

LOW-ENERGY AMPLITUDES IN THE NON-LOCAL CHIRAL QUARK MODEL*

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We apply chiral quark model with momentum dependent quark mass to two kinds of nonperturbative objects. These are: photon Distribution Amplitudes which we calculate up to twist-4 in tensor, vector and axial channels and pion–photon Transition Distribution Amplitudes together with related form factors. Where possible we compare our results with experimental data.

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1. Introduction

One of the biggest problems in particle physics is description of hadrons in terms of the fundamental degrees of freedom — quarks and gluons. This is, in fact, a non-perturbative problem and usually is formulated in terms of various distribution functions, which appear in QCD factorization theorems. The most famous example are Parton Distribution Functions which can be measured in the inclusive lepton–hadron deep inelastic processes. They are, however, one dimensional distributions only. Therefore, although they are by now sufficient for description of various processes at high energies, they simply give only limited information on the structure of hadrons.

On the other hand, one can study also hard *exclusive processes*, such as deeply virtual Compton scattering for instance. Then the factorization theorem states, in great simplicity, that the amplitude is given by the convolution

$$\mathcal{M} = (\text{soft}) \otimes (\text{hard}) , \quad (1)$$

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where *hard* is the part that can be calculated in perturbative QCD, while the *soft* part is of non-perturbative nature. In the following we will be mainly concentrated on the *soft* part. Although difficult to access experimentally, they can be obtained either by lattice calculations or — as we shall see — they can be estimated from theoretically justified low energy effective models.

The *soft* part parametrizes hadronic matrix elements of certain non-local operators on the light-cone. The simplest objects of this kind are Distribution Amplitudes (DA) which correspond to hadron-to-vacuum matrix elements of bi-local quark operators on the light-cone. In the case of the leading twist DAs, *i.e.* the ones giving main contribution to the amplitude, they describe (in the infinite momentum frame) the probability for a composite particle to dissociate into its constituents with given longitudinal momentum fractions. Distribution Amplitudes have been successfully used in theoretical description of hadronic form factors [1, 2] for many years. However, recent BaBar data for pion–photon transition form factor shows that probably the standard factorization formulae do not apply [3, 4]. We shall come back to the BaBar data in Section 4.

More general class of *soft* objects are Generalized Parton Distributions (GPD). They correspond to non-diagonal in momenta matrix elements, therefore they describe also the distributions of transverse momenta of the partons inside the hadron. GPDs appear in description of deeply virtual Compton scattering for instance, which is recently the subject of intensive theoretical and experimental studies. For a review of this issue see *e.g.* [7].

One can still define more general class of the objects than GPDs — so-called Transition Distribution Amplitudes (TDA). They parametrize matrix elements which are non-diagonal in momenta and in physical states. Such a family of objects was introduced for the first time in [13]. We shall discuss this class further in Section 4.

As already remarked above there is very little experimental data concerning the *soft* part of (1). On the other hand, it can be studied in effective models. This is, however, nontrivial not only because of complex non-local interactions at low energies. Even bigger problem is that in general effective models do not inherit all symmetries of the underlying theory. Soft objects considered here appear in the framework of QCD, therefore, they should possess several important properties, for example Lorentz and gauge invariance. They should also correctly reproduce quantum anomalies. The task to cope with all the constraints in the effective models is, therefore, nontrivial. We shall come back to this point in Section 2.

2. Non-local chiral quark model

Let us consider the scattering process involving the simplest possible hadronic state — the pion. On one hand, it is a bound state of quark–anti-quark pair and the Goldstone boson of spontaneously broken chiral symmetry on the other. Before we proceed let us briefly recall these very important aspects of QCD.

Here and in the following we assume only two quarks u and d which are massless, *i.e.* $m_u = m_d = 0$. Then the Lagrangian of QCD is invariant under separate rotations of left- and right-handed spinors, that is the symmetry group is $SU(2)_R \otimes SU(2)_L$ (the chiral symmetry group). It is generated by the chiral charges satisfying $SU(2)$ commutation relations $Q_{L,R}^a = \int d^3x \psi_{L,R}^\dagger(x) \gamma_5 \frac{\tau^a}{2} \psi_{L,R}(x)$, where τ^a are Pauli matrices, ψ denote iso-doublets. One can also define the combination of chiral fields transforming as vector and axial-vector. Then the corresponding combination of L, R charges $Q^a = Q_R^a + Q_L^a$ and $Q_5^a = Q_R^a - Q_L^a$ generate the $SU(2)_V \otimes SU(2)_A$ group¹. The most direct consequence of this symmetry would be a degeneracy of the states with different parity. However, such a behavior is not seen in the hadronic spectrum — on the contrary, we observe huge mass differences between parity partners. The most natural way of solving this discrepancy, is to postulate that although theory is chirally invariant, the vacuum state is not. This phenomenon is known as *spontaneous chiral symmetry breaking* ($S\chi SB$). According to Goldstone theorem we should then observe a triplet of massless pseudo-scalar particles — the Goldstone bosons. Indeed they can be apparently identified with pions (π^+ , π^0 , π^-), which are very light ($m_\pi \approx 140$ MeV) in comparison to other hadrons (*e.g.* $m_{\text{proton}} \approx 1$ GeV). Non-zero pion mass can be explained by finite (although small) current masses of u , d quarks, which explicitly break chiral symmetry from the very beginning.

Another important aspect of $S\chi SB$ is the existence of the quark condensates, *i.e.* the quantities

$$\langle 0 | \bar{q}q | 0 \rangle \equiv \langle \bar{q}q \rangle = \langle \bar{q}_R q_L \rangle + \langle \bar{q}_L q_R \rangle, \quad (2)$$

where q denotes either u or d quark field. It can be easily seen that the nonzero value of the quark condensate breaks chiral symmetry of the vacuum. Consider the commutator

$$\left[Q_5^a, \bar{\psi} \gamma_5 \tau^b \psi \right] = -\delta^{ab} \bar{\psi} \psi \quad (3)$$

¹ There is also similar global symmetry acting on the whole doublet. The axial symmetry $U(1)_A$ is, however, broken due to quantum anomaly.

and its vacuum expectation value. If the right-hand side is nonzero it implies that

$$Q_5^a |0\rangle \neq 0, \quad (4)$$

what is exactly the $S\chi$ SB condition. Therefore, the quark condensate can be viewed as an order parameter measuring the breakdown of chiral symmetry. Phenomenological value of the quark condensate is quite large $\langle \bar{q}q \rangle \sim (-250 \text{ MeV})^3$ (at renormalization scale about 1 GeV). Let us notice next that in QCD $\langle \bar{q}q \rangle$ is represented by a closed quark loop, *i.e.* it is proportional to the trace of fermionic propagator $\langle \bar{q}q \rangle \sim \text{Tr} \hat{S}(x, x)$, where the trace is over Dirac and color indices. However, if this quantity is non-zero there must be a non-slash term in the propagator — the mass term. This dynamically (due to $S\chi$ SB) generated mass is often referred to as *constituent quark mass*. Notice that the quark condensate is a purely non-perturbative quantity, since it is impossible to generate non-slash quark self energy by interactions of vector bosons. Rather it must be created by some kind of a scalar interactions. We shall come back to the issue of quark condensate later in Section 3.

Let us now switch to description of the interactions between the pions and quarks at low energies. It is clear from the above that such a model must incorporate $S\chi$ SB. It is convenient to start discussion by recalling the famous Nambu–Jona-Lasinio (NJL) model. It is an effective theory of quarks with four fermion couplings, appearing due to integrating out the gluonic degrees of freedom from the QCD action. In the standard NJL model couplings with more fermions are neglected. The Lagrange density for the simplest version of the NJL model reads

$$\mathcal{L}_{\text{NJL}} = \bar{\psi} i \not{\partial} \psi + \frac{G}{2} \left[(\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma_5 \vec{\tau} \psi)^2 \right], \quad (5)$$

where $\vec{\tau} = (\tau^1, \tau^2, \tau^3)$ are Pauli matrices and G is coupling constant. It can be checked using some algebra and the relation

$$e^{-i(\alpha \cdot \tau) \gamma_5} = (\cos |\alpha| - i \gamma_5 \hat{\alpha} \cdot \tau \sin |\alpha|),$$

where $\alpha^i = |\alpha| \hat{\alpha}^i$ that Lagrangian (5) is indeed chirally invariant. The most important feature of NJL model is that it incorporates the mechanism leading to $S\chi$ SB. One way to see this is to solve the corresponding lowest order Dyson–Schwinger equation for the quark propagator. Denoting quark self-energy by $\Sigma(p) \equiv M$ one obtains the following consistency condition (so-called gap equation)

$$M = -i8GN_c \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - M^2}. \quad (6)$$

It has two solutions: $M = 0$ (for massless quarks) and $M \neq 0$. The latter corresponds to the constituent quark mass which generates non-zero quark condensate breaking the chiral symmetry of the vacuum. Notice that the integral in (6) requires regularization. We shall discuss this later in this section. For a review of NJL model see [6] for example.

Mesons can be easily introduced into the just described theory as auxiliary fields σ and π^a — this can be done formally in the path integral formalism and is called bosonization procedure. The new Lagrange density reads

$$\mathcal{L}_{\text{NJL}'} = \bar{\psi} i \not{\partial} \psi + g \bar{\psi} [\sigma + i \gamma_5 \vec{\tau} \cdot \vec{\pi}] \psi + \frac{\mu^2}{2} (\sigma^2 + \vec{\pi}^2), \quad (7)$$

where $g^2 = \mu^2 G$. Notice that the fields σ, π^a are truly auxiliary — there are no corresponding kinetic terms, moreover, they are composed fields what can be immediately seen using equations of motion. NJL Lagrangian in the form (7) can also be used to show that the ground state which minimizes the energy is populated by the scalar quark condensate.

One can also look at the appearance of mesonic fields from a slightly different point of view. Consider the following effective Lagrange density

$$\mathcal{L} = \bar{\psi} (i \not{\partial} - M) \psi, \quad (8)$$

which leads to Dirac equation for the quark with constituent quark mass M . However (8) is obviously not chirally invariant. In order to fix this deficiency one has to introduce additional fields in the form

$$U^{\gamma_5}(x) = e^{\frac{i}{F_\pi} \vec{\tau} \cdot \vec{\pi}(x) \gamma_5} \approx 1 + \frac{i}{F_\pi} \gamma_5 \tau^a \pi^a(x) + \dots, \quad (9)$$

where $F_\pi \approx 93$ MeV is the pion weak decay constant, and couple them to quarks,

$$\mathcal{L} = \bar{\psi} (i \not{\partial} - M U^{\gamma_5}) \psi. \quad (10)$$

Then the axial transformations of quark fields can be absorbed by pion fields π^a . The Lagrange density (10) is a starting point for our further considerations and represents the simplest *local chiral quark model*. It describes quarks having dynamically generated constituent mass M and interacting with the external pion fields.

The effective theories just described are a non-renormalizable ones. The regularization introduced in order to make the loop integrals finite cannot be removed at the very end of the calculations and the observables depend on its actual form. Moreover, it is somehow (but not straightforwardly) related to the domain of applicability of the model. There are many ways of regularizing the loop integrals. One could use for example simple four-momentum cutoff or Pauli–Villars regularization. However, the point is that

the regularization scheme should respect all symmetries of the underlying theory, *i.e.* QCD. This is extremely important especially in the case of soft matrix elements as stated in Section 1. Therefore, four-momentum cutoff is excluded in the first place since it violates Lorentz invariance. Also very often used Pauli–Villars regularization is not the best method, because in order to get the results consistent with QCD one must keep it finite in some diagrams and remove in others (connected with anomalous processes). On the other hand, notice that in reality the constituent quark mass should not be a constant — it should vanish for large quark momenta due to asymptotic freedom. Therefore, in the following we assume that

$$M \equiv M(k) = M_0 F^2(k), \quad \text{where} \quad F(k) \xrightarrow[k \rightarrow \infty]{} 0, \quad F(0) = 1. \quad (11)$$

The constituent quark mass at zero momenta M_0 is chosen to be about $M_0 \sim 350$ MeV.

The interaction part of the effective action corresponding to (10) with assumption (11) can be written in momentum space as

$$S_{\text{int}} = M_0 \int \frac{d^4k d^4l}{(2\pi)^8} \bar{\psi}(k) F(k) U^{\gamma_5}(k-l) F(l) \psi(l). \quad (12)$$

The explicit shape of $F(k)$ cannot be obtained from the gap equation itself. However, the action (12) was actually obtained in the instanton model of the QCD vacuum, together with the expression for $F(k) = F_{\text{inst}}(k)$. Unfortunately $F_{\text{inst}}(k)$ turns out to be a highly non-trivial function of Euclidean momenta [8]. Therefore, instead of $F_{\text{inst}}(k)$ we shall use the following simple formula in Minkowski space [19]

$$F(k) = \left(\frac{-\Lambda_n^2}{k^2 - \Lambda_n^2 + i\epsilon} \right)^n, \quad (13)$$

which reproduces F_{inst} quite well when continued to Euclidean space. The parameter n is responsible for the actual shape of $F(k)$, therefore we can investigate the sensitivity of calculated quantities to the form of the cutoff function. The cutoff parameter Λ_n is adjusted in such a way that pion decay constant F_π given by the formula [21]

$$F_\pi^2 = \frac{N_c}{4\pi^2} \int_0^\infty dk_E^2 k_E^2 \frac{M^2(k_E) - k_E^2 M(k_E) M'(k_E) + k_E^4 M'(k_E)^2}{(k_E^2 + M^2(k_E))^2} \quad (14)$$

is equal to the experimental value. In the above equation k_E corresponds to Euclidean momentum, while the prime to differentiation with respect to k_E^2 . For example for constituent quark mass $M_0 = 350$ MeV and $n = 1$ (14) gives $\Lambda_1 = 836$ MeV.

Although momentum dependent quark mass seems to be the most natural regulator it introduces a serious difficulty. Namely Ward–Takahashi identities are not satisfied in such a model. It can be most easily seen by considering a divergence (in momentum space) of the vector current and applying Dirac equation. This violation turns out to be not very large, it can, however, spoil some important properties of soft matrix elements, like correct normalization for example. In order to fix this problem the standard vector γ^μ and axial $\gamma^\mu\gamma_5$ vertices have to be modified by adding new non-local terms. The problem is, however, that such a modification is not unique [21] (Ward identities fix only longitudinal part of the vertices). In this work we use the following modified vector and axial vertices:

$$\Gamma^\mu(k, p) = \gamma^\mu - \frac{k^\mu + p^\mu}{k^2 - p^2} (M(k) - M(p)) , \quad (15)$$

$$\Gamma_5^\mu(k, p) = \gamma^\mu\gamma_5 + \frac{p^\mu - k^\mu}{(p - k)^2} (M(k) + M(p))\gamma_5 , \quad (16)$$

reproducing Ward–Takahashi identities. The vector vertex does not introduce additional singularities, while the axial one has a pole corresponding to the massless pion as it should be [20]. Let us remark that Eq. (14) expressing F_π is determined unambiguously since it involves only the derivative of the axial current.

At the end of this section we remark that any non-local (*i.e.* with momentum dependent constituent quark mass $M(k)$) chiral quark model is determined by specifying both the $M(k)$ and the precise form of all vertices.

3. Photon Distribution Amplitudes

As already remarked in the Introduction the simplest *soft* objects are Distribution Amplitudes. In this section we present how the non-local chiral quark model can be applied to this class. However, instead of considering the hadronic DA we shall discuss less known photon DA. This is possible due to the fact that — besides standard perturbative part — photons possess also hadronic component. This fact is very well known from photoproduction processes, where photon structure function has to be taken into account (so-called resolved photoproduction).

Photon DAs appear for example in the description of vector mesons radiative decays. To be more specific consider for instance the process $D^{0*}(q+p) \rightarrow D^0(p) + \gamma(q)$ (Fig. 1 (a)). Using OPE one can then write the amplitude as products of factors that are divergent on the light-cone and finite photon-to-vacuum matrix elements. The latter can be identified with the photon DA as we shall see.

Before we give the more precise definition of the photon DA we should recall useful kinematical variables. One defines two null vectors $n = (1, 0, 0, -1)$ and $\tilde{n} = (1, 0, 0, 1)$. Then any four-vector v^μ can be decomposed into “plus”, “minus” and transverse components

$$v^\mu = v^+ \frac{\tilde{n}^\mu}{2} + v^- \frac{n^\mu}{2} + v_\perp^\mu. \quad (17)$$

Photon Distribution Amplitude is defined as a Fourier transform of the photon-to-vacuum matrix element of the non-local quark operator on the light-cone. In general this can be written as

$$\int \frac{d\lambda}{2\pi} e^{i(2u-1)\lambda P^+} \langle 0 | \bar{\psi}(\lambda n) \mathcal{O} \psi(-\lambda n) | \gamma(P) \rangle \sim F_{\mathcal{O}}(P^2) \times \left\{ \mathcal{O}_{\text{twist-2}} \phi_{\mathcal{O}}^{\text{twist-2}}(u, P^2) + \mathcal{O}_{\text{twist-3}} \phi_{\mathcal{O}}^{\text{twist-3}}(u, P^2) + \dots \right\}, \quad (18)$$

where $\mathcal{O} = \{\sigma^{\mu\nu}, \gamma^\mu, \gamma^\mu \gamma_5\}$ corresponds to different tensor nature of bilocal operators, $\mathcal{O}_{\text{twist-2}}, \mathcal{O}_{\text{twist-3}}, \dots$ denote appropriate tensor structures which are multiplied by photon DA $\phi_{\mathcal{O}}$ of given kinematical twist. Notice that we do not assume that the photon is on-shell. Then the decay constants $F_{\mathcal{O}}$ depend on photon virtuality P^2 and become a kind of “form factors” — we shall use this terminology in the following. For more precise definitions of the photon DAs refer to [9, 10, 17].

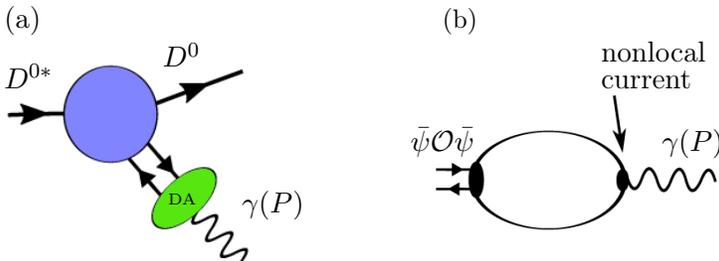


Fig. 1. (a) Bag diagram for the radiative D^{0*} vector meson decay. The lower blob corresponds to photon Distribution Amplitude. (b) Simple quark loop corresponding to photon DA in the quark model. Double external line represents bilocal quark operator on the light-cone.

Using the non-local chiral quark model described in Section 2 we calculated photon DAs up to twist-4 in tensor, vector and axial channels [12] in one loop approximation (Fig. 1 (b)). Our results are analytical up to the solution of a certain polynomial equation. Let us briefly summarize our results. For real photon the leading DA is the twist-2 tensor amplitude $\phi_{\sigma^{\mu\nu}} \equiv \phi_\Gamma$ (Fig. 2 (a)) corresponding to $\sigma^{\mu\nu}$ structure. We find it is almost

flat and non-vanishing in the end-points. Also the sensitivity to the n parameter, *i.e.* to the shape of $F(k)$ is rather small. In the vector channel one has to subtract the infinite, perturbative part when calculating the corresponding matrix element. Then we find in particular that leading twist vector DA vanishes in the end-points. It can be easily shown on general grounds that the vector “form factor” $F_{\gamma^\mu}(P^2) \equiv F_V(P^2)$ should be zero for the real photon. This property is maintained in our model only when we use modified vector vertex (Fig. 2 (b)), as described in Section 2. Higher twist amplitudes turn out to be rather strongly model dependent. Moreover, some of them contain Dirac delta functions in the end points, they should be therefore viewed rather as the generalized functions. Similar calculation was previously done in Ref. [10] and differs from ours in some points.

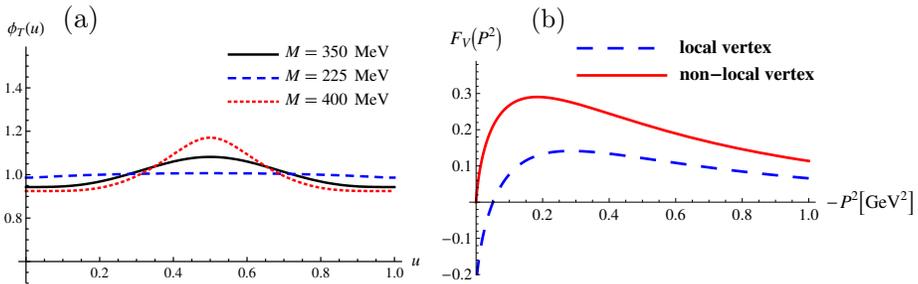


Fig. 2. (a) Twist-2 tensor photon Distribution Amplitude for $n = 1$ and several values of the constituent quark mass M_0 . (b) Vector form factor for $M_0 = 350$ MeV, $n = 1$, calculated using naive vector vertex γ^μ (dashed line) and the modified one Γ^μ (solid). Notice that the modified vertex assures that F_V vanishes for real photon as required by QED.

The left hand side of the definition (18) is dimensionfull, therefore we should have several quantities that set up the characteristic mass scale for photon DAs. Among others, it is a quark condensate, already discussed in Section 2. The non-local chiral quark model with (13) allows to obtain the following “analytical” expression for the quark condensate

$$\langle \bar{q}q \rangle = -\frac{N_c M_0^2 A_n^2}{4\pi^2} \sum_{i=1}^{4n+1} f_i \eta_i^{2n} (1 + \eta_i) \ln(1 + \eta_i), \quad (19)$$

where the complex numbers η_i are numerical solutions to the equation $z^{4n+1} + z^{4n} - (M_0/A_n)^2 = 0$, while f_i are defined as $f_i = \prod_{k \neq i}^{4n+1} (\eta_i - \eta_k)^{-1}$. For example for $M_0 = 350$ MeV and $n = 1$ we get $\langle \bar{q}q \rangle = (-253 \text{ MeV})^3$. It turns out that in general the values of $\langle \bar{q}q \rangle$ rather strongly depend on model parameters.

4. Pion–photon Transition Distribution Amplitudes

In this section we switch to more involved applications of the non-local chiral quark model. Transition Distribution Amplitudes, apart from being interesting on their own, can serve as a demanding testing ground for the model. The reason is that they involve diagrams responsible for axial anomaly. We shall come back to this point later in this section.

Transition Distribution Amplitudes were originally introduced in order to describe hadron–antihadron annihilation into two photons, *i.e.* the process $H\bar{H} \rightarrow \gamma^*\gamma$ or backward virtual Compton scattering $\gamma^*H \rightarrow \gamma H$ [13]. The amplitudes for these processes can be described in QCD analogously to the reactions $H\bar{H} \rightarrow \gamma^*$ and $\gamma^*H \rightarrow H$, respectively, with the restriction that Distribution Amplitudes for H should be replaced by a new object — Transition Distribution Amplitudes (Fig. 3 (a)). First estimates were done in Refs. [14–17].

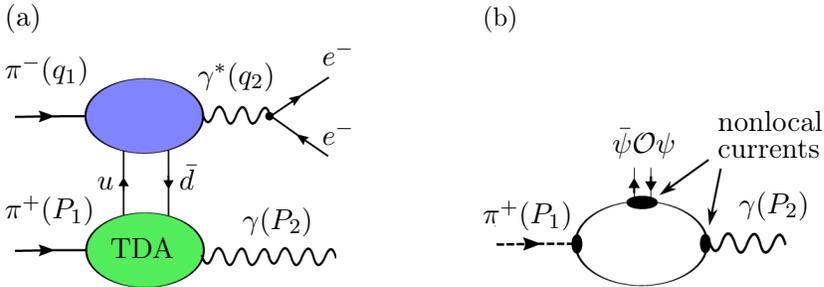


Fig. 3. (a) The bag diagram for the process $\pi^+\pi^- \rightarrow \gamma^*\gamma$. The lower bag represents Transition Distribution Amplitude while the upper corresponds to the hard process. (b) The quark loop corresponding to TDA, the bilocal operator is assumed to “live” on the light-cone. In order to recover correct normalization both vertices have to be non-local.

Before we give the general definition of TDAs we should define relevant kinematics. We consider pion with momentum P_1^μ transforming into the photon with momentum P_2^μ . We define the momentum transfer as $q^\mu = P_2^\mu - P_1^\mu$ and the momentum transfer squared $t = q^2$ which is assumed to be small. Using the average momentum $p^\mu = \frac{1}{2}(P_1^\mu + P_2^\mu)$ we define so-called skewedness $\xi = -q^+/2p^+$, which is a standard variable in the GPDs formalism. We consider chiral limit and real photons, *i.e.* $P_1^2 = P_2^2 = 0$.

The general definition of leading twist TDAs can be written as

$$\int \frac{d\lambda}{2\pi} e^{i\lambda X p^+} \langle \gamma(P_2, \varepsilon) | \bar{\psi}(\lambda n) \mathcal{O} \psi(-\lambda n) | \pi^+(P_1) \rangle = \mathcal{O}_{\text{twist-2}D}(X, \xi, t) + \dots, \tag{20}$$

where in practice $\mathcal{O} = \{\gamma^\mu, \gamma^\mu\gamma_5\}$. Dots stand for the other terms that can appear and are not related to TDA under consideration. For example in the axial channel, *i.e.* for $\mathcal{O} = \gamma^\mu\gamma_5$, pion DA accompanied by massless pole appears on the right-hand side. This reflects the fact that the axial current couples to a pion directly. In the following we denote vector TDA as $V(X, \xi, t)$ (*i.e.* for $\mathcal{O} = \gamma^\mu$) and the axial TDA as $A(X, \xi, t)$ (for $\mathcal{O} = \gamma^\mu\gamma_5$).

There is very important property that TDAs should possess, namely so-called polynomiality

$$\int_{-1}^1 dX X^n D(X, \xi, t) = a_n(t) \xi^n + a_{n-1}(t) \xi^{n-1} + \dots + a_0(t), \quad (21)$$

which follows simply from Lorentz invariance. In principle the zeroth moment is related to the corresponding form factor. Second very important constraint is the normalization of the vector TDA, which is fixed by the axial anomaly

$$\int_{-1}^1 dX V(X, \xi, t=0) = \frac{1}{2\pi^2}. \quad (22)$$

Above condition is model independent and can be derived using Ward–Takahashi identities that relate the two-photon matrix elements of the axial and pseudoscalar currents. The latter can be then identified with our matrix element (20) with $\mathcal{O} = \gamma^\mu$. There is no similar normalization condition for axial TDA. However, in the local models, *i.e.* with $M(k) \equiv M$ it turns out that

$$\int_{-1}^1 dX A_{\text{local}}(X, \xi, t=0) = \int_{-1}^1 dX V(X, \xi, t=0) = \frac{1}{2\pi^2}. \quad (23)$$

In the quark model, calculation of the TDAs reduces to performing quark loop shown in Fig. 3 (b). We present a typical results in Fig. 4 [18]. Notice first that curves obtained in non-local model are much more smooth than the ones obtained in the local model. Next, we find that the normalization condition (22) is recovered only when light-cone bilocal current in (20) is also modified according to (15). At the same time the normalization of the axial TDA is much lower than (23). This result is important because zeroth moments of vector and axial TDAs are directly related to the vector and axial form factors which can be estimated experimentally. To be more precise the relation is

$$\int_{-1}^1 dX \begin{Bmatrix} V(X, \xi, t) \\ A(X, \xi, t) \end{Bmatrix} = 2\sqrt{2}F_\pi \begin{Bmatrix} F_V^X(t) \\ F_A^X(t) \end{Bmatrix}, \quad (24)$$

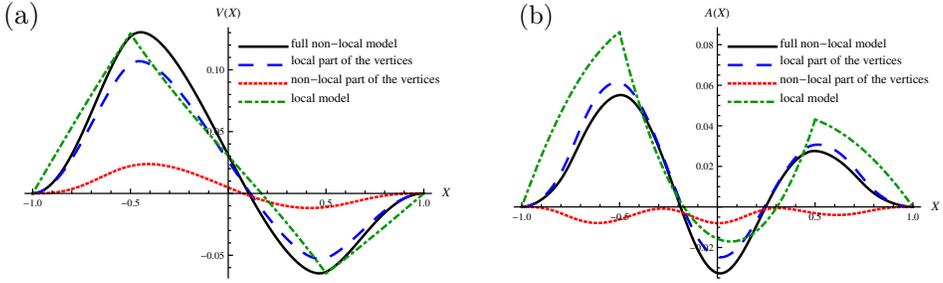


Fig. 4. (a) Vector Transition Distribution Amplitude for $M = 350$ MeV, $n = 1$, $t = -0.1$ GeV² and $\xi = 0.5$. Solid line corresponds to the full non-local model with non-local vertices and is a sum of the dashed line and the dotted. Dash-dotted line was obtained in local model, *i.e.* with $M(k) \equiv M$. (b) The same for the axial TDA. Here the addition coming from the non-local part of the vertices gives negative contribution.

where the superscript χ denotes that these quantities are defined in the chiral limit. The experimental values for $t = 0$ are (PDG)

$$F_V^{\text{exp}}(0) = 0.017 \pm 0.008, \quad (25)$$

$$F_A^{\text{exp}}(0) = 0.0115 \pm 0.0005, \quad (26)$$

$$(F_A(0)/F_V(0))_{\text{exp}} = 0.7_{-0.2}^{+0.6}. \quad (27)$$

On the other hand, the normalization (22) gives (model independent)

$$F_V^\chi(0) \approx 0.027, \quad (28)$$

what overshoots (25) more than one standard deviation. The results for axial form factor are model dependent. For reasonable model parameters we obtain:

M [MeV]	n	$F_A^\chi(0)$	$F_A^\chi(0)/F_V^\chi(0)$
225	1	0.0217	0.80
350	1	0.0168	0.62
350	5	0.0163	0.60
400	1	0.0161	0.60
400	5	0.0152	0.56

We see that indeed the assumption of non-locality (11) lowers the value of F_A^χ towards the experimental data.

Moreover, F_V^χ is directly related to so-called pion-photon transition form factor $F_{\pi\gamma}$ via the relation

$$F_{\pi\gamma}(t) = \sqrt{2}F_V^\chi(t). \quad (29)$$

This quantity describes the pion decay $\pi^0 \rightarrow \gamma^* \gamma$ process and was measured by CLEO [23], CELLO [22] and recently by BaBar [24] Collaborations. We compare our predictions to the experimental ones in Fig. 5. There is, however, important remark in order. Notice, that by definition TDAs are sensible only for small momentum transfers t , the precise range of application is, however, not known. Therefore, as an example we have chosen arbitrarily the range of 0–8 GeV². It is worth noting at this point that the new BaBar data are in disagreement with the standard QCD factorization formula, as it was already remarked in Introduction. In QCD, the pion–photon form factor can be described using pion DA and some perturbatively calculable factor, which leads to the certain asymptotic form. New BaBar data cover the range of 0–40 GeV² (in Fig. 5 we retained only the relevant low momentum data) and cross the asymptotic line already at about 10 GeV². One way to resolve this discrepancy is to note that pion DA which vanishes at the end-points was assumed in the standard factorization formula. In Refs. [3,4] the authors study the flat pion DA in order to describe the new BaBar data (but see also [5]).

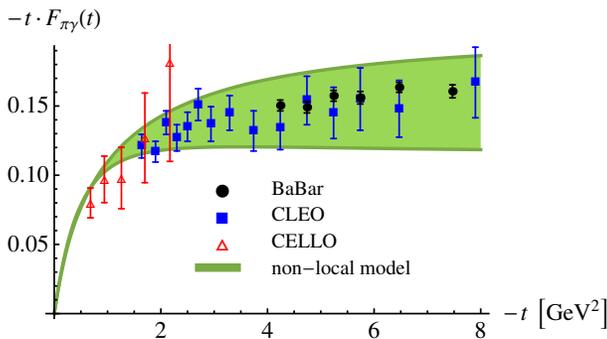


Fig. 5. The experimental data for pion–photon transition form factor $F_{\pi\gamma}$ times momentum transfer t . The shaded area represents the predictions from the non-local chiral quark model predictions for sensible model parameters. We get the best description of the low momentum transfer data for $M_0 \sim 300$ MeV.

5. Summary

Let us briefly summarize our presentation. In the beginning we recalled chiral quark models, starting from widely known Nambu–Jona-Lasinio model. We argued that spontaneous chiral symmetry breaking is their main ingredient. We also showed that they lead to nonzero quark condensates and in turn to dynamically generated constituent quark mass, which in general can depend on momentum. Next, we used a simple ansatz for this

dependence and applied the model to two low-energy objects: photon Distribution Amplitude and photon–pion Transition Distribution Amplitude. We find that they fulfill most symmetries required by QCD, provided we modify the vector and axial vertices in such a way that relevant currents are conserved. We find also that form factors which are calculated using Transition Distribution Amplitudes are realistic when compared to the experimental data.

At the end, we draw attention to important issues which was not covered by this presentation. First of all, in QCD all the low-energy quantities depend on some factorization scale μ and are a subject for corresponding QCD evolution. On the other hand, within effective models they are obtained at some fixed μ , which is in fact unknown (although can be roughly estimated). Therefore, before one makes a real use of them, the evolution has to be applied. The second remark is rather a technical one and concerns the cutoff A_n parameter in (13). One should not confuse it with the scale μ of the model, as discussed in [19].

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REFERENCES

- [1] A.V. Radyushkin, [hep-ph/0410276](#).
- [2] G.P. Lepage, S.J. Brodsky, *Phys. Rev.* **D22**, 2157 (1980).
- [3] A.V. Radyushkin, [arXiv:0906.0323 \[hep-ph\]](#).
- [4] M.V. Polyakov, *JETP Lett.* **90**, 228 (2009) [[arXiv:0906.0538 \[hep-ph\]](#)].
- [5] S.V. Mikhailov, N.G. Stefanis, [arXiv:0910.3498 \[hep-ph\]](#).
- [6] S.P. Klevansky, *Rev. Mod. Phys.* **64**, 3 (1992).
- [7] A.V. Radyushkin, A.V. Belitsky, *Phys. Rep.* **418**, 1 (2005).
- [8] D.I. Diakonov, V.Yu. Petrov, *Nucl. Phys.* **B245**, 259 (1984); **B272**, 457 (1986).
- [9] P. Ball, V.M. Braun, N. Kivel, *Nucl. Phys.* **B649**, 263 (2003).
- [10] A.E. Dorokhov, W. Broniowski, E.R. Arriola, *Phys. Rev.* **D74**, 054023 (2006).
- [11] V.Yu. Petrov, M.V. Polyakov, R. Ruskov, C. Weiss, K. Goeke, *Phys. Rev.* **D59**, 114018 (1999).
- [12] P. Kotko, M. Praszalowicz, “Photon distribution amplitudes in the non-local chiral quark model”, to be published.
- [13] B. Pire, L. Szymanowski, *Phys. Rev.* **D71**, 111501 (2005).
- [14] B.C. Tiburzi, *Phys. Rev.* **D72**, 094001 (2005).

- [15] A. Courtoy, S. Noguera, *Phys. Rev.* **D76**, 094026 (2007).
- [16] E.R. Arriola, W. Broniowski, *Phys. Lett.* **B649**, 49 (2007).
- [17] P. Kotko, M. Praszalowicz, *Acta Phys. Pol. B* **40**, 123 (2009).
- [18] P. Kotko, M. Praszalowicz, *Phys. Rev.* **D80**, 074002 (2009).
- [19] M. Praszalowicz, A. Rostworowski, *Phys. Rev.* **D64**, 074003 (2001).
- [20] B. Holdom, R. Lewis, *Phys. Rev.* **D51**, 6318 (1995).
- [21] R.D. Bowler, M. Birse, *Nucl. Phys.* **A582**, 655 (1995)
- [22] H.J. Behrend *et al.* [CELLO Collaboration], *Z. Phys.* **C49**, 401 (1991).
- [23] J. Gronberg *et al.* [CLEO Collaboration], *Phys. Rev.* **D57**, 33 (1998).
- [24] B. Aubert *et al.* [BaBar Collaboration], *Phys. Rev.* **D80**, 052002 (2009).