

DISCRETE MINIMAL FLAVOUR VIOLATION*

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We investigate the consequences of replacing the continuous flavour symmetry of minimal flavour violation by a discrete group. Goldstone bosons, resulting from the breaking of the continuous flavour symmetry, generically lead to bounds on new flavour structure many orders of magnitude above the TeV-scale. The absence of the latter for discrete symmetries constitute the *primary* motivation of our work. The four crystal-like groups $\Sigma(168)$, $\Sigma(72\varphi)$, $\Sigma(216\varphi)$ and $\Sigma(360\varphi)$ provide enough protection for a discrete TeV-scale MFV scenario in the case where $\Delta F = 2$ processes are generated by two subsequent $\Delta F = 1$ transitions.

PACS numbers: 11.30.Hv

1. Introduction

In the absence of Yukawa interactions the global flavour symmetry of the standard model (SM) is $G_F = U(3)^5 = U(3)_Q \times U(3)_{U_R} \times U(3)_{D_R} \times U(3)_L \times U(3)_{E_R}$. It was realized a long time ago [2] that these sort of flavour symmetries forbid flavour changing neutral currents (FCNC) at tree level. The idea behind Minimal Flavour Violation (MFV) [3] is that the Yukawa matrices are the sole sources,

$$G_F = U(3)^5 \xrightarrow{Y_{U,D,E}} U(1)_B \times U(1)_L, \quad (1)$$

that break this flavour symmetry. *Nota bene* the further breaking of this group down to $U(1)_{B-L}$ due to the chiral anomaly [7] is not central to this work. For notational simplicity we shall focus in this work on the quark sector. Results can easily be transferred to the lepton sector.

* Presented at the FLAVIANet Topical Workshop “Low energy constraints on extensions of the Standard Model”, Kazimierz, Poland, July 23–27, 2009.

It is observed that the flavour symmetry is restored when the following transformation properties, $Y_U \sim (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1})_{G_q}$, $Y_D \sim (\mathbf{3}, \mathbf{1}, \bar{\mathbf{3}})_{G_q}$, are assigned to the Yukawa matrices, where $G_q = \text{SU}(3)_Q \times \text{SU}(3)_{U_R} \times \text{SU}(3)_{D_R}$ is the quark flavour symmetry group¹. An *effective theory* constructed from the SM fields and the Yukawa matrices is then said to obey the principle of *Minimal Flavour Violation* [3], if all operators are invariant under G_q ². The MFV approach is in part motivated by the, so far, pertinent absence of FCNC. Relative bounds from K_0 , B_d -oscillations on the Wilson coefficients $C_{\text{SM}}/C_{\text{MFV}} \simeq (0.5\text{TeV})^2/m_W^2$. The reader is referred to the talks on MFV [4] and MFV-bounds [5] for further reference.

Even in the absence of the knowledge of the exact dynamics one delicate question might be raised: How is the G_F symmetry broken? If the symmetry is broken spontaneously, which is what has been proposed so far *e.g.* [8], this would then give rise to $3 \times 8 + 2 = 26$ CP-odd massless Goldstone bosons, bearing in mind possible U(1) anomalous contributions, in the quark sector associated with the breaking of $\text{U}(3)^3 \rightarrow \text{U}(1)_B$. Those Goldstone bosons set bounds on new flavour structure many orders of magnitude above the TeV-scale [1].

In this paper we aim to ameliorate this situation by replacing G_F by a discrete symmetry³. Spontaneous breaking of discrete symmetries do *not* lead to Goldstone bosons. The absence of the latter in this framework constitutes the *primary* motivation of our work. The main remaining issue is then to investigate whether the reduced symmetry provides enough protection for a discrete TeV-scale MFV-scenario. On the technical side the analysis of the effective field theory boils down to the classification of invariants of discrete $\text{SU}(3)$ subgroups.

2. Discrete minimal flavour violation

Replacing the continuous flavour symmetry with a discrete flavour symmetry requires the following additional information or assumptions:

- (a) The group $D_q = \mathcal{D}_{Q_L} \times \mathcal{D}_{U_R} \times \mathcal{D}_{D_R} \subset G_q$, $\mathcal{D} \subset \text{SU}(3)$,
 - (b) The representation $R_3(\mathcal{D}_{Q_L})(3\text{D irrep of families})$,
 - (c) Yukawa expansion $Y_{U/D} \rightarrow \kappa Y_{U/D}$, $\kappa \in \mathbf{R}$,
- (2)

where \mathcal{D} denotes a discrete groups. The three families transform under a 3D irreducible representation (irrep) of \mathcal{D} , which has to be specified since

¹ In this work we take a bit of a cavalier attitude towards U(1) factors. Some remarks on U(1) factors can be found in [1].

² Sometimes additional assumptions such as CP invariance [3] or no new Lorentz structures [6] are made.

³ Another alternative is to resort to the Higgs mechanism.

some groups have more than one (modulo the complex conjugate). The Yukawa expansion⁴ allows higher dimensional operators to be controlled, which otherwise give rise to rather anarchic flavour transitions [1].

The effective Lagrangian, in the absence of the knowledge of the dynamics of the underlying model, is parametrized as

$$\begin{aligned}\mathcal{L}^{\text{eff}} &= \sum_n C_{\text{dMFV}}^m (\mathcal{I}_n(\text{Quarks, Yukawas}) + \text{h.c.}), \\ C_{\text{dMFV}}^m &= \frac{c_n}{\Lambda^{\dim(\mathcal{I}_n)-4}},\end{aligned}\tag{3}$$

the sum of all combinations invariant under \mathcal{D}_q (2). The dimension of the operator (invariant) brings in a certain hierarchy in the infinite sum above⁵. The crucial technical point is that *finding the invariants is equivalent to finding the constant tensors of the symmetry group*. Let us here introduce the following (standard) notation for tensors: An index transforming under a **3** or **$\bar{3}$** representation shall be denoted by lower and upper indices respectively

$$\mathcal{I}^{(m,n)} \sim \mathcal{I}_{a_1 \dots a_m}^{b_1 \dots b_n}.\tag{4}$$

Non-constant tensors will be denoted by $\mathcal{T}^{(m,n)}$. In principle this tensor classification is not sufficient for our general problem since there are three different group factors (2). It will though prove sufficient here to contract the other indices⁶. We therefore contract the \mathcal{D}_{UR} index and directly write

$$(\Delta_U)_a{}^r \equiv (Y_U Y_U^\dagger)_a{}^r, \quad (\Delta \in \mathcal{T}^{(1,1)}).\tag{5}$$

The subscript U shall be dropped when there is no reason for confusion. In the reminder we shall use the following notation:

$$D_L = (d_L, s_L, b_L) \rightarrow D_i = (D_1, D_2, D_3), \quad (D_i \in \mathcal{T}^{(1,0)}).\tag{6}$$

The following operator classification

$$\begin{aligned}\mathcal{I}_n^{(2,2)} &= (\mathcal{I}_n)_{rs}^{ab} (\bar{D}^r \Delta_a^s D_b) && \in O^{\Delta F=1'}, \\ \mathcal{I}_n^{(3,3)} &= (\mathcal{I}_n)_{rst}^{abc} (\bar{D}^r \Delta_a^s D_b) \bar{D}^t D_c && \in O^{\Delta F=1}, \\ \mathcal{I}_n^{(4,4)} &= (\mathcal{I}_n)_{rstu}^{abcd} (\bar{D}^r \Delta_a^s D_b) (\bar{D}^t \Delta_c^u D_d) && \in O^{\Delta F=2},\end{aligned}\tag{7}$$

⁴ The authors [9] distinguish $\kappa \ll 1$ linear MFV *versus* $\kappa \sim \mathcal{O}(1)$ non-linear MFV where resummation (non-linear σ -model techniques) become imperative.

⁵ It has to be kept in mind that the association of C^n with the scale of new physics is generally obscured by loop factors, mixing angles and renormalisation group effects as in any bottom-up effective field theory approach.

⁶ A refined treatment is only necessary when there are new $\mathcal{I}^{(2,2)}$ invariants and those groups are not of interest to us anyway.

directly connects to the MFV operators [3]

$$\begin{aligned} O^{\Delta F=1'} &= (\bar{D}_L Y_U Y_U^\dagger Y_D \sigma \cdot F D_R), \\ O^{\Delta F=1} &= (\bar{D}_L Y_U Y_U^\dagger \gamma_\mu D_L) \bar{D}_L \gamma^\mu D_L, \\ O^{\Delta F=2} &= (\bar{D}_L Y_U Y_U^\dagger \gamma_\mu D_L)^2. \end{aligned} \quad (8)$$

Before entering into the realm of invariants let us briefly discuss the discrete SU(3) subgroups.

2.1. Discrete SU(3) subgroups

The discrete SU(3) subgroups were classified a long time ago [11] and further analysed as alternatives to SU(3)_F in the context of the eightfold way [12]. Explicit representations and Clebsch–Gordan coefficients were systematically worked out in a series of papers around 1980 [10], partly motivated as alternatives to SU(3)_{colour} for lattice QCD; and further elaborated very recently [13] in the context of family symmetries.

The discrete SU(3) subgroups are of two kinds. The analogues of crystal groups, of which we list here the maximal subgroups: $\Sigma(168)$, $\Sigma(360\varphi)$ and $\Sigma(216\varphi)$. The factor φ can in general either be one or three depending on whether the center of SU(3) is divided out or not. For the maximal subgroups it is three. The second kind are the infinite sequence of groups, sometimes called “dihedral like” or “triheral”, $\Delta(3n^2) \approx (\mathbb{Z}_n \times \mathbb{Z}_n) \rtimes \mathbb{Z}_3$ and $\Delta(6n^2) \approx (\mathbb{Z}_n \times \mathbb{Z}_n) \rtimes S_3$ for $n \in \mathbb{Z}$. The symbol “ \rtimes ” denotes a semidirect product. The largest irreps of $\Delta(3n^2)$ and $\Delta(6n^2)$ are 3-, respectively 6-dimensional; independent of n .

That the catalogue of [12] as compared to [11] is not complete already surfaced in the 1980 [10, 14] and it has recently been reemphasised [15] that the so-called (D)-groups [11] have not been discussed systematically in the literature. In appendix A.2.1 [1] we argue that the (D)-groups, or more precisely the six-parameter $D(n, a, b; d, r, s)$ matrix groups, are subgroups of $\Delta(6g^2)$, where g is the lowest common multiple of n , d and 2. This is sufficient for our consideration.

2.2. Invariants

In discussing invariants we are going to use the fact, based on the orthogonality theorem, that the number of times the identity appears in a Kronecker product $\mathbf{A} \times \mathbf{B} \times \mathbf{C} \times \dots = n_1 \mathbf{1} + \dots$, denoted by n_1 , is equal to the number of invariants that can be formed out of the irreps $\{\mathbf{A}, \mathbf{B}, \mathbf{C}, \dots\}$ [1].

New invariants $\mathcal{I}^{(4,4)}$ -level – no 27: The problem of finding all invariants of the $\Delta F = 2$ operator (8) is equivalent to finding the invariants of the following Kronecker product: $K^{\mathcal{D}_{\mathcal{L}}} = (\mathbf{\bar{3}} \times \mathbf{3} \times \mathbf{\bar{3}} \times \mathbf{3})_{\mathbf{s}} \times (\mathbf{\bar{3}} \times \mathbf{3} \times \mathbf{\bar{3}} \times \mathbf{3})_{\mathbf{s}}$.

The symbol “s” stands for the symmetric part. The restriction to the symmetric part can be justified by first considering the tensor products of the Yukawas and the quarks separately. The logic that we shall employ is that if the Kronecker product of the $\mathcal{D} \subset \text{SU}(3)$ decomposes any different from $\text{SU}(3)$ then there are necessarily further invariants. The following Kronecker products will prove sufficient to convey our argument:

$$(\mathbf{3} \times \bar{\mathbf{3}})_{\text{SU}(3)} = \mathbf{1} + \mathbf{8}, \quad (9)$$

$$(\mathbf{8} \times \mathbf{8})_{\text{SU}(3)} = (\mathbf{1} + \mathbf{8} + \mathbf{27})_s + (\mathbf{8} + \mathbf{10} + \bar{\mathbf{10}})_a. \quad (10)$$

The two equations above make it evident that a necessary condition for an identical decomposition is that the discrete group contains a 27D irrep. The trihedral groups $\Delta(3n^2)$ and $\Delta(6n^2)$ are not in this category since their largest irreducible representations (irreps) are at most 3-, respectively 6-dimensional. Going through the character tables in [12] and the more recent work [15] we realize that there is *no* discrete subgroup of $\text{SU}(3)$ which has a 27D irrep! Note, on even more general grounds that $\dim(\mathbf{27})^2 = 729$ almost saturates the relation between the order of the group and the sum of the dimension of its irreps squared [1] and leaves $|\Sigma(360\varphi)| = 1080 > 729$ as the only hypothetical candidate among the crystal-like groups.

No new invariants $\mathcal{I}^{(2,2)}$ -level for four groups: The good news is though that the four crystal-like groups $\Sigma(168)$, $\Sigma(72\varphi)$, $\Sigma(216\varphi)$ and $\Sigma(360\varphi)$ do have representations that decompose as (9) and are going to be interesting under the technical assumption of “family irreducibility” to be discussed below. Here we shall give an overview of the number of complex conjugate 3D irreps and the number of invariants (under certain symmetrisations): Note that we implicitly stated that there are groups where $\Delta F = 2$ operators are possible with $\mathcal{I}^{(2,2)}$ solely; without any Yukawa transitions. The group $\Sigma(60) \approx A_5$ is an example which is discussed in Appendix A of reference [1].

It is argued, with concrete examples, that new invariants necessarily upset the flavour hierarchy of MFV [1]. In particular the breaking of the continuous flavour group down to a discrete group implies that the mass-flavour basis transformation become observable and deprive the approach of its predictivity. To conclude, that the necessity of new invariants at the $\mathcal{I}^{(4,4)}$ -level and its connection with the well-tested $\Delta F = 2$ transitions, imply that no discrete flavour group is suitable appears too hasty. The generation mechanism of $\Delta F = 2$ operators has to be reflected upon. We distinguish the two cases where the $\Delta F = 2$ process is generated via two subsequent $\Delta F = 1$ parts and where this is not the case. We shall call the former “family irreducible” and the latter “family reducible”, *cf.* Fig. 1. The SM and the R-parity conserving MSSM are examples of the “family reducible”-type whereas the R-parity violating MSSM is of the “family irreducible”-type.

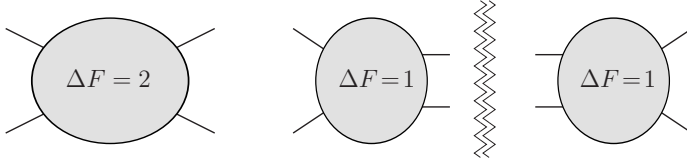


Fig. 1. Left: “family irreducible”, right: “family reducible”.

The composite technicolor model [16], in the absence of the knowledge of its non-perturbative dynamics, have to be counted into the latter class as well. “Family reducibility” essentially implies

$$(\Delta F = 2) \approx (\Delta F = 1) \times (\Delta F = 1) \Rightarrow \mathcal{I}^{(4,4)} \rightarrow \mathcal{I}^{(2,2)} \mathcal{I}^{(2,2)}, \quad (11)$$

factorisation of the invariant and proves to be a *sufficient* condition for a discrete TeV-scale scenario for the four crystal-like groups listed in Table I. To this end we would like to discuss two further points:

- $\Sigma(360\varphi)$ model-independent: The most suitable candidate, for a TeV-scale dMFV scenario is $\Sigma(360\varphi)$, since the first new invariants appear only at the $\mathcal{I}^{(4,4)}$ level *cf.* Table I. Yet: *How small does κ need to be in order for C_{dMFV} (3) to satisfy the same kind of experimental bounds as for C_{MFV} found in reference [3]?* The discussion in section 5.1 [1] suggests that $s \rightarrow d$ could be induced at first order in λ for new invariants, as compared to order $|V_{ts}V_{td}^*| \sim \lambda^5$ in MFV. A $\Delta S = 2$ transition could therefore be $\mathcal{O}(\lambda^6)$ as compared to $\mathcal{O}(\lambda^{10})$ in MFV. According to our reflection above we have to balance: $\text{MFV} : \text{DMFV} = \lambda^{10} : \lambda^6 \kappa^4 \Rightarrow \kappa_{\Sigma(360\varphi)} \simeq \lambda \simeq 0.2$.
- It has to be kept in mind that even when a model is in the “family irreducible”-class, the fact that the vertices are \mathcal{D}_q -invariant prevents the generation of non factorizable $\mathcal{I}^{(4,4)}$ -invariants. We argue that this is indeed the case for the R-parity violating MSSM (at least to leading order) [1].

Our aim, in this work, was to point out general issues of implementing MFV via a discrete group. We would hope that this work would be of some help for further investigations towards more specific models. It also has to be said that although MFV has very attractive features, *e.g.* a sufficiently stable proton in the R-parity violating MSSM [17], so far no explicit model for MFV has appeared in the literature. Moreover, the formulation (2) could be refined by constructing a discrete subgroup of G_q which does not factor into direct products of $\text{SU}(3)$ subgroups [1]. One might wonder what the consequences are, of such a non-trivial embedding, for the invariants.

TABLE I

Number of invariants for tensors of type $\mathcal{I}^{(n,n)}$, whose definition can be inferred from Eq. (7). The subscripts x, y indicate symmetrisations of x and y pairs of **3, 3** indices. $\mathcal{I}_{2,1}^{(3,3)}$ and $\mathcal{I}_{2,2}^{(4,4)}$ correspond to the (symmetric) contractions of $\Delta F = 1$ and $\Delta F = 2$ in (7).

Group	order	(3, 3)	$\mathcal{I}^{(2,2)}$	$\mathcal{I}^{(3,3)}$	$\mathcal{I}_{2,1}^{(3,3)}$	$\mathcal{I}^{(4,4)}$	$\mathcal{I}_{3,1}^{(4,4)}$	$\mathcal{I}_{2,2}^{(4,4)}$
SU(3)	∞	1	2	6	5	23	15	14
$\Sigma(360\varphi)$	1080	2	2	6	5	28	18	17
$\Sigma(216\varphi)$	648	3	2	7	6	40	27	23
$\Sigma(168)$	168	1	2	7	6	44	29	25
$\Sigma(72\varphi)$	216	4	2	11	8	92	55	43

R.Z. would like to thank many colleagues for inspiring discussions and remarks and at last R.Z. would like to thank the organizers of the Kazimierz workshop for the organisation and a very pleasant atmosphere. R.Z. acknowledges the support of an advanced STFC fellowship.

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