d\tau\int_\varepsilon^{\bar{z}}\frac{dz}{z^5}\,, \tag{6}$$

where the range of z is given by $0 \leq z \leq \bar{z}$. If there is no IR cut-off in the AdSBH background, $\bar{z}$ becomes $z_{\rm h}$. Anyway, it is also possible to introduce the IR cut-off to the AdSBH. In this case, $\bar{z}$ is given by the minimum value in $z_{\rm IR}$ and $z_{\rm h}$. For $z_{\rm IR} > z_{\rm h}$, since the IR cut-off is located behind the black hole horizon, the background is just an AdSBH solution. If $z_{\rm IR} < z_{\rm h}$, it becomes tAdS including the non-trivial effect of the temperature, where we call it thermal AdSBH (tAdSBH). As will be shown, the free energy of tAdSBH is always bigger that the one of tAdS. Therefore, there is no the Hawking–Page transition in the latter case.

To see this more explicitly, we should calculate the free energy difference between two backgrounds, in which according to the AdS/CFT correspondence the free energy, F, of the boundary gauge theory can be obtained from the bulk gravitational the regularized action V, $F = TV$. The difference of the regularized actions is given by

$$\Delta V_g = \lim_{\varepsilon \to 0} \left[V_2(\varepsilon) - V_1(\varepsilon)\right] = \begin{cases} \frac{L^3 \pi z_{\rm h}}{\kappa^2} \frac{1}{2 z_{\rm h}^4}\,, & z_{\rm IR} < z_{\rm h} \\ \frac{L^3 \pi z_{\rm h}}{\kappa^2} \left(\frac{1}{z_{\rm IR}^4} - \frac{1}{2 z_{\rm h}^4}\right)\,, & z_{\rm IR} > z_{\rm h}\,. \end{cases} \tag{7}$$

This is the result in the hard wall model. When ΔV_g is positive(negative), tAdS (the black hole) is stable. Thus, at $\Delta V_g = 0$ there exists a Hawking–Page transition as previously mentioned. In the first case $z_{\rm IR} < z_{\rm h}$, there is no Hawking–Page transition and the thermal AdS is always stable. In the second case $z_{\rm IR} > z_{\rm h}$, the Hawking–Page transition occurs at

$$T_0 = \frac{2^{1/4}}{\pi z_{\rm IR}} \tag{8}$$

and at low temperature $T < T_0$ (at high temperature $T > T_0$) the thermal AdS (the AdS black hole) geometry becomes a dominant background.

Now, we consider the quark number density dependence on the deconfinement temperature. In QCD, quark chemical potential introduced as $\mu_q \bar{\psi} \gamma_0 \psi$, and so according to an AdS/CFT dictionary we need to introduce a bulk U(1) field in AdS_5 whose boundary value is μ_q

$$S = \int d^5 x \sqrt{g} \left[-\frac{1}{2\kappa^2} \left(\mathrm{R} + \frac{12}{L^2} \right) + M_5 \ \mathrm{Tr} \frac{1}{4} F^2 \right], \tag{9}$$

where following [20] we generalize the symmetry to $\mathrm{U}(N_f)$ and $F = dV$ is the field strength. In this note, we will consider the vector field as a fluctuation, which corresponds to the probe approximation. The equation of motion for the time component of the U(1) vector field is given by

$$\partial_z \left[\frac{1}{z} \, \partial_z V_\tau(z) \right] = 0\,, \tag{10}$$

and a solution is given by

$$V_\tau = c_1 + c_2 z^2 \,. \tag{11}$$

Since the factors $g^{\tau\tau}g^{zz}$ in the equation of motion for tAdS and AdSBH backgrounds are the same, we conclude that the equations of motion of both backgrounds are the same and the corresponding forms of the solutions are also the same. That is, we can use the same form of the solution V_τ in both background tAdS and AdSBH. According to the AdS/CFT correspondence, the coefficient of the non-normalizable term, c_1, is proportional to coupling with the dual operator of the boundary theory. Since the time component of the U(1) vector field is dual to the quark number current, c_1 must correspond to the quark chemical potential. Meanwhile, the coefficient of the normalizable term, c_2, corresponds to the expectation value of the dual operator so that c_2 is interpreted as the quark number density, $c_2 = 12\pi^2\rho_q/N_c$ [20].

Following the same procedure used in the previous section, we arrive at

$$V_{v1} = \pi z_{\rm h} M_5 N_f L^5 c_2^2\, z_{\rm IR}^2 \tag{12}$$

for the tAdS and

$$V_{v2} = \begin{cases} \pi z_{\rm h} M_5 N_f L^5 c_2^2 z_h^2\,, & z_{\rm h} < z_{\rm IR}\,, \\ \pi z_{\rm h} M_5 N_f L^5 c_2^2 z_{\rm IR}^2\,, & z_{\rm h} > z_{\rm IR}\,, \end{cases} \tag{13}$$

for AdSBH. From these results, the differences of the action reads

$$\Delta V_v = \begin{cases} -\pi z_{\rm h} M_5 N_f L^5 c_2^2 (z_{\rm IR}^2 - z_h^2)\,, & z_{\rm h} < z_{\rm IR}\,, \\ 0\,, & z_{\rm h} > z_{\rm IR}\,. \end{cases} \tag{14}$$

The final result for $z_{\rm h} < z_{\rm IR}$ is

$$\Delta V = \frac{L^3\pi z_{\rm h}}{\kappa^2}\left[\frac{1}{z_{\rm IR}^4} - \frac{1}{2z_{\rm h}^4} - \frac{L^4 N_f c_2^2}{48N_c}\left(z_{\rm IR}^2 - z_h^2\right)\right]. \tag{15}$$

In the above, the last term $z_{\rm IR}^2 - z_{\rm h}^2$, which is from quark number density, is positive, and so the critical temperature at finite density is always lower than that of the pure gravity theory. Now, we consider a case where $z_{\rm h} \ll z_{\rm IR}$ to see the quark number dependence of $T_{\rm c}$ clearly. We note here that typically $z_{\rm h}^2/z_{\rm IR}^2 \approx 0.6$. In this case, the critical temperature is given by

$$T_{\rm c}(\rho_q) = \frac{2^{1/4}}{\pi}\left(\frac{1}{z_{\rm IR}^4} - \frac{L^4 N_f z_{\rm IR}^2}{48N_c}\, c_2^2\right)^{1/4}\,, \tag{16}$$

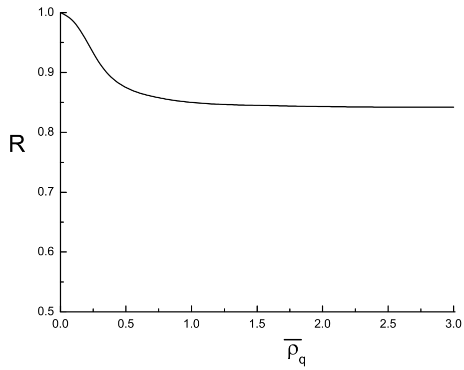

Fig. 1. Density dependence of $T_{\rm c}$ from (15). Here $R \equiv T_{\rm c}(\rho_q)/T_0$ and $\bar{\rho}_q = \rho_q z_{\rm IR}^3$.

which is a consistent result with the lattice QCD data. Note here again that $c_2 \sim \rho_q$ [20]. In Fig. 1, we plot Eq. (15), together with the lattice result [21, 22], where $R \equiv T_{\rm c}(\rho_q)/T_0$ and $\bar{\rho}_q = \rho_q z_{\rm IR}^3$. Our result in Fig. 1 is consistent with lattice QCD results at low density.

Here, We have studied the effects of matters on the deconfinement transition in the context of a Hawking–Page type analysis. We observed, as it should be, that the corrections from the quark density to the deconfinement temperature decreases the deconfinement temperature. In the low density regime, our result shows a similar behavior calculated in lattice QCD.

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