# PROBING NEW PHYSICS WITH $b \rightarrow s \ell \ell$ AND $b \rightarrow s \nu \bar{\nu}$ TRANSITIONS* 

Michael Wick<br>Physik Department, Technische Universität München<br>85748 Garching, Germany

(Received February 2, 2010)

The rare decay $B \rightarrow K^{*}(\rightarrow K \pi) \mu^{+} \mu^{-}$is regarded as one of the crucial channels for $B$ physics since its angular distribution gives access to many observables that offer new important tests of the Standard Model (SM) and its extensions. We point out a number of correlations among various observables which will allow a clear distinction between different New Physics (NP) scenarios. Furthermore, we discuss the decay $B \rightarrow K^{*} \nu \bar{\nu}$ which allows for a transparent study of $Z$ penguin effects in NP frameworks in the absence of dipole operator contributions and Higgs penguin contributions. We study all possible observables in $B \rightarrow K^{*} \nu \bar{\nu}$ and the related $b \rightarrow s$ transitions $B \rightarrow K \nu \bar{\nu}$ and $B \rightarrow X_{s} \nu \bar{\nu}$ in the context of the SM and various NP models.

PACS numbers: 13.20.He, 14.80.Ly

## 1. Introduction

The decays $B \rightarrow K^{*}(\rightarrow K \pi) \mu^{+} \mu^{-}$and $B \rightarrow K^{*} \nu \bar{\nu}$ are to some extent complementary: combined with its charge conjugated counterpart, the charged final state of the first decay gives access to a vast number of observables sensitive to CP violation. Although the second decay yields only two observables, it offers a unique possibility to a transparent study of $Z$ penguin effects. This is due to neutrinos in the final state, which goes along with an absence of non-perturbative contributions related to low energy QCD dynamics and photon exchanges. What these decays have clearly in common is the important role they will play in the upcoming experiments such as LHCb and SuperB facilities.

[^0]
## 2. The $B \rightarrow K^{*}(\rightarrow K \pi) \boldsymbol{\mu}^{+} \boldsymbol{\mu}^{-}$decay

A prediction for the observables of $B \rightarrow K^{*}(\rightarrow K \pi) \mu^{+} \mu^{-}[1-3]$ involves mainly three theoretical ingredients: effective Hamiltonian, form factors and QCD factorization. While the effective Hamiltonian governing this decay is discussed in $[1,8]$, we emphasize here that we also include scalar operators and lepton mass effects. The $B \rightarrow K^{*}$ matrix elements of the relevant operators can be expressed in terms of seven form factors depending on the momentum transfer $q^{2}$ between the $B$ and the $K^{*}$ mesons. The well established technique of QCD sum rules on the light cone (LCSR) that is applied here combines classic QCD sum rules with information on light cone distribution amplitudes in order to determine the form factors. The result is a set of form factors fulfilling all correlations required in the heavy quark limit. In addition to terms proportional to form factors, the $B \rightarrow K^{*} \mu^{+} \mu^{-}$ amplitude contains certain "non-factorizable" contributions, which do not correspond to form factors. We include these QCD factorization corrections to NLO in $\alpha_{s}$ but LO in $1 / m_{b}$. The resulting angular distribution of $\bar{B}^{0} \rightarrow$ $\bar{K}^{* 0}\left(\rightarrow K^{-} \pi^{+}\right) \mu^{+} \mu^{-}$gives rise to twelve angular coefficient functions $I_{i}^{(a)}$ :

$$
\begin{equation*}
\frac{d^{4} \Gamma}{d q^{2} d \cos \theta_{l} d \cos \theta_{K^{*}} d \phi a}=\frac{9}{32 \pi} I\left(q^{2}, \theta_{l}, \theta_{K^{*}}, \phi\right) \tag{1}
\end{equation*}
$$

where

$$
\begin{align*}
I\left(q^{2}, \theta_{l}, \theta_{K^{*}}, \phi\right)= & I_{1}^{s} \sin ^{2} \theta_{K^{*}}+I_{1}^{c} \cos ^{2} \theta_{K^{*}}+\left(I_{2}^{s} \sin ^{2} \theta_{K^{*}}+I_{2}^{c} \cos ^{2} \theta_{K^{*}}\right) \cos 2 \theta_{l} \\
& +I_{3} \sin ^{2} \theta_{K^{*}} \sin ^{2} \theta_{l} \cos 2 \phi+I_{4} \sin 2 \theta_{K^{*}} \sin 2 \theta_{l} \cos \phi \\
& +I_{5} \sin 2 \theta_{K^{*}} \sin \theta_{l} \cos \phi+\left(I_{6}^{s} \sin ^{2} \theta_{K^{*}}+I_{6}^{c} \cos ^{2} \theta_{K^{*}}\right) \cos \theta_{l} \\
& +\left(I_{7} \sin \theta_{l}+I_{8} \sin 2 \theta_{l}\right) \sin 2 \theta_{K^{*}} \sin \phi \\
& +I_{9} \sin ^{2} \theta_{K^{*}} \sin ^{2} \theta_{l} \sin 2 \phi \tag{2}
\end{align*}
$$

The corresponding expression for the CP-conjugated mode $B^{0} \rightarrow K^{* 0}$ $\left(\rightarrow K^{+} \pi^{-}\right) \mu^{+} \mu^{-}$is

$$
\begin{equation*}
\frac{d^{4} \bar{\Gamma}}{d q^{2} d \cos \theta_{l} d \cos \theta_{K^{*}} d \phi}=\frac{9}{32 \pi} \bar{I}\left(q^{2}, \theta_{l}, \theta_{K^{*}}, \phi\right) \tag{3}
\end{equation*}
$$

The function $\bar{I}\left(q^{2}, \theta_{l}, \theta_{K^{*}}, \phi\right)$ is obtained from (2) by the replacements [7]

$$
\begin{equation*}
I_{1,2,3,4,7}^{(a)} \longrightarrow \bar{I}_{1,2,3,4,7}^{(a)}, \quad I_{5,6,8,9}^{(a)} \longrightarrow-\bar{I}_{5,6,8,9}^{(a)} \tag{4}
\end{equation*}
$$

where $\bar{I}_{i}^{(a)}$ equals $I_{i}^{(a)}$ with all the weak phases conjugated. The minus sign in (4) is a result of our convention that, while $\theta_{K^{*}}$ is the angle between the
$\bar{K}^{* 0}$ and the $K^{-}$flight direction or between the $K^{* 0}$ and the $K^{+}$, respectively, the angle $\theta_{l}$ is measured between the $\bar{K}^{* 0}\left(K^{* 0}\right)$ and the lepton $\mu^{-}$ in both modes. The angle between $(K, \pi)$ and $\left(\mu^{+}, \mu^{-}\right)$planes in the $B$ rest frame is denoted by $\phi$. A more detailed exposition of the kinematics can be found in [1].

Instead of using the angular coefficient functions as fundamental observables, we use straightforward combinations of the $I_{i}^{(a)}$ and $\bar{I}_{i}^{(a)}$ to reduce the theoretical error and separate CP-conserving and CP-violating NP effects. The twelve CP averaged angular coefficients [1,3]

$$
\begin{equation*}
S_{i}^{(a)}=\left(I_{i}^{(a)}+\bar{I}_{i}^{(a)}\right) / \frac{d(\Gamma+\bar{\Gamma})}{d q^{2}}, \tag{5}
\end{equation*}
$$

as well as the twelve CP asymmetries are given in terms of angular coefficient functions

$$
\begin{equation*}
A_{i}^{(a)}=\left(I_{i}^{(a)}-\bar{I}_{i}^{(a)}\right) / \frac{d(\Gamma+\bar{\Gamma})}{d q^{2}} . \tag{6}
\end{equation*}
$$

Since this is a complete set of accessible observables, all the previously considered observables, for example the forward backward symmetry, can be expressed straightforwardly in terms of the new observables. While in [1] several different models including the Littlest Higgs Model with T-Parity (LHT) are studied, we illustrate here the discriminating power of our set of observables using examples of the effects in models based on the MSSM. The most interesting CP asymmetries are $A_{7,8,9}$, which are not suppressed by small strong phases [6] and thus potentially of $O(1)$.

In Fig. 1, we show the effects in a Flavor Blind MSSM [5], which is a modification of the Minimal Flavor Violating MSSM with additional flavorconserving CP-violating phases in the soft terms. The other two scenarios are general MSSM frameworks with different mass insertions switched on.


Fig. 1. Dependence of the CP asymmetries $A_{7,8,9}$ on $q^{2}$ in three variations of the MSSM: the Flavor Blind MSSM ((a), orange) with $\operatorname{Arg}\left(\mu A_{\tilde{t}}\right)=50^{\circ}$, the MSSM with complex $\left(\delta_{d}\right)_{32}^{\mathrm{LR}}$ mass insertion ( $(\mathrm{b})$, red) and the MSSM with complex $\left(\delta_{u}\right)_{32}^{\mathrm{LR}}$ mass insertion ((c), green). For further information on model parameters see [1].

One can see that the effects in different observables are highly model dependent and give, combined with the other CP symmetries and asymmetries, an extraordinary tool to discriminate between different models or parameter configurations.

## 3. The $B \rightarrow K \nu \bar{\nu}, B \rightarrow K^{*} \nu \bar{\nu}$ and $B \rightarrow X_{s} \nu \bar{\nu}$ decays

The effective Hamiltonian for $b \rightarrow s \nu \bar{\nu}$ transitions is generally given by

$$
\begin{equation*}
H_{\mathrm{eff}}=-\frac{4 G_{F}}{\sqrt{2}} V_{t b} V_{t s}^{*}\left(C_{\mathrm{L}}^{\nu} O_{\mathrm{L}}^{\nu}+C_{\mathrm{R}}^{\nu} O_{\mathrm{R}}^{\nu}\right)+\text { h.c. } \tag{7}
\end{equation*}
$$

with the operators

$$
\begin{align*}
O_{\mathrm{L}}^{\nu} & =\frac{e^{2}}{16 \pi^{2}}\left(\bar{s} \gamma_{\mu} P_{\mathrm{L}} b\right)\left(\bar{\nu} \gamma^{\mu}\left(1-\gamma_{5}\right) \nu\right)  \tag{8}\\
O_{\mathrm{R}}^{\nu} & =\frac{e^{2}}{16 \pi^{2}}\left(\bar{s} \gamma_{\mu} P_{\mathrm{R}} b\right)\left(\bar{\nu} \gamma^{\mu}\left(1-\gamma_{5}\right) \nu\right) \tag{9}
\end{align*}
$$

There are four observables of interest that are generated by the quark-level transition $b \rightarrow s \nu \bar{\nu}$ (see Table I). These are the three branching ratios and one additional polarization ratio in the case of $B \rightarrow K^{*} \nu \bar{\nu}$, measuring the fraction $F_{\mathrm{L}}$ of longitudinally polarized $K^{*}$ mesons [4]. This polarization fraction can be extracted from the angular distribution in the invariant mass of neutrino-antineutrino pair and the angle between the $K^{*}$ flight direction in the $B$ rest frame and the $K$ flight direction in the $K \pi$ rest frame.

A major source of uncertainties of the $b \rightarrow s \nu \bar{\nu}$ based decays are the QCD/hadronic ingredients entering the calculation. A well known problem in the inclusive decay is the $m_{b}$ dependence, which leads to considerable uncertainties. The traditional approach is to normalize the decay rate to the semileptonic, inclusive $b \rightarrow c$ decay. On the other side, this introduces again uncertainties through the dependence of the semileptonic phase space factor on the charm quark mass. Instead of this normalization, we use the $b$ mass evaluated in the $1 S$ scheme [11], being known at a precision of $1 \%$. For the $B \rightarrow K \nu \bar{\nu}$ decay we use the form factors given in [9], being valid in the full physical range, while we use the already mentioned set of [1] for the decay $B \rightarrow K^{*} \nu \bar{\nu}$. These improvements combined with an up-to-date top mass [10] lead to a significantly lower predictions for $\mathrm{BR}\left(B \rightarrow K^{*} \nu \bar{\nu}\right)$ and a considerably more accurate prediction for $\mathrm{BR}\left(B \rightarrow X_{s} \nu \bar{\nu}\right)$, than the ones present in the literature.

In Table I we give a summary of our SM predictions.
The four observables accessible in the three different $b \rightarrow s \nu \bar{\nu}$ decays are dependent on the two in principle complex Wilson coefficients $C_{\mathrm{L}}^{\nu}$ and $C_{\mathrm{R}}^{\nu}$.

TABLE I
SM predictions and experimental bounds (all at the $90 \%$ C.L.) for the four $b \rightarrow s \nu \bar{\nu}$ observables.

| Observable | Our SM prediction | Experiment |
| :--- | :---: | :---: |
| $\operatorname{BR}\left(B \rightarrow K^{*} \nu \bar{\nu}\right)$ | $\left(6.8_{-1.1}^{+1.0}\right) \times 10^{-6}$ | $<80 \times 10^{-6}[14]$ |
| $\operatorname{BR}\left(B^{+} \rightarrow K^{+} \nu \bar{\nu}\right)$ | $(4.5 \pm 0.7) \times 10^{-6}$ | $<14 \times 10^{-6}[15]$ |
| $\operatorname{BR}\left(B \rightarrow X_{s} \nu \bar{\nu}\right)$ | $(2.7 \pm 0.2) \times 10^{-5}$ | $<64 \times 10^{-5}[16]$ |
| $\left\langle F_{\mathrm{L}}\left(B \rightarrow K^{*} \nu \bar{\nu}\right)\right\rangle$ | $0.54 \pm 0.01$ | - |

However, only two real combinations of these complex quantities enter in the observables [4, 13]:

$$
\begin{equation*}
\varepsilon=\frac{\sqrt{\left|C_{\mathrm{L}}^{\nu}\right|^{2}+\left|C_{\mathrm{R}}^{\nu}\right|^{2}}}{\left|\left(C_{\mathrm{L}}^{\nu}\right)^{\mathrm{SM}}\right|}, \quad \eta=\frac{-\operatorname{Re}\left(C_{\mathrm{L}}^{\nu} C_{\mathrm{R}}^{\nu *}\right)}{\left|C_{\mathrm{L}}^{\nu}\right|^{2}+\left|C_{\mathrm{R}}^{\nu}\right|^{2}} \tag{10}
\end{equation*}
$$

Measurements of the four observables are then transparently represented as bands in the $\varepsilon-\eta$ plane. To illustrate the theoretical cleanliness of the various observables, we show in Fig. 2 the combined constraints after hypothetical


Fig. 2. Left: Hypothetical constraints on the $\varepsilon-\eta$-plane, assuming all the four $b \rightarrow s \nu \bar{\nu}$ observables have been measured with infinite precision. The error bands include uncertainties due to the form factors in the case of exclusive decays, uncertainties in the CKM elements and in the SM Wilson coefficient. The band with a dashed line in the middle (green) represents $\mathrm{BR}\left(B \rightarrow K^{*} \nu \bar{\nu}\right)$, the band with a solid line (black) $-\mathrm{BR}(B \rightarrow K \nu \bar{\nu})$, the vertical band (red) $-\mathrm{BR}\left(B \rightarrow X_{s} \nu \bar{\nu}\right)$, and the horizontal band (orange) - $\left\langle F_{\mathrm{L}}\right\rangle$. The shaded area is ruled out experimentally at the $90 \%$ confidence level. The $68 \%$ (red) and $95 \%$ (green) areas describe the projected sensitivity at SuperB with $75 a b^{-1}$ integrated luminosity [12]. Right: dependence of $F_{\mathrm{L}}$ on the normalized momentum transfer $s_{B}=q^{2} / m_{B}^{2}$ for different values of $\eta$, from top to bottom: $\eta=0.5,0,-0.2,-0.4,-0.45$.
measurements of the observables. Apart from the model independent analysis, an application to specific models shows that NP effects in the LHT and the Randall-Sundrum model with custodial protection of left-handed $Z$-couplings to down type quarks are small, as opposed to the MSSM with a generic flavor violating soft sector. Taking into account the strong constraints from $B \rightarrow X_{s} \gamma$ and $B_{s} \rightarrow \mu^{+} \mu^{-}$, it turns out that dominantly chargino contributions lead to sizeable effects.

## 4. Conclusions

Correlations between observables of the decay $B \rightarrow K^{*}(\rightarrow K \pi) \mu^{+} \mu^{-}$ are highly model-dependent, and thus allow to distinguish between different models of New Physics. Theoretical uncertainties in the branching ratios of $B \rightarrow K^{*} \nu \bar{\nu}, B \rightarrow K \nu \bar{\nu}$ and $B \rightarrow X_{s} \nu \bar{\nu}$ are comparable to or smaller than projected experimental uncertainties at SuperB. It will be fascinating to confront these decays with actual experimental data.

## REFERENCES

[1] W. Altmannshofer, P. Ball, A. Bharucha, A.J. Buras, D.M. Straub, M. Wick, J. High Energy Phys. 0901, 019 (2009) [arXiv:0811.1214 [hep-ph]].
[2] U. Egede, T. Hurth, J. Matias, M. Ramon, W. Reece, J. High Energy Phys. 0811, 032 (2008) [arXiv:0807. 2589 [hep-ph]].
[3] C. Bobeth, G. Hiller, G. Piranishvili, J. High Energy Phys. 0807, 106 (2008) [arXiv:0805. 2525 [hep-ph]].
[4] W. Altmannshofer, A.J. Buras, D.M. Straub, M. Wick, J. High Energy Phys. 0904, 022 (2009) [arXiv:0902.0160 [hep-ph]].
[5] W. Altmannshofer, A.J. Buras, P. Paradisi, Phys. Lett. B669, 239 (2008).
[6] C. Bobeth, G. Hiller, G. Piranishvili, J. High Energy Phys. 0712, 040 (2007).
[7] F. Kruger, L.M. Sehgal, N. Sinha, R. Sinha, Phys. Rev. D61, 114028 (2000) [Erratum D63, 019901 (2001)] [arXiv:hep-ph/9907386].
[8] C. Bobeth, A.J. Buras, F. Kruger, J. Urban, Nucl. Phys. B630, 87 (2002).
[9] P. Ball, R. Zwicky, Phys. Rev. D71, 014015 (2005).
[10] [Tevatron Electroweak Working Group and CDF and D0 collaborations], arXiv:0808. 1089 [hep-ex].
[11] A.H. Hoang, arXiv:hep-ph/0008102.
[12] F. Renga, talk given at the SuperB Physics Workshop in Warwick, 13-17 April 2009.
[13] D. Melikhov, N. Nikitin, S. Simula, Phys. Lett. B428, 171 (1998).
[14] [BABAR Collaboration] B. Aubert et al., Phys. Rev. D78, 072007 (2008).
[15] [BELLE Collaboration] K.F. Chen et al., Phys. Rev. Lett. 99, 221802 (2007).
[16] [ALEPH Collaboration] R. Barate et al., Eur. Phys. J. C19, 213 (2001).


[^0]:    * Presented at the FLAVIAnet Topical Workshop, "Low energy constraints on extensions of the Standard Model", Kazimierz, Poland, July 23-27, 2009.

