

APPLICATION OF CHIRAL QUARKS TO HIGH-ENERGY PROCESSES AND LATTICE QCD*

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Results of the chiral quark models for the soft matrix elements involving pions and photons, relevant for high-energy processes, are reviewed. We discuss quantities related to the generalized parton distributions of the pion: the parton distribution functions, the parton distribution amplitudes, and the generalized form factors. The model predictions are compared to the data or lattice simulations, with good agreement. The QCD evolution from the low quark model scale up to the experimental scales is a crucial ingredient of the approach.

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The low-energy properties of the pion are dominated by the spontaneous breakdown of the chiral symmetry, which is a key dynamical factor. It allows to model the soft matrix element in a genuinely dynamical way [1–24]. There are two basic elements in our analysis: the low-energy dynamical quark model itself, and the QCD *evolution*, bringing the predictions from the low quark-model scale to higher scales of the experiment or lattice data. This talk is based on Refs. [25, 26], where the details can be found.

The theoretical framework is conveniently set by the Generalized Parton Distributions (GPDs) [27–35]. For the case of the pion, considered here, the GPD for the non-singlet channel is defined as

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$$\varepsilon_{3ab} \mathcal{H}^{q,\text{NS}}(x, \zeta, t) = \int \frac{dz^-}{4\pi} e^{ixp^+ z^-} \langle \pi^b(p') | \bar{\psi}(0) \gamma^+ \psi(z) \tau_3 | \pi^a(p) \rangle \Big|_{z^+=0, z^\perp=0},$$

with similar expressions for the singlet quarks and gluons. We have omitted the gauge link operators, absent in the light-cone gauge. The kinematics is determined by $p' = p + q$, $p^2 = p'^2 = m_\pi^2$, $q^2 = -2pq = t$, and $\zeta = q^+/p^+$, which denotes the momentum transfer passed along the light cone. Formal properties of GPDs can be compactly written in the symmetric notation involving $\xi = \zeta/(2 - \zeta)$, $X = (x - \zeta/2)/(1 - \zeta/2)$, where one has

$$H^{I=0}(X, \xi, t) = -H^{I=0}(-X, \xi, t), \quad H^{I=1}(X, \xi, t) = H^{I=1}(-X, \xi, t).$$

For $X \geq 0$ one finds where $q(x)^i$ denote the parton distribution functions (PDFs). The following *sum rules* hold:

$$\begin{aligned} \int_0^1 dX H^{I=1}(X, \xi, t) &= 2F_V(t), \\ \int_{-1}^{1-1} dX X H^{I=0}(X, \xi, t) &= 2\theta_2(t) - 2\xi^2\theta_1(t), \end{aligned}$$

where $F_V(t)$ denotes the electromagnetic form factor, while $\theta_1(t)$ and $\theta_2(t)$ are the gravitational form factors [36]. Other important formal properties are the *polynomiality* conditions [27], the *positivity bounds* [37,38], and a low-energy theorem [39] relating the GPD to the pion *distribution amplitude*. We stress that all these required properties are satisfied in our calculation [25].

With $\zeta = t = 0$, the GPDs becomes the usual PDFs. In the Nambu–Jona-Lasinio model [1] $q(x) = 1$. This result holds at the low-energy quark-model scale, which needs to be determined. At this scale the quarks are the only degrees of freedom. Thus, all observables are saturated with the quark contribution. In particular, this holds for the momentum sum rule. From experiment, the momentum fraction carried by the valence quarks is [42,43]

$$\langle x \rangle_v = 0.47(2) \quad \text{at} \quad Q^2 = 4 \text{ GeV}^2.$$

We evolve this value backward with the LO DGLAP equations down to the scale where the quarks carry 100% of the momentum, $\langle x \rangle_v = 1$. This procedure yields the *quark model scale*

$$Q_0 = 313_{-10}^{+20} \text{ MeV},$$

where the range reflects the uncertainty in $\langle x \rangle_v$.

The results of this method for the non-singlet PDF of the pion are shown in the left panel of Fig. 1. We have evolved the quark model result from the scale Q_0 up to the scale $Q = 4 \text{ GeV}$ corresponding to the E615 experiment [40]. We notice a very good agreement. In the right panel we present the same quantity evolved to the scale of $Q = 350 \text{ MeV}$ and confronted with the *transverse lattice* data [41], designed to work at low-energy scales. Again, the agreement is remarkable.

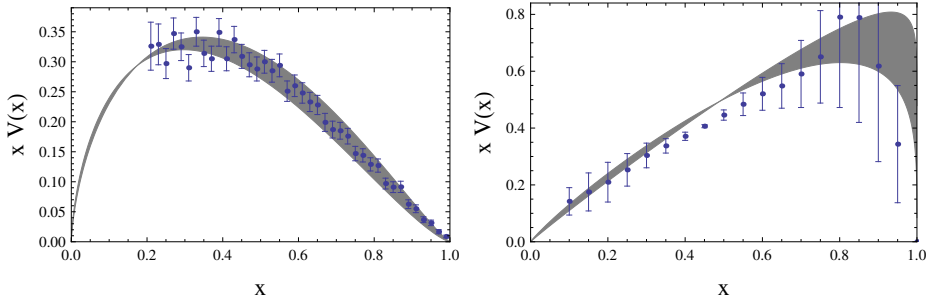


Fig. 1. Left: chiral quark model prediction for the valence PDF of the pion, evolved to the scale of $Q = 4 \text{ GeV}$ (band). The width of the band indicates the uncertainty in the initial scale Q_0 . The data points come from the E615 experiment [40]. Right: the same, evolved to the scale $Q = 0.35 \text{ GeV}$ and compared to the data from the transverse lattice calculations of Ref. [41].

Next, we look at the DA. Here the evolution is carried out with the LO ERBL equations. The results, displayed in Fig. 2, again are in fair agreement with the data, especially for the lattice case shown in the right panel.

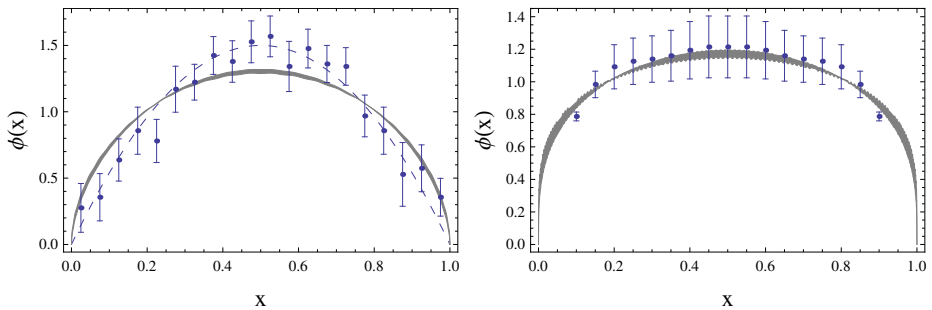


Fig. 2. Left: chiral quark model prediction for the pion DA evolved to $Q = 2 \text{ GeV}$ (band) and compared to the E791 data [44]. The width of the band indicates the uncertainty in the initial scale Q_0 . The dashed line shows the asymptotic form $\phi(x, \infty) = 6x(1-x)$. Right: the same, evolved to $Q = 0.5 \text{ GeV}$ and compared to the transverse lattice data [41].

In Ref. [25] we provide formulas for the GPDs in the NJL model and in the Spectral Quark Model [46]. These expressions have a rather non-trivial structure, *not exhibiting factorization* in the t and x variables, while satisfying all the formal requirements mentioned above. Since there is no data for the full kinematic range for the GPDs, we only present the results for the generalized form factors, for which there is recent information from the lattice QCD [45, 47]. The vector form factor and the quark part of the gravitational form factor of the pion, obtained in the Spectral Quark Model, are compared to these lattice data in Fig. 3. We note a very good agreement. In the Spectral Quark Model the expressions are particularly simple

$$F_V^{\text{SQM}}(t) = \frac{m_\rho^2}{m_\rho^2 - t}, \quad \frac{\theta_{1,2}^{\text{SQM}}(t)}{\theta_{1,2}^{\text{SQM}}(0)} = \frac{m_\rho^2}{t} \log \left(\frac{m_\rho^2}{m_\rho^2 - t} \right).$$

We note the longer tail of the gravitational form factor in the momentum space, meaning a more compact distribution in the coordinate space. Explicitly, we find a quark-model formula $2\langle r^2 \rangle_\theta = \langle r^2 \rangle_V$.

Finally, we compare our model values for the higher-order form factors at $t = 0$ to the lattice data provided in Sec. 7 of Ref. [45], given below in parenthesis. After the evolution to the lattice scale of $Q = 2 \text{ GeV}$ we find

$$\begin{aligned} \langle x \rangle &= 0.28 \pm 0.02 \text{ (} 0.271 \pm 0.016 \text{)}, \\ \langle x^2 \rangle &= 0.10 \pm 0.02 \text{ (} 0.128 \pm 0.018 \text{)}, \\ \langle x^3 \rangle &= 0.06 \pm 0.01 \text{ (} 0.074 \pm 0.027 \text{)}. \end{aligned}$$

The model error bars come from the uncertainty of Q_0 . We note that the model predictions fall within the error bars. It should be mentioned that while high energies are needed to test leading twist parton distributions, transverse and Euclidean lattices can directly evaluate them *per se*.

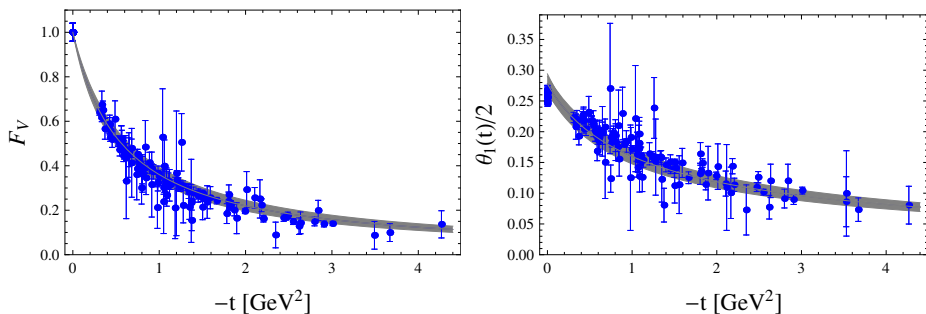


Fig. 3. Left: the electromagnetic form factor. Right: the quark part of the gravitational form factor, $\theta_1(t)/2$, both computed in the Spectral Quark Model and compared to the lattice data from Ref. [45]. The band around the model curves indicates the uncertainty in the quark momentum fraction.

In [48] we provide a handy way of undertaking evolution for generalized form factors which are currently becoming directly available on Euclidean lattices.

To briefly summarize, the chiral quark models supplied with the QCD evolution work well for a wide variety of quantities related to the GPDs of the pion and provide valuable insight into the non-perturbative dynamics behind the soft matrix elements.

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