Vol. 2 (2009) *Acta Physica Polonica B Proceedings Supplement* No 3

LONG LIVED OSCILLONS*

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(Received November 4, 2009)

Oscillons are well localized, almost periodic and surprisingly long living states in classical field theories. We present a short overview of their basic properties and dynamics in $1+1$ dimension. During collisions with kinks they behave as massive bodies which can reflect the kink or increase kink's kinetic energy. Oscillons can also undergo so-called negative radiation pressure.

PACS numbers: 11.10.Lm, 11.27.+d

1. Introduction

Recent years have brought much attention to long lived, quite localized and almost periodic solutions which can be encountered in various relativistic scalar field theories [1]. This objects reveal many properties of breathers from integrable models like sine-Gordon. However, in contrary to breathers, oscillons loose energy due to nonlinear radiation and finally vanish. Their lifetime is surprisingly long, compared to specific time scale in a given theory. Oscillons can be created from generic initial conditions in generic scalar field theories. Their existence is, therefore, of great importance in case of many relaxation problems as well as phase transitions and perhaps even quantum fluctuations.

Early papers were mostly focused on stability of oscillons in multidimensional ϕ^4 model. The lifetime of oscillons created from Gaussian initial conditions was measured. It highly depended on dimensionality of space-time. In three spatial dimensions the oscillons could live up to 10^4 oscillations, loosing its energy very slowly. During that time the frequency of these oscillation raised until it reached a certain critical value. Than a relatively fast decay of the oscillon was observed. In two spatial dimensions the oscillons

* Seminar presented at the XLIX Cracow School of Theoretical Physics, "Non-perturbative Gravity and Quantum Chromodynamics", Zakopane, May 31–June 10, 2009.

could live some 10^2 or even 10^4 times longer. A very interesting thing which was found was the fractal dependence of the lifetime of the oscillons and the width of initial Gaussian [2]. In the plots of a lifetime against the width of initial Gaussian, sharp picks could be observed, revealing that some initial data could give rise to oscillons with exceptional long lifetime. The structure of the pick revealed also the feature of selfsimilarity. $1+1$ dimensional oscillons lose their energy very slowly and no final fast decay was observed.

More recent papers focused on emergence of oscillons in different theories and from different initial data. For example oscillons were observed in abelian and non-abelian Higgs model. In all cases the presence of massive scalar field seems to essential. Oscillons could be created as a result of collision of topological defects (kink–antikink or vortex–antivortex) [3]. They can be also created form thermal fluctuations. It is also worth to emphasize that in case of thermal fluctuations a synchronization between created oscillons was observed.

In our paper we present some of the features of the oscillons widely discussed in literature and stress out some unexpected preliminary results of our own work. Our results in more detail will be publish in future papers. First we present a basic mathematical description of these objects, their structure and radiation created by oscillations. The following section is focused on collision between an oscillon and a kink in the ϕ^4 theory. Next we describe the possibility of so called negative radiation pressure in case of oscillons. The presented mechanism responsible for this phenomenon could be generalized to other oscillating object such as a floating body.

2. Oscillons

As we stressed out in the introduction oscillons can be observed in various nonlinear scalar field theories. One of the simplest example, very often studied is the ϕ^4 model described by following Lagrangian:

$$\mathcal{L} = \tfrac{1}{2}\,\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{2}\,\left(\phi^2-1\right)^2 . \tag{1}$$

In this model there are two *vacua* $\phi_v = \pm 1$. In $1+1$ dimensions (or more precisely when $\partial_y\phi = \partial_z\phi = 0$) this model reveals also the famous kink and anti-kink solutions

$$\phi_k = \pm\tanh x\,. \tag{2}$$

In papers [4, 5] the authors observed oscillating objects which were created from low velocity collisions of kink–antikink pair. These objects were later identified as oscillons. Therefore, in some sense oscillons can be interpreted as a bound state of topological defects. Numerical simulation of evolution of gassian initial conditions:

$$\phi(x,t=0) = 1 - 0.4\exp(-0.5\,x^2)\,, \qquad \dot\phi(x,t=0) = 0 \tag{3}$$

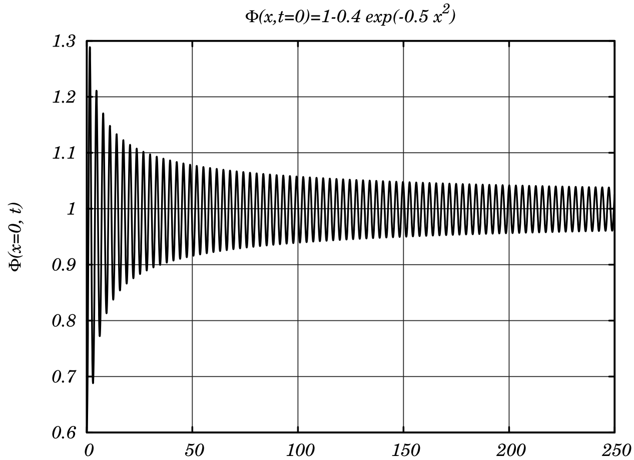

Fig. 1. Field value at the mass center for initial conditions $\phi(x,t=0) = 1 - 0.4 \times e^{-0.5\,x^2}$. Relatively fast decay is visible.

showed that the Gaussian is a large source of radiation and very quickly looses its amplitude and shape (Fig. 1). This is not a surprise because small perturbation from the vacuum $\phi = 1+u$ solution can be linearized obtaining Klein–Gordon equation

$$u_{tt} - u_{xx} - 4u = 0\,. \tag{4}$$

It is well known that each solution can be expressed as a superposition of traveling waves $u_k(x,t) = \cos(kx+\omega t)$, where $k = \pm\sqrt{\omega^2-4}$ is a wave number. Therefore, no localized stationary oscillating solutions are possible. The remaining oscillation at the center will tend from above to the threshold frequency $\omega_{\rm tr} = 2$. However, time evolution of a Gaussian with the same amplitude but different width:

$$\phi(x,t=0) = 1 - 0.4\exp(-0.1x^2)\,, \qquad \phi(x,t=0) = 0 \tag{5}$$

is very different. The oscillating object is now a very small source of radiation (Fig. 2), and after thousand oscillations the amplitude changes almost insignificantly. Moreover, the measured frequency of these oscillations is $\omega = 1.90$ which is significantly below the mass threshold $\omega_{\rm tr}$. This shows that nonlinearities are crucial in this example. For given height of the Gaussian there is one value of width for which the evolution leads to almost periodic, metastable state.

The later solution resembles many similarities to *breathers* known from the sine-Gordon theory described by equation:

$$\phi_{tt} - \phi_{xx} = \sin\phi\,. \tag{6}$$

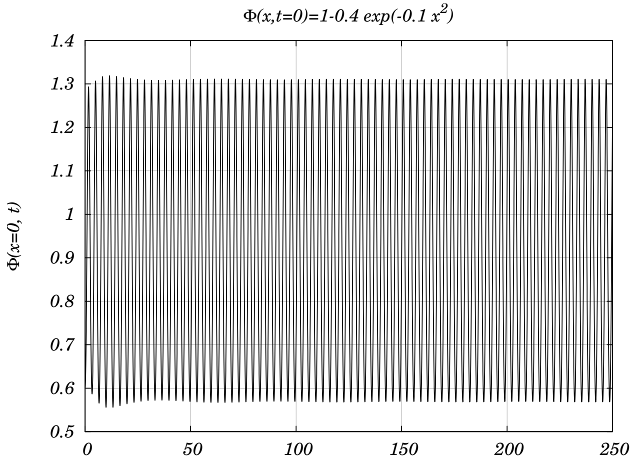

Fig. 2. Field value at the mass center for initial conditions $\phi(x,t=0) = 1 - 0.4 \times e^{-0.1\,x^2}$. Decay is almost invisible.

Sine–Gordon is a very unusual equation and the solution describing breather can be found in an exact form:

$$\phi_c(x,t) = -4\arctan\left[\frac{c}{\sqrt{1-c^2}}\frac{\sin\left(\sqrt{1-c^2}t\right)}{\cosh cx}\right]. \tag{7}$$

In ϕ^4 finding such an expression is not possible. One of the methods to find the solution of an oscillon is to make a Fourier decomposition of the solution:

$$\phi(x,t) = 1 + \sum_n u_n(x)\cos n\Omega t\,. \tag{8}$$

After plugging this ansatz into the ϕ^4 equation one obtains a system of ordinary differential equations:

$$\Delta u_n + (n^2\Omega^2 - 4)u_n = F_n(u_0, u_1, \ldots)\,, \tag{9}$$

where

$$F_n(u_0,u_1,\ldots) = 3\sum_{m,p}(\delta_{n,m+p} + \delta_{n,m-p})u_m u_p$$
$$+\tfrac{1}{2}\sum_{m,p,k}(\delta_{n,m+p+k} + \delta_{n,m-p+k} + \delta_{n,m+p-k} + \delta_{n,m-p-k})u_m u_p u_k\,. \tag{10}$$

Oscillon is a symmetric solution. This sets N conditions. It also radiates very slowly, therefore, the next conditions would be to minimize the outgoing radiation. Only for integrable systems, such as sine-Gordon model, one

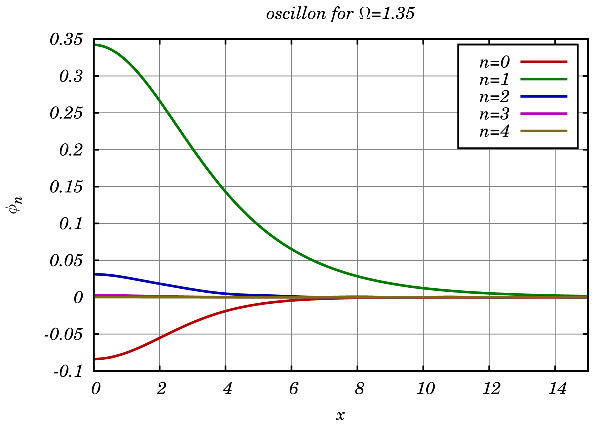

Fig. 3. First Fourier constituents of an oscillon. Note that the radiation tail is invisible in this scale.

can completely get rid off the radiation (or more precisely the Fourier constituents vanish exponentially). In ϕ^4 we can only minimize this radiation. The obtained solution is periodic in time but because of this radiation tail it has infinite energy. Moreover, the series is asymptotic, therefore, only the first few elements can be taken into account. The profiles were also found analytically using perturbation method with parameter $\epsilon = 2 - \Omega$. Some mathematical tricks can also give a decent evaluation of the outgoing radiation amplitude. Kruskal and Segur [6] showed (and later Forgacs *et al.* [7,8] proved more rigorously), that in $1+1d$ ϕ^4 the energy of the oscillon changes with time as

$$E(t) \sim (\ln t)^{-1} \tag{11}$$

for large values of t.

3. Oscillon interactions

Oscillons live for relatively long time in comparison to characteristic time in a given theory $\sim 1/m$.

If some phenomenon lasts much shorter than the lifetime of the oscillon we do not need to consider its radiation and asymptotic stability. These phenomena include collisions, phase transitions and interaction with waves. In sine-Gordon theory breathers interact with other objects elastically due to the integrability of the theory. Oscillons do not interact elastically.

3.1. Collisions with kinks

We have studied collisions between a stationary oscillon which was hit by a kink. Similar problem was previously studied by Hindmarsh in [9] when an oscillon hit a domain wall in $2+1$ dimensions. Our numerical simulations showed that in 1+1 dimensions the situation differs significantly from the one described in [9]. The difference comes from the fact that in $1+1$ dimensions both a kink and an oscillon are particle-like quite well localized objects and their energies can be of the same order. In $2+1$ dimensions oscillon is a cylindrically symmetric particle-like object while domain walls are more string-like objects which carry nonvanishing energy density along some line in a plane. Domain wall has, therefore, more energy than oscillon.

Hindmarsh showed that during the collisions some part of the oscillon can go through the domain wall but some part of the oscillon is reflected. This reflected part is also an oscillon but with much smaller amplitude. Due to the difference in mass of these objects the change in motion of the domain wall was negligible.

We studied a similar problem in but $1+1$ dimensions. A moving kink collided with a stationary oscillon. Initial conditions were

$$\phi(x,t=0) = A\exp(-ax^2) + \tanh\left(\gamma(x-x_0-vt)\right) \tag{12a}$$

and

$$\phi_t(x,t=0) = -\frac{v\gamma}{\cosh^2\left(\gamma(x-x_0-vt)\right)}\,. \tag{12b}$$

For small velocity collisions ($v < 0.18$) the kink was reflected from the oscillon (Fig. 4). During the collision some radiation was emitted. In ϕ^4 the kink has one linearized so-called oscillational (or shape) mode, which was also excited during the collision. This result is very different from the similar process in sine-Gordon equation (collision of a kink and a breather). Due to the integrability of sine-Gordon equation only perfectly elastic collision are possible. There cannot be any radiation emitted during such process.

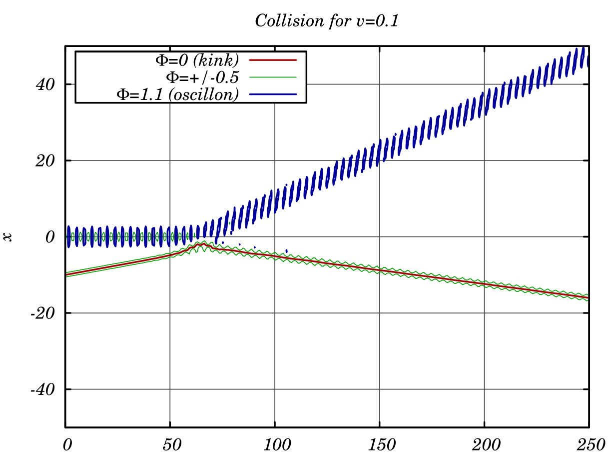

Fig. 4. Low velocity collision. Kink is reflected from the oscillon.

Moreover, since the breather can be interpreted as a bound state of kink and antikink the final state looks as if the colliding kink went through the breather without almost any interaction. After the collision the breather was slightly moved but remained motionless and the kink is moving with the same velocity.

Higher velocity collisions in ϕ^4 model gave also some interesting results (Fig. 5). For certain range of velocities, the kink after the collision was moving faster than before the collision (Fig. 6). For ultra-fast collisions the kink could move faster or slower than before the collision depending on the initial phase of the oscillon. In Fig. 5 a shadowed region indicates all possible values of final velocity of the kink. For velocities between 0.18 and 0.40 kink always gain kinetic energy. These results mean that during the collisions oscillon can transform some of its oscillating energy into kinetic energy. So, in a sense, they could behave similarly to a rotating body.

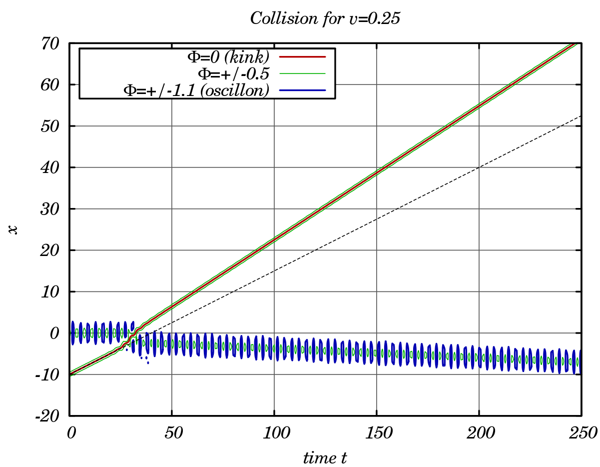

Fig. 5. Collision for higher velocity — after collision the kink gains kinetic energy.

This could be qualitatively described by effective theory using only few degrees of freedom: a position of the kink $X(t)$ and a position of the oscillon $Y(t)$. For small amplitude oscillon, we can assume that only the basic frequency dominates so we can neglect the rest of the harmonics. Our last degree of freedom would be the amplitude of the oscillon $A_1(t)$. We assume that the profile of the oscillon does not change with time. This obviously is not true since amplitude and width of the oscillon are dependent, but we want to give only a qualitative description.

We can write the field as

$$\phi(x,t) = \psi\,(x-X(t))+A_1(t)\Phi\,(x-Y(t)) + \text{higher harmonics} + \text{radiation}\,. \tag{13}$$

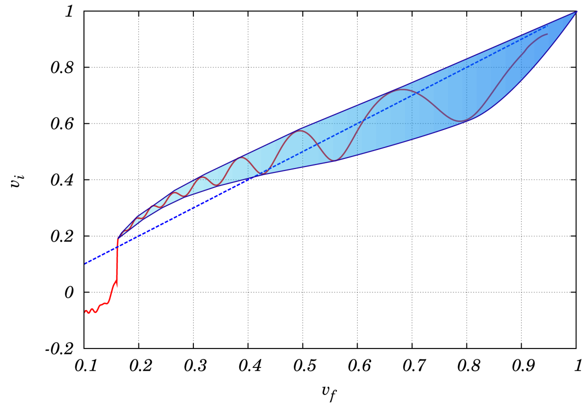

Fig. 6. Kink velocity after collision v_f *versus* kink initial velocity v_i.

Substitution to the Lagrangian gives

$$\mathcal{L} = \mathcal{L}_k + \mathcal{L}_{\text{osc}} + \mathcal{L}_{\text{int}} \,,$$

where

$$\mathcal{L}_{\text{osc}} = \frac{m}{2}\left(\dot{A}^2 - \left(4 + \frac{M}{m}\right)A^2\right) - \gamma_3 A^3 - \gamma_4 A^4 + \tfrac{1}{2}\,A^2 M \dot{Y}^2 \,,$$

For slow kink:

$$\mathcal{L}_k = -\tfrac{1}{2}\,M_k \dot{X}^2$$

and the interaction part:

$$\mathcal{L}_{\text{int}} = \int dx\; A\dot{Y}\dot{X}\Phi'\Psi' - \dot{A}\dot{X}\Phi\Psi' - 3A^2\Phi^2\Psi^2 - 2A^3\Phi^3\Psi - 2A\Phi\Psi^3 + 2A\Phi\Psi \,. \quad (14)$$

After integrations we obtain a system of equations which gives similar results as the full PDE such as the gain of velocity, the maximal reflection velocity and phase dependence.

3.2. Negative radiation pressure

We have also studied an interaction between an oscillon and a planar wave in ϕ^4 model. Oscillon can serve as a simple example of oscillating object interacting with radiation like *Q-Ball* or a floating body. We found out that in special case the oscillon can undergo a negative radiation pressure that is instead of being pushed by radiation it is being pulled towards the source of the radiation. In Fig. 7 we have plotted a measured (during numerical simulations) acceleration of an oscillon (with basic frequency being

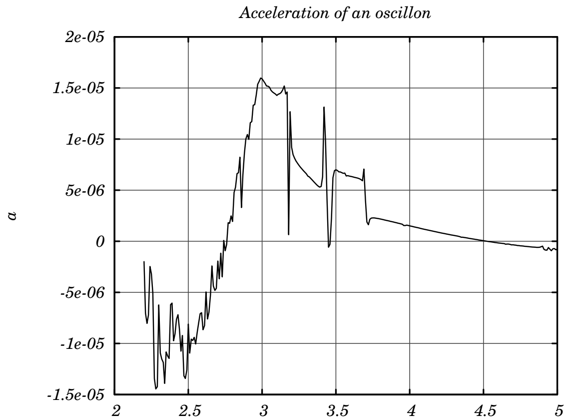

Fig. 7. Measured acceleration of an oscillon in a presence of travelling wave.

$\Omega = 1.7$) under influence of a wave travelling from $+\infty$ with the same amplitude $A = 0.1$. Negative values of the acceleration indicates that the oscillon was moving in the same direction as the wave. In other words the oscillon was pushed by the radiation. Positive values indicates that the oscillon was accelerating in posite direction — it was pulled by the radiation towards the source of radiation. This is an example of the negative radiation pressure. From the figure one can clearly see that the positive radiation pressure was observed for small frequencies close to the mass threshold ($\omega_{\mathrm{tr}} = 2$) and above some $\omega = 4.5$. One can also notice many resonances, especially in lower part of the spectrum. One of the largest resonances is seen around $\omega = 3.4$ which can be identified with 2Ω. This phenomenon of the negative radiation pressure was also described and explained for ϕ^4 kink [10]. Also it was numerically seen in case of relativistic vortices in Goldstone's and abelian Higgs models. Until now two mechanism of negative radiation pressure were presented: nonlinear selfinteracting field (ϕ^4) and in case of two (or more) interacting fields with different masses (vortices, toy model) [11]. In those mechanisms an incoming wave, due to the interaction with an object, transforms part of its energy into modes carrying more momentum. In ϕ^4 kink, in the first order of wave amplitude, is transparent and higher nonlinear corrections must be taken into account to find the force, which radiation exerts on the kink. A wave with twice the frequency of incident wave is created due to the nonlinear term ϕ^2 in the equation. Knowing the dispersion relation one can calculate that the force pushes the kink towards the source of radiation.

A different mechanism stands behind the negative radiation pressure in case of vortices. Lagrangian in Goldstone's model can be written as

$$\mathcal{L} = \tfrac{1}{2}\,\partial_\mu\phi^*\partial^\mu\phi - \tfrac{1}{2}\,(\phi^*\phi - 1)^2\,. \tag{15}$$

The potential has a form of so-called *Mexican Hat*. The vacuum manifold is a circle $|\phi| = 1$. Changing the field along this vacuum manifold costs no potential energy and small excitation of such type is a massless Goldstone's mode. Excitation in perpendicular direction costs potential energy and can be called massive amplitude's mode. An interaction of these two types of fields can result in surplus of momentum behind a vortex. This happens in case when a vertex is hit with an amplitude wave and one can observe a negative radiation pressure. When the vortex is hit with a Goldstone's wave it experiences positive radiation pressure. This mechanism was much more clearly presented on a toy model [11]. We believe that this case can be the most frequent example of NRP it should be observed in real physical situations.

Oscillons provide a third mechanism which is responsible for the negative radiation pressure.

This phenomenon can be understood in the following way:
A wave ξ with certain frequency ω hits an oscillon which oscillates with frequency $0, \Omega, 2\Omega, \ldots$. Due to the nonlinear interaction (ϕ^2, ϕ^3) modes with frequencies $\omega_{nm} = n\omega + m\Omega, n, m \in \mathbb{Z}$ appear.
Suppose that Ω/ω is not a rational number (to avoid resonances). For small amplitude of the wave we can linearize the equation obtaining

$$\xi_{tt} - \xi_{xx} + [V_0(x) + V_1(x)\cos\Omega t + V_2(x)\cos 2\Omega t + \cdots]\,\xi = 0\,, \qquad (16)$$

where the $V_i(x)$ are functions of u_m. The most dominating part is V_1 so neglecting the rest we obtain

$$\xi_{tt} - \xi_{xx} + V_1(x)\cos\Omega t\,\xi = 0\,.$$

The solution can be sought in the following form:

$$\xi = \sum_m \xi_m(x)\, e^{i(\omega + m\Omega)t}$$

which leads to the following set of equations:

$$\left[-\frac{d^2}{dx^2} - (\omega + m\Omega)^2\right]\xi_m + \tfrac{1}{2}\, V_1(x)\,(\delta_{n,m+1} + \delta_{n,m-1})\,\xi_n = 0\,, \qquad (17)$$

which can be solved with two-point boundary conditions (for some large $|x|$).

We want only one wave going towards the oscillon (for frequency ω), and the rest should be going outwards. What we could expect is that the most dominating waves will have frequencies $\omega \pm \Omega$, but if $\omega - \Omega$ would be below the mass threshold the wave with such frequency cannot propagate. For such frequencies we set vanishing boundary conditions.

Generally, it is easier to create a wave propagating in the same direction as the initial wave.

If two waves carry the same amount of energy, the one with higher frequency carries more momentum. Therefore, the wave with frequency $\Omega + \omega$ would carry more momentum than the wave with frequency ω. This leads to creation of momentum surplus behind the oscillon and a force pushing the oscillon towards the source of radiation — an example of the negative radiation pressure

However, if $\omega - \Omega$ is larger than the mass threshold than the dominating frequency would be $\omega - \Omega$ which is smaller than ω and the waves created with this frequency would have less momentum than the initial waves. The lack of the momentum must be compensated with increase of oscillon momentum, this time in the direction agreeable with the initial wave.

Numerical simulations of the full PDE as well as the solution to the system of equations 17 are in agreement with above considerations. The above procedure neglected resonances which are especially important for low frequencies and can even change the sign of the acceleration.

4. Conclusion

In the present paper we have presented some preliminary results concerning dynamics of oscillons. We have shown that during collisions with kinks, oscillons behave like massive bodies. Contrary to breathers, oscillons do not collide elasticly. Sometimes a kink can be reflected back from the oscillon. For higher velocity collisions a kink can take some of the oscillating energy from the oscillon.

Oscillons also can undergo the negative radiation pressure. A presence of basic frequency of the oscillon is crucial. During interaction with monochromatic wave a whole ladder of frequencies is created. The largest amplitudes have waves with frequency being a sum and a difference of the basic frequency of the oscillon and on the initial wave. If the difference is below the mass threshold that wave cannot propagate and the only dominant wave is the one with frequency being the sum.This wave could carry more momentum the the initial wave brought and the motion of the oscillon must compensate this. This leads to the appearance of the negative radiation pressure. For larger frequencies also the second wave could propagate and change the sign of the force exerted on the oscillon.

This mechanism of negative radiation pressure could be possible in case of other oscillating objects in theories with mass gap.

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