

# RANDALL–SUNDRUM MODELS AND PRECISION OBSERVABLES\*

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I present a review of phenomenological implications of the Randall–Sundrum (RS) model with bulk fermions and brane-localised Higgs boson. Modifications to the  $W$ -boson mass, corrections to the Peskin–Takeuchi parameters and to the  $Zb\bar{b}$  couplings will be discussed. From these observables severe bounds on the mass scale of Kaluza–Klein (KK) modes arise. Constraints from all three observables are very sensitive to the exact value of the Higgs boson mass and the bounds can be significantly lowered by allowing for a heavy Higgs boson ( $m_h \sim 1$  TeV). Consequences thereof, as well as other approaches like “little RS” models and models with custodial symmetry will also be briefly discussed.

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## 1. Introduction

Extra dimensional models with a warped background were proposed ten years ago by Randall and Sundrum (RS) [1] in order to solve the gauge hierarchy problem. In these models the fifth dimension is an  $S_1/Z_2$  orbifold, which is warped due to a non-factorizable metric

$$ds^2 = e^{-2\sigma(\phi)} \eta_{\mu\nu} dx^\mu dx^\nu - r^2 d\phi^2, \quad \sigma(\phi) = kr|\phi|, \quad (1)$$

where  $k$  denotes the curvature and  $r$  the radius of the extra dimension. The extra dimension is bound by two branes, the ultra-violet brane (UV) at  $\phi = 0$  and the infra-red (IR) brane at  $\phi = \pi$ . Due to the warp factor  $e^{-2\sigma(\phi)}$ , energy scales in this model depend on the position along the fifth dimension. This allows to address the gauge hierarchy problem if the Higgs field is located at the IR brane. Without additional constraints, all other SM fields are allowed to probe the fifth dimension. In this setup<sup>1</sup> the localisation

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<sup>1</sup> I will refer to this model as the *minimal model* in contrast to extensions with custodial protection.

of quark fields along the extra dimension provides an attractive explanation of the flavour puzzle. Section 2 reviews the theoretical framework of this model.

Many qualitative and quantitative studies have been performed in the last years and allow for a detailed understanding of the limits on the KK mass scale. In Sections 2 and 3 the results of these computations will be summarised with particular emphasis on electroweak precision observables. Corrections to the  $S, T$ , and  $U$  parameter, to the  $Zb\bar{b}$  vertex, and to the mass of the  $W$  boson as well as possible constraints from these observables will be examined. Consistency with the bounds coming from the current experimental status of these observables can be achieved within the minimal model, but also models with an extended gauge group received increasing attention in the last years, as they provide an elegant solution to the tension coming especially from the constraints from  $Zb\bar{b}$  and  $T$ . I will discuss pros and cons of the different solutions to round off the review. The results of this proceedings are based on [2] and a recently published study of flavour observables in the context of the minimal model [3].

## 2. The minimal model

In the minimal realization of the RS scenario all SM fields except for the Higgs are five-dimensional (5D) fields. In order to solve the gauge hierarchy problem, the Higgs must be confined to (or localised close to) the IR brane, where the UV cutoff becomes of  $\mathcal{O}$  (few TeV), due to the warp factor  $\varepsilon \equiv e^{-kr\pi} \approx 10^{-16}$ .

Introducing a coordinate  $t = \varepsilon e^{\sigma(\phi)}$  along the extra dimension [4], which runs from  $t = \varepsilon$  on the UV brane to  $t = 1$  on the IR brane, the KK decompositions of the left-handed (right-handed) components of the five-dimensional  $SU(2)_L$  doublet (singlet) quark fields read

$$q_L(x, t) \propto \text{diag}[F(c_{Q_i}) t^{c_{Q_i}}] \mathbf{U}_q q_L^{(0)}(x) + \mathcal{O}\left(\frac{v^2}{M_{\text{KK}}^2}\right) + \text{KK modes},$$

$$q_R^c(x, t) \propto \text{diag}[F(c_{q_i}) t^{c_{q_i}}] \mathbf{W}_q q_R^{(0)}(x) + \mathcal{O}\left(\frac{v^2}{M_{\text{KK}}^2}\right) + \text{KK modes}, \quad (2)$$

where  $q = u, d$  stands for up- and down-type quarks, respectively. The fields are three-component vectors in flavour space. 5D fields on the left-hand side refer to interaction eigenstates, while the four-dimensional (4D) fields appearing on the right-hand side are mass eigenstate. The superscript “(0)” denotes the so-called “zero modes”, which correspond to the light SM fermions. Heavy KK fermions will not play a role in the following.

The “zero-mode” profiles  $F(c_{Q_i, q_i})$  are exponentially suppressed in the volume factor  $L \equiv -\ln \varepsilon$  if the bulk mass parameters  $c_{Q_i} = +M_{Q_i}/k$  and  $c_{q_i} = -M_{q_i}/k$  are smaller than the critical value  $-1/2$ , in which case  $F(c) \sim e^{L(c+\frac{1}{2})}$  [4, 5]. Here  $\mathbf{M}_Q$  and  $\mathbf{M}_{u,d}$  are the mass matrices of the 5D  $\text{SU}(2)_L$  doublet and singlet fermions. This mechanism explains in a natural way the large hierarchies observed in the spectrum of the quark masses [5, 6], which follow from the eigenvalues of the effective Yukawa matrices

$$\mathbf{Y}_q^{\text{eff}} = \text{diag}[F(c_{Q_i})] \mathbf{Y}_q \text{diag}[F(c_{q_i})] = \mathbf{U}_q \boldsymbol{\lambda}_q \mathbf{W}_q^\dagger, \quad (3)$$

and are up to  $\mathcal{O}(1)$  factors

$$m_{u_i} \sim \frac{v}{\sqrt{2}} |F(c_{Q_i}) F(c_{u_i})|, \quad m_{d_i} \sim \frac{v}{\sqrt{2}} |F(c_{Q_i}) F(c_{d_i})|. \quad (4)$$

The 5D Yukawa matrices  $\mathbf{Y}_q$  are assumed to have  $\mathcal{O}(1)$  complex entries, and  $\boldsymbol{\lambda}_q$  are diagonal matrices with entries  $(\lambda_q)_{ii} = \sqrt{2} m_{q_i}/v$ . The unitary matrices  $\mathbf{U}_q$  and  $\mathbf{W}_q$  appearing in (2) and (3) have a hierarchical structure given by quotients of the zero-mode profiles.

The profiles of the SM weak gauge bosons receive  $t$ -dependent corrections due to electroweak symmetry breaking, whereas the massless gluon and photon modes remain flat along the extra dimension. In order to compute the full contribution to tree-level processes one has to consider the whole tower of KK modes. The sum over the KK tower of gauge bosons can be evaluated by generalising a method developed in [7]. Dropping irrelevant  $\mathcal{O}(\varepsilon^2)$  constant terms, one finds for the sum over massive and massless KK gauge bosons respectively

$$\begin{aligned} \sum_n \frac{\chi_n(t) \chi_n(t')}{m_n^2} = & \left\{ \frac{1}{2\pi m_{W,Z}^2} + \frac{1}{4\pi M_{\text{KK}}^2} \left[ L t_{<}^2 - L (t^2 + t'^2) + 1 - \frac{1}{2L} + \mathcal{O}\left(\frac{m_{W,Z}^2}{M_{\text{KK}}^2}\right) \right] \right. \\ & \left. \left[ \frac{1}{4\pi M_{\text{KK}}^2} \left[ L t_{<}^2 - t^2 \left( \frac{1}{2} - \ln t \right) - t'^2 \left( \frac{1}{2} - \ln t' \right) + \frac{1}{2L} \right] \right] \right\}, \end{aligned} \quad (5)$$

where  $t_{<}^2 \equiv \min(t^2, t'^2)$ .

In principle, the terms proportional to  $t$  and  $t'$  could cause dangerously large FCNCs, but the corresponding vertices receive suppressions from the zero-mode profiles of the associated fermions, mitigating these effects. This mechanism is known as the RS–GIM mechanism [8–10].

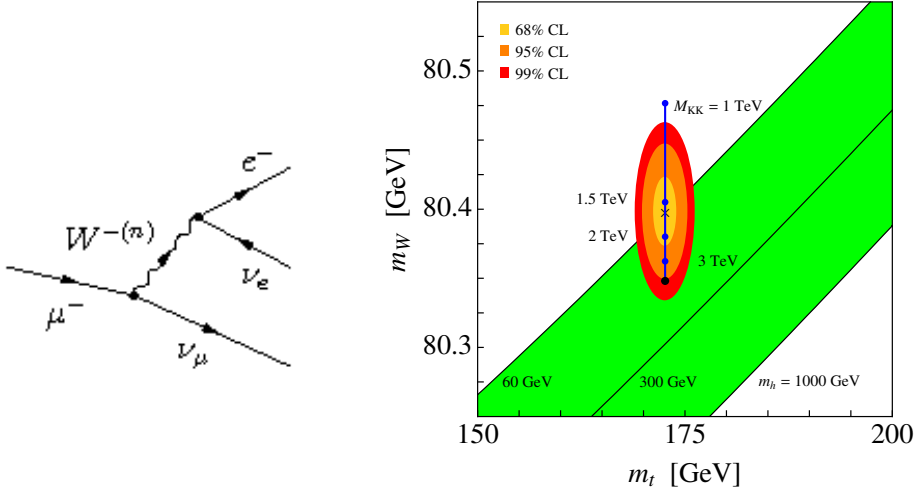


Fig. 1. The left panel shows the tree-level diagram for  $\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$ , including the KK modes  $W^{-(n)}$ . The right panel shows different probability regions from the direct measurement of  $m_W$  and  $m_t$  at LEP2 and Tevatron, the SM prediction based on the value of  $G_F$  as a black dot and the SM expectation for  $m_h \in [60, 1000]$  GeV as a shaded inclined band. The RS prediction approaches the SM for increasing  $M_{KK}$  as one follows the vertical line crossing the ellipse.

### 3. Modification to the $W$ -boson mass

In the SM the value from the direct measurement of the  $W$ -boson mass, following from the latest results of LEP2 and the Tevatron [11, 12], differs from the indirect extraction from precise measurements of  $\alpha$ ,  $G_F$ , and  $\sin^2 \theta_W$  by roughly 50 MeV. In the RS model,  $G_F$ , extracted from muon decay, receives a universal contribution<sup>2</sup> from the exchange of the KK excitations of the  $W$  boson. The process is illustrated in the left panel of Fig. 2. With (5) one finds

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2} \left[ 1 + \frac{m_W^2}{2M_{KK}^2} \left( 1 - \frac{1}{2L} \right) + \mathcal{O} \left( \frac{m_{W,Z}^4}{M_{KK}^4} \right) \right]. \quad (6)$$

This translates into a modification for the mass of the  $W$  boson through the SM relation

$$(m_W^2)_{\text{ind}} \equiv \frac{\pi\alpha}{\sqrt{2}G_F \sin^2 \theta_W}, \quad (7)$$

<sup>2</sup> In general, the  $t$ -dependent part of (5) also contributes, but they are strongly suppressed due to the UV localisation of the leptons.

so that

$$(m_W)_{\text{ind}} = m_W \left[ 1 - \frac{m_W^2}{4M_{\text{KK}}^2} \left( 1 - \frac{1}{2L} \right) + \mathcal{O} \left( \frac{m_{W,Z}^4}{M_{\text{KK}}^4} \right) \right]. \quad (8)$$

The plot on the right panel of Fig. 2 shows that the RS prediction can therefore explain the difference for KK mass scales slightly above 1.5 TeV, while allowing for a heavier Higgs mass,  $m_h = 400$  GeV ( $m_h = 1000$  GeV), the KK mass scale can even be lowered to 1.5 TeV (1 TeV).

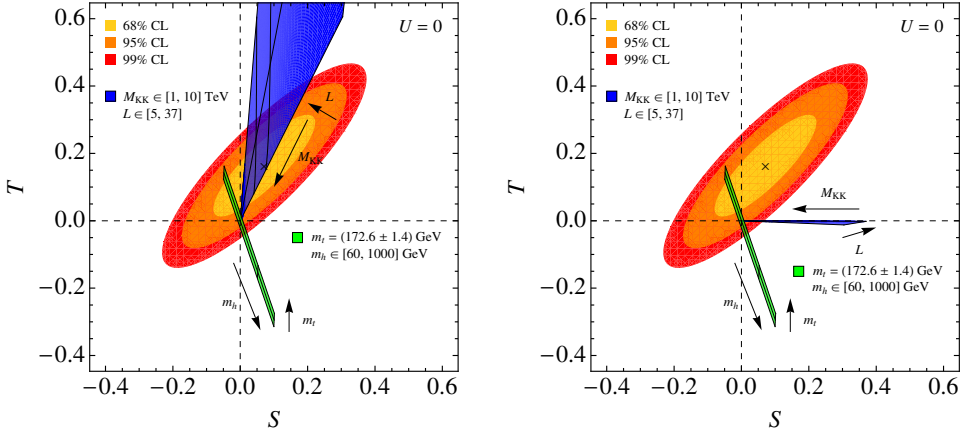


Fig. 2. The left/right plot shows different probability regions from a global fit to LEP and SLC measurements for  $S$  and  $T$  in the RS model with/without custodial protection. The inclined narrow stripes indicate the SM corrections for increasing  $m_h \in [60, 1000]$  GeV and  $m_t = (172.6 \pm 1.4)$  GeV. The RS corrections are indicated by the darkest shaded area and depend on the value of  $L \in [5, 37]$  and  $M_{\text{KK}} \in [1, 10]$  TeV.

#### 4. $S$ , $T$ , and $U$ parameters

Shifts from the SM values of the  $S, T$  and  $U$  parameters induced by new physics indicate deviations from the electroweak radiative corrections expected in the SM. In general, theories with additional heavy bosons call for an extension of this setup [13]. But the additional parameters include second derivatives of vacuum polarisation amplitudes and therefore turn out to be very small. Measurable corrections are only found in  $S$  and  $T$  [14, 15]

$$S = \frac{2\pi v^2}{M_{\text{KK}}^2} \left( 1 - \frac{1}{L} \right), \quad T = \frac{\pi v^2}{2 \cos^2 \theta_W M_{\text{KK}}^2} \left( L - \frac{1}{2L} \right). \quad (9)$$

As one can see from the left panel of Fig. 3, the correction to  $T$  strongly constrains the parameter space of the minimal RS model and pushes the KK mass scale up to  $M_{\text{KK}} > 4.0$  TeV.

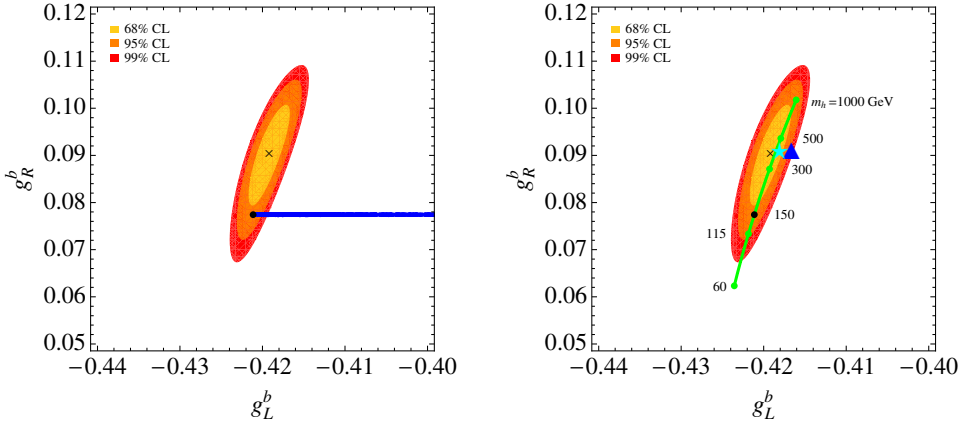


Fig. 3. Both plots show probability regions for the experimentally extracted value<sup>3</sup>. The SM prediction is indicated by the black dot. RS points lie on the narrow horizontal stripe. The right panel shows that following the inclined line, one can shift this stripe vertically by increasing  $m_h \in [60, 1000]$  GeV. The triangle and star indicate reference points at  $M_{\text{KK}} = 1.5$  TeV and  $M_{\text{KK}} = 3$  TeV, respectively.

There are three ways to solve this issue<sup>4</sup>. As a first option, one could assume a large Higgs mass. This corresponds to a negative shift  $\Delta T \sim \log m_h/m_h^{\text{ref}}$  and can lower the bound, for  $m_h = 1$  TeV, to  $M_{\text{KK}} > 2.6$  TeV<sup>5</sup>. A relaxation can also be achieved by lowering the volume factor to about  $L = 5$ . That means abandoning the solution to the full hierarchy problem and requires a UV completion at  $\Lambda_{\text{UV}} \approx 10^3$  TeV, but lowers the bound on the KK mass scale to  $M_{\text{KK}} > 1.5$  TeV. These models are called “little RS” scenario and were first proposed in [16]. A third possibility is to introduce an extended bulk symmetry group, a so-called custodial symmetry  $\text{SU}(3)_c \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_X$ . The tree-level corrections to  $S$  and  $T$  then read [17]

$$S = \frac{2\pi v^2}{M_{\text{KK}}^2} \left(1 - \frac{1}{L}\right), \quad T = -\frac{\pi v^2}{4 \cos^2 \theta_W M_{\text{KK}}^2} \frac{1}{L}, \quad (10)$$

<sup>3</sup> For details see [2].

<sup>4</sup> In fact, there is at least another one provided by large brane-localised kinetic terms which is, however, not discussed in this review.

<sup>5</sup> Where the reference value is set to  $m_h = 150$  GeV.

and are displayed in the right panel of Fig. 3. While this can provide a KK mass scale as low as  $M_{\text{KK}} = 2.4$  TeV, it should be mentioned that in this case a large Higgs mass would spoil the electroweak fit.

## 5. $Zb\bar{b}$ couplings

Another strong bound comes from the non-universal corrections to the coupling of the  $Z$  to bottom quarks [18]. The corresponding couplings read

$$g_L^b = (g_L^b)_{\text{SM}} \left[ 1 - \frac{m_Z^2}{2M_{\text{KK}}^2} \frac{F^2(c_{Q_3})}{3 + 2c_{Q_3}} \left( L - \frac{5 + 2c_{Q_3}}{2(3 + 2c_{Q_3})} \right) \right] + \mathcal{O} \left( \frac{m_b^2}{M_{\text{KK}}^2} \right), \quad (11)$$

$$g_R^b = (g_R^b)_{\text{SM}} \left[ 1 - \frac{m_Z^2}{2M_{\text{KK}}^2} \frac{F^2(c_{d_3})}{3 + 2c_{d_3}} \left( L - \frac{5 + 2c_{d_3}}{2(3 + 2c_{d_3})} \right) \right] + \mathcal{O} \left( \frac{m_b^2}{M_{\text{KK}}^2} \right). \quad (12)$$

Unfortunately, as one can see in the left panel of Fig. 3 the right-handed coupling remains practically unaffected, while large corrections to the left-handed coupling are possible. In order to reverse this feature one can rescale the zero-mode profiles  $F(c_{Q_3})$  and  $F(c_{b_R})$ . From the quark mass relations (4) follows that this redistribution requires a large value for  $c_{u_3}$ . However, if  $c_{u_3}$  becomes to large one has to sacrifice the explanation of the quark mass hierarchy relying on order one bulk mass parameters. While this problem does not appear in custodially protected models since corrections to the left-handed  $Z$  couplings basically vanish, the issue can also be solved within the minimal model if one assumes a heavy Higgs boson. The effect of a large Higgs mass is displayed on the right panel of Fig. 3. Good agreement with the experimental value can be achieved for  $m_h = 400$  GeV.

## 6. Concluding remarks

The  $W$ -boson mass difference between direct and indirect measurements can be explained within the minimal RS model with reasonably low KK mass scale. The tensions in  $T$  and  $Zb\bar{b}$  call for large  $M_{\text{KK}} > 4$  TeV, but can be resolved by introducing a heavy Higgs boson allowing for a KK mass scale as low as  $M_{\text{KK}} > 2.6$  TeV. Since the cutoff on the IR brane is around the TeV scale, a Higgs mass of this order is naturally expected in this model. A considerably lower Higgs mass would introduce a little hierarchy problem. An alternative way to deal with these issues is to introduce a custodial protection. However, a possible problem of the latter model is that in the presence of a heavy Higgs boson a good agreement with electroweak fits is challenging.

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