QUANTIZATION OF NONRELATIVISTIC PHASE SPACE AND THE STANDARD MODEL*

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We start from recalling the limited and descriptive character of our theories, and point out the existence of a tension between relativity and quantum physics. It is then argued that some quantum features of the Standard Model may be understood when the concept of arena used for the description of physical processes is changed from relativistic 'space-time' to nonrelativistic 'phase-space + time'. The phase-space form $\boldsymbol{x}^2+\boldsymbol{p}^2$, which constitutes a natural generalization of 3D invariants $\boldsymbol{x}^2$ and $\boldsymbol{p}^2$, is linearized *á la* Dirac, with x_k and p_j satisfying standard commutation relations. This leads to a quantum-level structure related to phase space, and to the appearance of new quantum numbers. The latter are identified with internal quantum numbers characterizing the structure of a single quark–lepton generation. The approach provides a preonless interpretation of the Harari–Shupe model, and leads both to a different view on the concept of quark mass and to the emergence of quark-confining strings.

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1. Introduction

The Standard Model (SM) gives us a successful description of a plethora of experimental results. Yet, many questions exist for which it provides no answer. For example, we do not know the origin of the $\mathrm{U}(1)\otimes\mathrm{SU}(2)_\mathrm{L}\otimes\mathrm{SU}(3)$ gauge group structure, we are ignorant of the mechanism leading to the emergence of SM parameters (masses, mixing angles), *etc.* Clearly, in order to find answers to such questions one must go beyond SM itself. Going outside the framework of a given theory leaves us in a totally unchartered

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territory, however. Which way should one choose there, which assumptions accept? If we do not have a solid guiding principle, we are bound to go astray already after the first few steps ...

2. Physics and reality

Thirty years ago Werner Heisenberg, speaking about the development of particle theory, said [1]:

> *I believe that certain erroneous developments in particle theory — and I am afraid that such developments do exist — are caused by a misconception by some physicists that it is possible to avoid philosophical arguments altogether. Starting with poor philosophy, they pose the wrong questions.*

Thus, it is appropriate to start with a brief discussion of the philosophical status of our theories. In particular, one should realize that we all have the tendency to commit a philosophical error known under the name of *the fallacy of misplaced concreteness* (term introduced by A.N. Whitehead). We commit this error when we mistake a theory for a physical or 'concrete' reality. In fact, the actual relationship between our theories and physical reality we want to describe is more like that schematically shown in Fig. 1. In other words, a physical theory provides only *a description of some aspects* of reality. Accordingly, theories must not be identified with the latter. In principle, we know this well. Yet, we all tend to fall into the trap. For example, if we think of absolute time as a property of physical reality, we are prevented from moving on to a different description — that provided by special relativity, in which 'time' ceases to be absolute and changes when the frame used for the description of phenomena is altered.

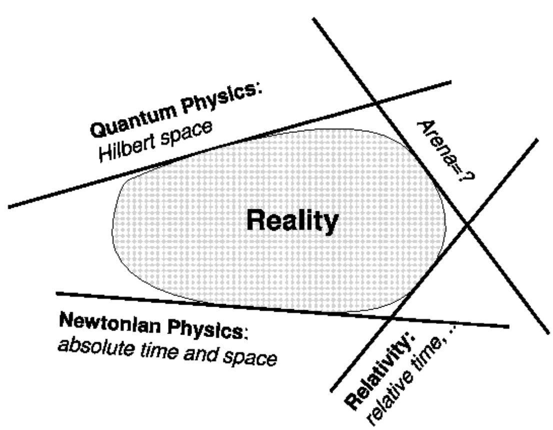

Fig. 1. Physical theories and reality.

Since all physical theories have been constructed to provide descriptions of certain aspects of reality only, they should all have limited domains of applicability. Going outside these domains leads to artifacts. As a second example, therefore, let us think of quantum physics as identical to physical reality. This may lead to such an idea as the Many Worlds theory, which may be regarded as an artifact. Unfortunately, we have no means of knowing in advance where the applicability of theory ends. This must be checked experimentally.

The above discussion demonstrates that when trying to describe physical reality one should not be bound too strongly by contemporary theories. Their assumptions are not harmful in their domain of applicability, but — essentially by definition — are harmful beyond it. Thus, it is important to analyze the assumptions themselves and judge which ones seem to be really sound, much sounder than the other ones, and which ones may be omitted as not needed for the intended goal.

At present, one of our goals is to see if the basic symmetries of the Standard Model could be derived from some single principle. In order to approach that question, let us first note that SM belongs to the quantum description of reality. Yet, other aspects of physical reality may be described in a classical way. Since both descriptions deal with aspects of the *same* reality (Fig. 1), the two descriptions must be related. Thus, it is possible that classical considerations might shed some light on the quantum aspects of SM.

This anticipated connection between the quantum and classical descriptions of Nature may be viewed upon in various ways. One may accept a reductionist-like attitude in which the concepts of quantum physics (*e.g.* quantum numbers of elementary particles) are thought of as more fundamental than those of classical physics. According to this view, one should start with quantum concepts and somehow build classical concepts (such as geometry) out of them. The whole idea may be called the philosophy of 'emergent space' and was succinctly expressed by J.A. Wheeler, as '*Day One: Quantum Principle, Day Two: Geometry*'.

A somewhat more symmetric (and presumably more holistic) attitude may be found in these words of Penrose [2]: '*I do not believe that a real understanding of the nature of elementary particles can ever be achieved without a simultaneous deeper understanding of the nature of spacetime'*. At least two things are being said here: first, that the observed properties of elementary particles (quantum numbers, masses, *etc.*) are related to the properties of the macroscopic classical arena on which their time evolution is described, and second, that the understanding of the geometrical and quantum aspects of Nature should be deepened *simultaneously*.

The view that the properties of elementary particles and the properties of macroscopic arena are closely related was recently studied from a novel angle in a series of papers [3–5]. The general idea developed in these papers is that the standard description of physical reality — in which, roughly speaking, one identifies the macroscopic arena with the ordinary 3D space — may (and should) be replaced by a different description, in which the arena is identified with phase space. In the following, we shall first argue step by step that this seems to be the right thing to do, and then see to what consequences such an assumption leads.

3. Quantum numbers of elementary particles and relativity

According to the Standard Model, there are three generations of fundamental elementary fermions, each generation composed of two leptons and two triplets of quarks. The particles of a given generation differ in their properties, with the differences corresponding to different eigenvalues of the internal quantum numbers of (weak) isospin and color. Both, the origin of the structure of a single generation and why it is repeated three times are unknown.

The name 'internal' quantum numbers was introduced to differentiate them from 'spatial' quantum numbers, which are closely related to the properties of macroscopic 3D space, while the internal ones are not. The spatial quantum numbers are: spin J — related to ordinary 3D rotations, parity P — related to 3D reflections, and charge-conjugation parity C — related to time reflection (recall the Stückelberg–Feynman interpretation of antiparticles as particles moving 'backward' in time). We observe that all these spatial quantum numbers are nonrelativistic in origin. In particular, we stress that the emergence of antiparticles is not a relativistic phenomenon as the Dirac equation might suggest: the antiparticles appear also when one linearizes the nonrelativistic Schrödinger equation [6], and are related to complex conjugation (as is time reversal).

In view of the nonrelativistic origin of all spatial quantum numbers, it seems, therefore, natural to suspect that the minimal extension of the concept of arena needed for a similar understanding of internal quantum numbers should be nonrelativistic as well. In other words, while for the description of classical relativistic motions one certainly needs the theory of relativity, the latter is presumably not needed for the description of particle quantum numbers. This is in perfect agreement with our previous discussion concerning the descriptive character of our theories.

More precisely, one should realize that Nature *is not* four-dimensional. It is only *a description of its certain aspects* that may be formulated in terms of the 4D Minkowskian space. Indeed, it is known that the connection between space and time is more subtle than the standard form of special relativity suggests. This standard form emerges when distant clocks are synchronized according to the Einstein radiolocation prescription, in which the *one-way* velocity of light is assumed equal to c. Yet, the latter assumption cannot be checked experimentally (as opposed to *two-way* velocity of light which is experimentally equal to c), as this would require the existence of *already* synchronized distant clocks. Thus, different synchronization prescriptions are possible, including even a description in which absolute simultaneity reappears [7]. In other words, special relativity is a clock gauge theory, with a fixed symmetric gauge (*i.e.* the velocity of light 'to' assumed equal to the velocity of light 'from') adopted by Einstein for aesthetic and simplicity reasons. It is true that this symmetric gauge simplifies the description very much. Clearly, however, a fixed gauge should not be mistaken for physical reality.

Returning to the quantum issues, there exists a known tension between quantum physics and the theory of relativity. And although the relativistic field theory does unite special relativity and quantum physics, this 'marriage' of quantum and relativistic ideas is — in the opinion of many theorists — somewhat uneasy [8]. The actual wording may take various forms, *e.g.* '*The construction of a fully objective theory of state-vector reduction which is consistent with the spirit of relativity is a profound challenge, since "simultaneity" is a concept (...) foreign to relativity*' [9]. Furthermore, the existence of 'nonlocality without signaling', which creates a conflict between quantum physics and the spirit of relativity, has been experimentally verified over the distances of the order of a hundred km (*e.g.* see references in [10]). This quantum concept of 'nonlocality without signaling' is completely foreign to relativity. In retrospect, the concept of locality on which the theory of relativity is founded should be, therefore, seen as an idealization of certain aspects of Nature. The tension existing between quantum physics and relativity is a symptom of the fact that we are dealing with descriptions, and not with physical reality, *i.e.* that our theories are incomplete idealizations (see Fig. 1). Or, in the words of Gisin [10]: '*...who can doubt that relativity is incomplete? And likewise who can doubt that quantum mechanics is incomplete? Indeed, these are two scientific theories and Science is nowhere near its end ...*'.

We conclude that in an approach that aims at understanding internal *quantum* numbers it is well justified to dismiss the use of relativistic concepts, at least at the beginning. In other words, the word 'spacetime' in the quotation from Penrose might be without much harm replaced with 'space+time'. Yet, if the internal quantum numbers are to be connected with the properties of the macroscopic 'space', the ordinary 3D space clearly has to be somehow extended into a broader 'arena'.

4. Phase space as an arena

The existence of a relation between particle properties (*e.g.* mass), and the 'emergent space' was of great concern already to Max Born. Over half a century ago he wrote [11]: '*I think that the assumption of the observability of the 4-dimensional distance of two events inside atomic dimensions (no clocks or measuring rods) is an extrapolation ...*'. Then, he continued with the discussion of a difference between the position and momentum spaces for the observed particles. First, he noted that different particles correspond to different discrete values of mass m, thus making p^2 observable (via $p^2 = m^2$). Then, he stressed that the corresponding invariant in position space (with x^2 of atomic size) seems to be '*no observable at all*'.

And yet, as he pointed out, the laws of nature such as

$$\begin{aligned} \dot{x}_k &= \frac{\partial H}{\partial p_k}\,, \qquad \dot{p}_k = -\frac{\partial H}{\partial x_k}\,, \\ [x_k, p_l] &= i\hbar\delta_{kl}\,, \\ L_{kl} &= x_k p_l - x_l p_k\,, \end{aligned} \tag{1}$$

are invariant under 'reciprocity' transformations:

$$x_k \to p_k\,, \qquad p_k \to -x_k\,. \tag{2}$$

Noting that the reciprocity symmetry does not seem to apply to elementary particles, he concluded: '*This lack of symmetry seems to me very strange and rather improbable*'.

The concept of reciprocity suggests the introduction of a new constant of dimension (GeV$/c$)/cm, so that all positions and momenta be of the same dimension and may be transformed into one another. This leads to the idea that it is the nonrelativistic phase space which might be considered as the macroscopic arena of physical events. Hence, instead of a nonrelativistic 'space+time' picture, we might use a 'phase-space+time' description (see Fig. 1).

We observe that such an extension of the concept of arena agrees well with the classical Hamiltonian description, in which positions and momenta are *independent* variables. With quantum mechanics 'living in phase space' [12], identifying the arena with phase space seems to be an even more natural thing to do (see also [13]). We also note that the above proposal constitutes an absolutely minimal generalization of the standard view of position space as the arena. In particular, no additional hidden dimensions are introduced in this way.

With the macroscopic arena thus enlarged with respect to the standard 3D arena of positions, one expects the emergence of additional quantum numbers. Obviously, if one accepts the philosophy of 'emergent space', the arrow of implications should be ultimately reversed. Then, one should start from the quantum numbers of elementary particles and somehow build the 'Emergent Phase Space'.

5. Phase space quantization and the Standard Model

The simplest and fully symmetric joint treatment of the basic O(3) invariants in position and momentum spaces is obtained by adding them together:

$$\boldsymbol{x}^2 + \boldsymbol{p}^2 \,. \tag{3}$$

It clearly includes Born's reciprocity and leads to O(6).

In the quantum case, $\boldsymbol{x}$ and $\boldsymbol{p}$ are operators satisfying standard commutation relations. Thus, we are dealing with a case of noncommuting geometry in six dimensions, in which three dimensions are interpreted as positions, and the remaining three dimensions — as momenta. As is well known from the case of the 3D harmonic oscillator, if one admits only those O(6) transformations which leave the commutation relations invariant, the O(6) symmetry group is reduced to U(1) $\otimes$ SU(3). Here, the U(1) factor describes Born's reciprocity transformations and their squares, *i.e.* the 3D reflections, while the SU(3) factor takes care of standard rotations in particular.

The appearance of the U(1)$\otimes$ SU(3) group and the presence of the same group in the Standard Model raises the question whether the internal symmetry group of the latter is related to phase space symmetries. A direct confirmation of this suggestion would require the introduction of a connection to the SM gauge prescription. Yet, the latter may appear only after or alongside the emergence of space. Consequently, at present it lies beyond our reach. Still, a different corroboration of our proposal may be obtained: in the following, we will show that the structure of quantum numbers obtained at the deeper level of the phase-space approach parallels that observed in the real world.

In order to reach this possibly deeper and more quantum-like level of the phase-space approach, let us note that in the 3D case the Dirac linearization procedure, *i.e.*

$$\boldsymbol{p}^2 = (\boldsymbol{p}\cdot\boldsymbol{\sigma})(\boldsymbol{p}\cdot\boldsymbol{\sigma})\,, \tag{4}$$

leads to the appearance of Pauli matrices $\boldsymbol{\sigma}$, *i.e.* to the concept of spin. Thus, linearization leads to the emergence of discrete structures and related quantum numbers, *i.e.* it provides a way of quantization. Let us, therefore, linearize $\boldsymbol{x}^2 + \boldsymbol{p}^2$ à la Dirac and consider $\boldsymbol{A}\cdot\boldsymbol{p} + \boldsymbol{B}\cdot\boldsymbol{x}$ with anticommuting matrices A_k and B_l $(k, l = 1, 2, 3)$. We use the following representation

$$\begin{aligned} A_k &= \sigma_k \otimes \sigma_0 \otimes \sigma_1 \,, \\ B_k &= \sigma_0 \otimes \sigma_k \otimes \sigma_2 \,, \\ B_7 &= \sigma_0 \otimes \sigma_0 \otimes \sigma_3 \,, \end{aligned} \tag{5}$$

(B_7 is the seventh anticommuting element of the Clifford algebra in question). One finds

$$R^{\rm tot} \equiv (\boldsymbol{A}\cdot\boldsymbol{p}+\boldsymbol{B}\cdot\boldsymbol{x})(\boldsymbol{A}\cdot\boldsymbol{p}+\boldsymbol{B}\cdot\boldsymbol{x}) = (\boldsymbol{p}^2+\boldsymbol{x}^2)+\sum_1^3 \sigma_k\otimes\sigma_k\otimes\sigma_3\,. \qquad (6)$$

The first term on the r.h.s., *i.e.* $R \equiv \boldsymbol{p}^2 + \boldsymbol{x}^2$, is standard: it emerges because elements A_k and B_l anticommute. The second term, *i.e.* $R^\sigma \equiv \sum_1^3 \sigma_k \otimes \sigma_k \otimes \sigma_3$, appears because x_k and p_k do not commute. In order to simplify our expressions and remove the rightmost σ_3 factor in R^σ, we multiply both sides of (6) by B_7 and introduce operator

$$Y \equiv \frac{1}{3}R^\sigma B_7 = \frac{1}{3}\sum_1^3 \sigma_k\otimes\sigma_k\otimes\sigma_0 \equiv \sum_1^3 Y_k\,. \qquad (7)$$

Since operators Y_k commute among themselves, they may be simultaneously diagonalized. One then gets the pattern shown in Table I (since the matrices are 8×8, this pattern is obtained twice).

TABLE I

Decomposition of the eigenvalue of Y into eigenvalues of Y_k.

Color	0	1	2	3
Y	-1	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$
Y_1	$-\frac{1}{3}$	$-\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$
Y_2	$-\frac{1}{3}$	$+\frac{1}{3}$	$-\frac{1}{3}$	$+\frac{1}{3}$
Y_3	$-\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	$-\frac{1}{3}$

The generators of SO(6) are constructed as antisymmetric bilinears of A_k, B_l. In particular, the generator of standard rotation has the explicit form

$$S_k = \tfrac{1}{2}\left(\sigma_k\otimes\sigma_0 + \sigma_0\otimes\sigma_k\right)\otimes\sigma_0 \qquad (8)$$

and corresponds to simultaneous (the same size and sense) rotations in momentum and position subspaces. Introducing $I_3 = B_7/2$, we observe that

$$[S_k, Y] = [S_k, I_3] = 0\,. \qquad (9)$$

Thus, Y and I_3 are invariant under standard 3D rotations. Likewise, they are also invariant under 3D reflections. Hence, Y and I_3 constitute candidates for internal quantum numbers. In [4] it was conjectured that electric charge Q is proportional to operator $R^{\rm tot}B_7$, evaluated for the lowest level of R:

$$Q = \frac{1}{6}(R_{\rm lowest} + R^\sigma)B_7 = I_3 + \frac{Y}{2}\,, \qquad (10)$$

with $R_{\text{lowest}} = (\boldsymbol{p}^2 + \boldsymbol{x}^2)_{\text{lowest}} = 3$, $I_3 = \pm 1/2$ being (weak) isospin, and Y — (weak) hypercharge. The above equation yields the charges of all eight leptons and quarks from a single SM generation (*cf.* Table I) and is known as the Gell-Mann–Nishijima relation. In our phase-space approach this law of nature is *derived* as a property of phase space.

There is one unexpected byproduct of the above scheme. Namely, one may consider partners of I_3, *i.e.* $I_k = \sigma_0 \otimes \sigma_0 \otimes \sigma_k/2$ for $k = 1, 2$. It is then straightforward to check that the SU(2) raising and lowering operators $I_1 \pm i\, I_2$ — which connect sectors of different I_3 — commute with ordinary rotations, but do not commute with 3D reflection. While the pattern of parity violation present in SM is clearly more complicated than this, it is nevertheless interesting that the lack of invariance under 3D reflections appears automatically and in the roughly right place.

6. Harari–Shupe rishons

The way in which hypercharge Y is built up of 'partial hypercharges' Y_k (shown in Table I), or rather an equivalent of this scheme, was proposed thirty years ago by Harari and Shupe [14]. The Harari–Shupe model describes the structure of a single SM generation with the help of a composite (*i.e.* preon) model. It builds all eight fermions of a single generation from only two spin-1/2 'rishons' V and T of charges 0 and $+1/3$, respectively, (see Table II).

TABLE II

Rishon structure of leptons and quarks with isospin $I_3 = +1/2$.

	ν_e	$u_{\rm R}$	$u_{\rm G}$	$u_{\rm B}$	e^+	$\bar{d}_{\rm R}$	$\bar{d}_{\rm G}$	$\bar{d}_{\rm B}$
	VVV	VTT	TVT	TTV	TTT	TVV	VTV	VVT
Q	0	$+\frac{2}{3}$	$+\frac{2}{3}$	$+\frac{2}{3}$	+1	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$
Y	−1	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	+1	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$

The rishon model is algebraically very economical, yet it poses several problems. These include, among others: the issue of preon confinement at extremely small distance scales (when confronted with the uncertainty principle), the apparent absence of spin-3/2 fundamental particles, and the lack of explanation why the ordering of three rishons leads to SU(3). (Note also that rishons — assumed to be spin-1/2 objects — obey strange statistics: in the three states of different color the rishons are not (anti)symmetrized but ordered, *e.g.* VTT, TVT, and TTV.)

The phase-space approach solves all these problems of the Harari–Shupe model with a single stroke, while exactly reproducing its successful part [4]. The resolution is made possible by the fact that 'phase-space rishons' are not spin-1/2 particles, but only the algebraic components of the charge operator. Consequently, there is no problem (1) of preon confinement, (2) of where are the fundamental spin-3/2 particles, nor (3) of 'strange statistics'.

Furthermore, the ordering of rishons is naturally connected with SU(3) [4]. The 'ordered rishon structure' (such as VTT) may be easily understood in phase-space terms. Thus, the position of a rishon corresponds to one of three directions in our macroscopic 3D space: VTT corresponds, therefore, to the partial hypercharge eigenvalue of $-1/3$ in direction (x, p_x) and to the same eigenvalue of $+1/3$ in both remaining directions, (y, p_y) and (z, p_z). Since any discussion of rotations requires three directions, the concept of spin cannot be applied to a single rishon. It is thus most appropriate to quote here the following words of Heisenberg [1]: '*... the antinomy of the smallest dimensions is solved in particle physics in a very subtle manner, of which neither Kant nor the ancient philosophers could have thought: The word "dividing" loses its meaning*'.

7. Transformations in phase space

In order to grasp the meaning of the relation between quarks and leptons in phase-space terms, consider first [4] a transformation of A_k and B_l generated by F^σ_{-2}, one of six 'genuine' SO(6) generators $F^\sigma_{\pm n}$ $(n = 1, 2, 3)$:

$$\begin{aligned} F^\sigma_{-n} &= \tfrac{1}{2}\,(\sigma_0 \otimes \sigma_n - \sigma_n \otimes \sigma_0) \otimes \sigma_3\,, \\ F^\sigma_{+n} &= \tfrac{1}{2}\,\varepsilon_{nkl}\,\sigma_k \otimes \sigma_l \otimes \sigma_3\,. \end{aligned} \tag{11}$$

For the F^σ_{-2}-generated transformations one gets:

$$\begin{aligned} A'_k &= A_1 \cos\phi - A_3 \sin\phi\,, & B'_1 &= B_1 \cos\phi + B_3 \sin\phi\,, \\ A'_2 &= A_2\,, & B'_2 &= B_2\,, \\ A'_3 &= A_3 \cos\phi + A_1 \sin\phi\,, & B'_3 &= B_3 \cos\phi - B_1 \sin\phi\,, \end{aligned} \tag{12}$$

i.e. $\boldsymbol{A}$ and $\boldsymbol{B}$ rotate in opposite senses. Setting $\phi = \pm\pi/2$, we obtain:

$$Y = Y_1 + Y_2 + Y_3 \to Y' = -Y_3 + Y_2 - Y_1\,. \tag{13}$$

From Table I it then follows that lepton and quark 2 are exchanged, while the remaining two quarks are untouched. An analogous result is obtained for the F^σ_{+2}-generated transformation.

The corresponding transformations in phase space may be obtained from the condition of the invariance of

$$\boldsymbol{A} \cdot \boldsymbol{p} + \boldsymbol{B} \cdot \boldsymbol{x}\,. \tag{14}$$

For the phase-space counterpart of the F^{σ}_{-2}-generated transformations, one then gets

$$\begin{aligned} [x'_k, x'_l] = [p'_k, p'_l] &= 0\,, \\ [x'_k, p'_l] &= i\Delta_{kl}\,, \end{aligned} \tag{15}$$

with

$$\Delta = \begin{bmatrix} \cos 2\phi & 0 & \sin 2\phi \\ 0 & 1 & 0 \\ -\sin 2\phi & 0 & \cos 2\phi \end{bmatrix}. \tag{16}$$

The case of lepton-quark 2 interchange (*i.e.* $\phi = \pm\pi/2$) corresponds then (for the counterparts of both F^{σ}_{-2}- and F^{σ}_{+2}- generated transformations) to:

$$\text{(quark)} \quad \Delta = \begin{bmatrix} -1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad \leftrightarrow \quad \Delta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{(lepton)}\,. \tag{17}$$

Thus, one may say that a quark is a lepton rotated in phase space.

Taking into account the remaining types of genuine SO(6) transformations, one obtains four sets of generalized commutation relations:

$$\begin{array}{cccc} \text{lepton} & \text{quark 1} & \text{quark 2} & \text{quark 3} \\ [x_1, p_1] = i & [x_1, p_1] = i & [p_1, x_1] = i & [p_1, x_1] = i \\ [x_2, p_2] = i & [p_2, x_2] = i & [x_2, p_2] = i & [p_2, x_2] = i \\ [x_3, p_3] = i & [p_3, x_3] = i & [p_3, x_3] = i & [x_3, p_3] = i \end{array}\,. \tag{18}$$

When a reflection in phase space (*e.g.* $p'_k = p_k$, $x'_k = -x_k$, but $i = i$) is performed, the number of sets is doubled from four to eight, with i in Eqs. (18) changed to $-i$. The two sectors corresponding to the imaginary unit on the r.h.s. of commutation relations being i and $-i$ correspond to doublets of weak isospin. In conclusion, we obtain 8 disjoint sectors corresponding to 8 particles of a single generation of the Standard Model. Antiparticles are obtained by complex conjugation, which may be defined as $p'_k = p_k$, $x'_k = -x_k$, and $i = -i$, which is clearly different from phase-space reflection.

Note that each of the three rightmost sets of commutation relations in Eq. (18) is not rotationally invariant. In my opinion, this by itself does not pose any problem since in our experiments we *never* deal with individual quarks. The only condition that must be satisfied is that those systems which are composed of quarks and to which we have experimental access (*i.e.* mesons, baryons, color-singlet combinations of quark currents with meson quantum numbers) must be described by structures covariant under rotations. Thus, the conjecture is that quarks must conspire when forming mesons and baryons. The question then is whether in our scheme such a conspiracy can be achieved.

8. Hints of conspiracy

The first hint of quark conspiracy comes from the consideration of the concepts of lepton and quark mass. In order to discuss the issue of mass, a study of the Clifford algebra of nonrelativistic phase space is needed. This 64-element algebra may be decomposed into its even and odd parts (with their elements being linear combinations of products of an even or odd number of A_m and B_n). The unit element and the 15 SO(6) generators may be projected upon the $I_3 = \pm 1/2$ subspaces, thus forming 32 even elements. The remaining 32 odd elements may be divided into two parts, of which one constitutes Hermitian conjugate of the other. The U(1)⊗SU(3) structure of one such odd part is presented in Table III. In this table, we show 16 odd elements with left and right eigenvalues of $I_{3\mathrm{l}} = +1/2$ and $I_{3\mathrm{r}} = -1/2$, respectively. In the columns marked Y_l and Y_r, the left and right eigenvalues of Y are given. A detailed explanation of entries in this table is given in [5]. Table III is relevant for the discussion of the concept of mass. Namely, the algebraic counterpart of lepton mass should be odd (just like the odd A_m is associated with p_m) and must be identified (up to Hermitian conjugation) with the only $Y = -1$ scalar element in Table III, *i.e.* with G_0. It may be verified that the $F_{\pm 2}$-generated transformation from the lepton to the quark sector changes G_0 into $G_{\{22\}}$, which is a member of the SU(3) sextet, and contains SO(3) singlet and tensor pieces. In particular, therefore, the algebraic counterpart of quark mass appears non-invariant under rotations. Yet, the sum over the three colors of such quark mass elements, *i.e.* $\sum_k G_{\{kk\}}$, is rotationally invariant, just as the idea of quark conspiracy suggests.

TABLE III

Odd elements of Clifford algebra according to their U(1)⊗SU(3) properties.

U(1)	SU(3)	Elem.	Y_l	Y_r
+1	3*	$U_k^\dagger$	$+\frac{1}{3}$	$+\frac{1}{3}$
−1	3	V_k	$+\frac{1}{3}$	−1
−1	3	W_k	−1	$+\frac{1}{3}$
+1	6	$G_{\{kl\}}$	$+\frac{1}{3}$	$+\frac{1}{3}$
−3	1	G_0	−1	−1

The above concept of quark mass is different from that assumed in the Standard Model. It should be stressed, however, that the application of the standard concept of mass to quarks leads to problems and conceptual inconsistencies. Such problems stem from the fact that in many standard calculations quarks are treated as free objects, despite the necessity of being treated as confined ones. For example, it is such treatment of quarks that led to the long-standing puzzle of the violation of Hara's theorem in the EM gauge-invariant quark model description of the weak radiative hyperon decays (see [15]). Now, in the standard field-theoretical description of baryons, the violation of Hara's theorem requires a violation of electromagnetic gauge invariance. The only known and consistent way of avoiding this unwanted and seemingly impossible result of the quark model is to evaluate these decays at the hadronic level, keeping the symmetries of the quark-level algebra of currents, but avoiding altogether the use of the standard concept of quark mass (see [16]).

The second hint of conspiracy comes from an analysis of the concept of additivity for composite systems [17]. Consider a system composed of ordinary particles such as leptons, hadrons, or classical particles. In the classical case, we know from our macroscopic experience that the momentum of the whole system is obtained by simply *adding* the momenta of its components. This prescription is carried over without any change to all standard quantum formalisms. In the phase-space approach, the role of some components of momenta is taken over by appropriate components of positions (see Eq. (18)). A more detailed analysis [17] shows that for the set of 'canonical momenta' $\boldsymbol{p}^{Q1}, \boldsymbol{p}^{Q2}, \boldsymbol{p}^{Q3}$ of the three colored quarks one may use the following representation

$$P^Q \equiv \begin{bmatrix} \boldsymbol{p}^{Q1} \\ \boldsymbol{p}^{Q2} \\ \boldsymbol{p}^{Q3} \end{bmatrix} = \begin{bmatrix} p_1^1 & -x_3^1 & +x_2^1 \\ +x_3^2 & p_2^2 & -x_1^2 \\ -x_2^3 & +x_1^3 & p_3^3 \end{bmatrix}, \tag{19}$$

where p_k^i and x_k^i denote physical momenta and positions of a quark of color i, while for antiquarks one has the form

$$P^{\overline{Q}} = \begin{bmatrix} p_1^1 & +x_3^1 & -x_2^1 \\ -x_3^2 & p_2^2 & +x_1^2 \\ +x_2^3 & -x_1^3 & p_3^3 \end{bmatrix}, \tag{20}$$

with reversed signs of all position coordinates. The structure of the *relative* signs in front of these coordinates is representation independent. A look at Eqs. (19), (20) shows that if one accepts the idea that the additivity of momenta for ordinary particles follows from a more general concept of the additivity of canonical momenta, then the additivity of quark canonical momenta leads to the appearance of translationally invariant expressions in quark–antiquark and three quark systems. Thus, the approach has the capacity of explaining the appearance of quark-confining string-like structures.

9. Summary

To sum up, let us recall three basic ingredients of our approach:

- the suggestion that the relativistic description is largely irrelevant at the quantum level, as the nonrelativistic character of all spatial quantum numbers and the existence of a tension between relativity and quantum physics seem to show,
- the idea of bringing more symmetry between the position and momentum coordinates, argued to be especially natural in quantum physics, which led us to the view of 'phase space' as the 6D macroscopic arena on which time evolution acts, and
- Dirac-like linearization of the relevant O(6) invariant after accepting the 'natural' noncommuting geometry in this 6D space.

The phase-space approach provides a possible theoretical explanation of the origin of the SM symmetry group and the structure of a single SM generation. In particular, it resolves all the main problems of the Harari–Shupe model.

Clearly, one should not have the impression that our approach reduces the whole physics to the 3D harmonic oscillator. Rather, it has been argued that any quantum approach which involves more symmetry between momenta and positions should contain the discussed idea as a substructure, just like the Dirac approach contains Pauli matrices, *i.e.* the nonrelativistic concept of spin, as a substructure. In short, just as the geometry of ordinary 3D space is related to spin and parity, so the geometry of 6D phase-space seems to be (additionally) related to the concepts of isospin and color.

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