

CONSTRAINTS ON MINIMAL FLAVOUR VIOLATION*

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We present an updated phenomenological analysis of the minimal flavour violating (MFV) effective theory. We evaluate the bounds on the scale of new physics derived mainly from recent measurements of B meson observables and we use such bounds to derive a series of model-independent predictions within MFV for future experimental searches.

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1. Introduction

The Standard Model (SM) accurately describes high energy physical phenomena up to the electro-weak (EW) scale $\mu_W \sim 100$ GeV. It is however known to be incomplete due to the lack of description of gravity, proper unification of forces as well as neutrino masses. In view of these shortcomings, it can be regarded as a low-energy effective description of physics below a UV cut-off scale Λ . But if it is an effective theory, at what scale Λ below the unification or the Planck scale does it break down? The EW hierarchy problem suggests that new physics (NP) should appear around or below $\Lambda \lesssim 1$ TeV, whereas excellent agreement between SM predictions and experiment on ε_K , $A_{CP}(B_d \rightarrow \Psi K_s)$ and Δm_d and $B \rightarrow X_s \gamma$ constrains a general flavour violating NP to appear above $\Lambda \gtrsim 2 \times 10^5$ TeV, 2×10^3 TeV and 40 TeV respectively. The resulting tension between the two estimates of the NP scale illustrates what is often called the new physics flavour problem.

The Minimal Flavour Violation (MFV) hypothesis [1, 2] aims to solve the issue by demanding that all flavour symmetry breaking in and also beyond the SM is proportional to the SM Yukawas, namely the Cabibbo–Kobayashi–Maskawa (CKM) matrix is the only source of flavour mixing and CP violation even beyond the SM.

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MFV establishes solid links among different flavour observables at low energy and allow to probe and constrain the scale of MFV NP Λ . The $\Delta F = 2$ processes for example put [3] as lower bound $\Lambda > 5.1$ TeV at 95% probability. $\Delta F = 1$ processes as well independently constraint the NP scale. In the following we present an update analysis on the $\Delta F = 1$ sector from Ref. [4].

2. Updating analysis of $\Delta F = 1$ constraints

In the SM the effective weak Hamiltonian describing $\Delta F = 1$ FCNC processes among down-type quark flavours $q_i - q_j$ can be written as [2]

$$\mathcal{H}_{\text{eff}}^{\Delta F=1} = \frac{G_F \alpha_{\text{em}}}{2\sqrt{2}\pi \sin^2 \theta_W} V_{ti}^* V_{tj} \sum_n C_n \mathcal{Q}_n + \text{h.c.}, \quad (1)$$

where G_F is the Fermi constant, α_{em} is the fine structure constant, θ_W is the Weinberg angle and V_{ij} are the CKM matrix elements. The short distance SM contributions are encoded in the Wilson coefficients C_n , computed via perturbative matching procedure at the EW scale. MFV NP manifests itself in the shifts of the individual Wilson coefficients in respect to the SM values $C_n(\mu_W) = C_n^{\text{SM}} + \delta C_n$. These shifts can be translated in terms of the tested NP energy scale Λ as $\delta C_n = 2a\Lambda_0^2/\Lambda^2$, where $\Lambda_0 = \lambda_t \sin^2(\theta_W)m_W/\alpha_{\text{em}} \sim 2.4$ TeV is the corresponding typical SM effective energy scale. The value of the free variable a depends on the details of a particular MFV NP model. In general $a \sim 1$ for tree level NP contributions, while $a \sim 1/16\pi^2$ for loop suppressed NP contributions. In our numerical results we put a to unity.

In order to address low energy phenomenology, one needs to evaluate the appropriate matrix elements of the corresponding effective dimension 6 operators $\mathcal{Q}_n$. At low and at large $\tan \beta$ we consider the following operators

$$\mathcal{Q}_{7\gamma} = \frac{2}{g^2} m_j \bar{d}_{iL} \sigma_{\mu\nu} d_{jR} (e F_{\mu\nu}), \quad \mathcal{Q}_{8G} = \frac{2}{g^2} m_j \bar{d}_{iL} \sigma_{\mu\nu} T^a d_{jR} (g_s G_{\mu\nu}^a), \quad (2)$$

$$\mathcal{Q}_{9V} = 2\bar{d}_{iL} \gamma_\mu d_{jL} \bar{\ell} \gamma_\mu \ell, \quad \mathcal{Q}_{10A} = 2\bar{d}_{iL} \gamma_\mu d_{jL} \bar{\ell} \gamma_\mu \gamma_5 \ell, \quad (3)$$

$$\mathcal{Q}_{\nu\bar{\nu}} = 4\bar{d}_{iL} \gamma_\mu d_{jL} \bar{\nu}_L \gamma_\mu \nu_L, \quad \mathcal{Q}_{S-P} = 4(\bar{d}_{iL} d_{jR})(\bar{\ell}_R \ell_L). \quad (4)$$

In our analysis we consider the most theoretically clean observables in order to derive reliable bounds on possible NP contributions. In particular, we use the inclusive branching ratio of the radiative $B \rightarrow X_s \gamma$ decay, measured with a lower cut on the photon energy. The latest HFAG value averaged over different measurements [8] is $\text{Br}(B \rightarrow X_s \gamma)_{E_\gamma > 1.6 \text{ GeV}}^{\text{exp}} = 3.52(23)(9) \times 10^{-4}$, where the first error is statistical and the second systematic. Theoretically, the SM value is known to better than 8% and the expansion in terms of δC_n evaluated at the weak scale is [6]

$$\text{Br}(B \rightarrow X_s \gamma)_{E_\gamma > 1.6 \text{ GeV}}^{\text{th}} = 3.16(23) \times 10^{-4} (1 - 2.28\delta C_{7\gamma} - 0.71\delta C_{8G} + 1.51\delta C_{7\gamma}^2 + 0.78\delta C_{8G}\delta C_{7\gamma} + 0.25\delta C_{8G}^2), \quad (5)$$

where the central value and its error have been adjusted to take into account the CKM matrix element determination from the UUT analysis [3].

A completely different combination of operators contributes to the helicity suppressed decay $B_s \rightarrow \mu^+ \mu^-$. Experimentally the best upper bound on the branching ratio was recently put by the CDF Collaboration [7] $\text{Br}(B_s \rightarrow \mu^+ \mu^-)^{\text{exp}} < 4.7 \times 10^{-8}$ at 90% C.L., which is only one order of magnitude above the SM prediction. The theoretical error of this prediction is around 23% and is dominated by the lattice QCD determination of the B_s decay constant. Again using UUT CKM inputs, the expansion in terms of δC_i reads

$$\text{Br}(B_s \rightarrow \mu^+ \mu^-)^{\text{th}} = 3.8(9) \times 10^{-9} (1 - 2.1\delta C_{10A} - 2.3\delta C_{S-P} + 1.1\delta C_{10A}^2 + 2.4\delta C_{S-P}\delta C_{10A} + 2.7\delta C_{S-P}^2). \quad (6)$$

The branching ratio $\text{Br}(B \rightarrow X_s \ell^+ \ell^-)$ is measured by the B factories [9] in several bins of di-lepton invariant mass squared (q^2). The errors vary from almost 90% in the first bin where only Belle has obtained a relevant signal, to around 30% in the other bins.

The latest calculations estimate the theoretical error at around 7% for the bins below the charmonium region and around 10% for the high q^2 bin [10]. The relevant formulae can be found in Refs. [4, 10].

Much more experimental information is available for exclusive channels where the $B \rightarrow K^{(*)} \ell^+ \ell^-$ branching ratios as well as several angular distributions have already been measured [11]. Theoretically however, despite considerable theoretical progress on exclusive channels in the recent years [12], a reliable determination can only be expected from fundamentally non-perturbative methods, such as lattice QCD. In the meantime, any phenomenological implications based on existing form factor estimates should be treated with care. We therefore consider separately the impact of the measured forward-backward asymmetry in $B \rightarrow K^* \ell^+ \ell^-$ in the low and high q^2 bins on the derived NP bounds and predictions.

Finally MFV NP contributions to the Z -penguin operators can be constrained using the first experimental hints [13] of the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}(\gamma))^{\text{exp}} = 147(120) \times 10^{-12}$ and comparing them to the theoretical predictions, which are brought under control by the use of experimental data on $K\ell 3$ decays [14] resulting in only 11% theoretical error. In presence of MFV NP the corresponding expression reads

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}(\gamma))^{\text{th}} = 7.53(82)(1 + 0.93\delta C_{\nu\bar{\nu}} + 0.22\delta C_{\nu\bar{\nu}}^2) \times 10^{-11}. \quad (7)$$

3. $\Delta F = 1$ fit results

We perform a correlated fit of subsets of observables turning on NP contributions and extract probability bounds on the shifts of the Wilson coefficients away from their SM values. The compilation of bounds on the MFV NP scale in respect to all the probed operators is summarised in a Table I. In the above conservative estimates we take into account all the possible fine-tunings and cancellations among the various operator contributions, including discrete ambiguities in cases where the NP contributions might flip the sign of the SM pieces.

TABLE I

Summary of the lower bounds at 95% probability on the MFV NP scale.

Operator	Λ bound [TeV]
$\mathcal{Q}_{7\gamma}$	1.6(5.2)
$\mathcal{Q}_{8G}$	1.2(3.1)
$\mathcal{Q}_{9V}$	1.4
$\mathcal{Q}_{10A}$	1.5
$\mathcal{Q}_{S-P}$	1.2
$\mathcal{Q}_{\nu\bar{\nu}}$	1.5

The strongest bounds come naturally from the $B \rightarrow X_s \gamma$ decay rate and affect $\mathcal{Q}_{7\gamma, 8G}$. However, the effect of the discrete ambiguity of the $C_{7\gamma}$ sign flip is large and only when discarding this solution, the resulting bounds on $\Lambda > 5.2(3.1)$ for $\mathcal{Q}_{7\gamma, 8G}$ are competitive with the ones on $\Delta F = 2$ operators [3]. On the other hand $\delta C_{9V, 10A}$ are mainly bounded by $B \rightarrow X_s \ell^+ \ell^-$ and using only presently available inclusive information the highly correlated bounds are around 1.5 TeV. The large ambiguities [4] could be removed in the future once the experimental information concerning the lowest q^2 region in $B \rightarrow X_s \ell^+ \ell^-$ rate and especially the forward-backward asymmetry (FBA) improves. At the moment, only the asymmetry in the exclusive $B \rightarrow K \ell^+ \ell^-$ has been measured in two q^2 bins. Its impact on the $C_{9V} - C_{10A}$ and $C_{10A} - C_{S-P}$ correlations is shown in Fig. 1.

Another presently available observable is the longitudinal polarisation of the K^* . It is, however, not very sensitive to MFV NP contributions and therefore does not improve the constraints. Finally, as expected, $\mathcal{Q}_{S-P}$ and $\mathcal{Q}_{\nu\bar{\nu}}$ operators are mainly bounded from single observables ($B_s \rightarrow \mu^+ \mu^-$ and $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ respectively) leading to robust bounds around 1.2 TeV and 1.5 TeV respectively. The bound on C_{S-P} is particularly important, as it already rules out significant contributions of the scalar operator to $B \rightarrow X_s \ell^+ \ell^-$ related observables.

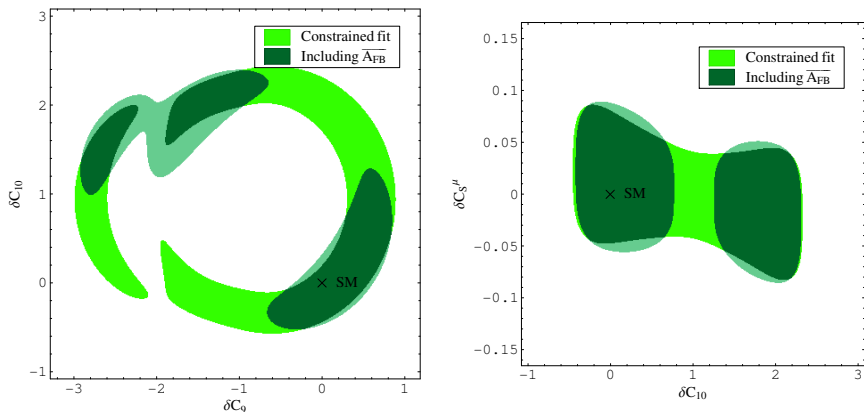


Fig. 1. Interesting correlation plots for pairs of shifts in the Wilson coefficients. Allowed regions at 68% C.L. obtained with (dark shaded) and without (light shaded) exclusive A_{FB} observables.

4. Discussion and outlook

In summary, immense experimental and theoretical progress in the area of flavour physics in the last decade has made it possible to constrain in a model independent way the complete set of possible beyond SM contributions to $\Delta F = 1$ and $\Delta F = 2$ processes due to possible MFV NP both at small and large $\tan\beta$. Bounds coming from $\Delta F = 2$ phenomenology are already very constraining, pushing the effective MFV NP scale beyond 5 TeV. In $\Delta F = 1$ sector, at present only the bounds coming from $B \rightarrow X_s \gamma$ are of comparable strength. However most uncertainties are dominated by experiments and one can look forward for the results of full dataset analyses by the B factories.

Using the derived bounds on the MFV NP contributions in $\Delta F = 1$ processes we are able to make predictions for other potentially interesting observables to be probed at LHCb or a future Super Flavour Factory. As already mentioned, angular distributions like the FBA probe different combinations of the operators and would provide complimentary bounds. At the moment, considering bounds from inclusive measurements alone, no firm constraints on the FBA or its zero can be imposed within MFV models. This conclusion reinforces the importance of these observables and their potentiality of discovering relevant deviations.

Another set of observables displays interesting sensitivity to the $\tan\beta$ enhanced C_{S-P} contributions. For example lepton flavour universality ratios $\Gamma(B \rightarrow K^{(*)} \mu^+ \mu^-) / \Gamma(B \rightarrow K^{(*)} e^+ e^-)$ are very close to 1 with the SM as well as MFV models with low $\tan\beta$. However, even at large $\tan\beta$ the present constraints do not allow deviations from 1 larger than 10% for these ratios.

Finally, the derived bounds allow to construct tests, which are potentially able to rule out MFV. Besides the interesting CP violation signals already emerging in the B_s sector [15], in $\Delta F = 1$ sector the relation $\text{Br}(B_s \rightarrow \ell^+ \ell^-) / \text{Br}(B_d \rightarrow \ell^+ \ell^-) \simeq f_{B_s} m_{B_s} / (m_{B_d} f_{B_d}) |V_{ts}/V_{td}|^2$ leads to one of the most stringent tests of the MFV scenario, both at small and large $\tan \beta$ values. Also interesting in this respect is the FBA in $B \rightarrow K \ell^+ \ell^-$ decay, which is already restricted to be below 1% within MFV models regardless of $\tan \beta$.

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