# NEW PHYSICS BOUNDS FROM THE COMBINATION OF CKM-UNITARITY AND HIGH ENERGY DATA*

Martín González-Alonso

Departament de Física Teòrica and IFIC, Universitat de València-CSIC
Apt. Correus 22085, 46071 València, Spain

Through an effective field theory approach, we analyse the new physics (NP) corrections to muon and beta decays and their effects on the extractions of $V_{ud}$ and $V_{us}$. Assuming nearly flavour blind NP interactions, we find that the only quantity sensitive to NP is $\Delta_{\mathrm{CKM}} \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1$, that receives contributions from four short distance operators. The phenomenological bound $\Delta_{\mathrm{CKM}} = (-1 \pm 6) \times 10^{-4}$ provides strong constraints on all four operators, corresponding to an effective scale $\Lambda > 11$ TeV (90% C.L.). Depending on the operator, this constraint is at the same level or better than that generated by the $Z$ pole observables.

PACS numbers: 12.90.+b, 12.15.Hh, 11.30.Hv

## 1. Introduction

Thanks to the precise experimental measurements [1] and the theoretical improvements [2], semi-leptonic (SL) decays of light quarks are a deep probe of the nature of weak interactions [3, 4]. In particular, the precise determination of the elements $V_{ud}$ and $V_{us}$ of the Cabibbo–Kobayashi–Maskawa (CKM) [5] matrix enables tests of the CKM unitarity condition[1] $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$, at the level of 0.001 or better. Assuming that NP contributions scale as $\alpha/\pi(M_W^2/\Lambda^2)$, this test probes energy scales $\Lambda$ on the order of the TeV, which will be directly probed at the LHC.

While the consequences of Cabibbo universality tests have been considered in some explicit Standard Model (SM) extensions [6], our goal is to study it in a model-independent way. We have to analyse the NP contributions to the muon decay (where the $G_{\mathrm{F}}$ is extracted) and to the different channels that are used to extract the product $G_{\mathrm{F}}V_{ud,us}$. Currently

---

* Presented at the FLAVIAnet Topical Workshop "Low energy constraints on extensions of the Standard Model", Kazimierz, Poland, July 23–27, 2009.

[1] $V_{ub} \sim 10^{-3}$ contributes negligibly to this relation.

the determinations of $V_{ud}$ and $V_{us}$ are dominated by super-allowed nuclear beta decays [7] ($V_{ud} = 0.97425(22)$) and $K_{l3}$ decays [8] ($V_{us} = 0.2252(9)$) respectively, although experimental and theoretical improvements in other channels can make them competitive in the near future.

## 2. Weak scale effective Lagrangian

In order to analyse in a model-independent framework NP contributions to both electroweak precision observables (EWPO) and beta decays we take the SM (including the Higgs) as the low-energy limit of a more fundamental theory, and more specifically we assume that: *(i)* there is a gap between the weak scale $v$ and the NP scale $\Lambda$ where new degrees of freedom appear; *(ii)* the NP at the weak scale is weakly coupled, so the electroweak (EW) gauge symmetry is linearly realized; *(iii)* the violation of total lepton and baryon number is suppressed by a scale much higher than $\Lambda \sim$ TeV. These assumptions lead us to an effective non-renormalizable Lagrangian of the form [10]:

$$\mathcal{L}^{(\text{eff})} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \frac{1}{\Lambda^3}\mathcal{L}_7 + \dots , \tag{1}$$

where $\mathcal{L}_n = \sum_i \alpha_i^{(n)} O_i^{(n)}$, being $\mathcal{O}_i^{(n)}$ local gauge-invariant operators of dimension $n$ built out of SM fields. It can be shown that under the above assumptions, there are no corrections to the SM Lagrangian at dimension five, whereas seventy seven operators appear at dimension six [9, 10], where we truncate the expansion. In order to be consistent with this truncation we will work at linear order in the NP corrections.

For the EWPO *and* beta decays it can be shown that we only need a twenty-five operator basis, with twenty one U(3)$^5$ invariant and four non-invariant[2] (we will see the usefulness of this separation later). Only nine of these operators contribute to the beta and muon decays:

$$O_{ll}^{(1)} = \tfrac{1}{2}(\bar{l}\gamma^\mu l)(\bar{l}\gamma_\mu l) , \qquad\qquad O_{ll}^{(3)} = \tfrac{1}{2}(\bar{l}\gamma^\mu \sigma^a l)(\bar{l}\gamma_\mu \sigma^a l) , \tag{2}$$

$$O_{lq}^{(3)} = (\bar{l}\gamma^\mu \sigma^a l)(\bar{q}\gamma_\mu \sigma^a q) , \tag{3}$$

$$O_{\varphi l}^{(3)} = i(h^\dagger D^\mu \sigma^a \varphi)(\bar{l}\gamma_\mu \sigma^a l) + \text{h.c.} , \quad O_{\varphi q}^{(3)} = i(\varphi^\dagger D^\mu \sigma^a \varphi)(\bar{q}\gamma_\mu \sigma^a q) + \text{h.c.} , \tag{4}$$

$$O_{qde} = (\bar{l}e)(\bar{d}q) + \text{h.c.} , \tag{5}$$

$$O_{lq} = (\bar{l}_a e)\epsilon^{ab}(\bar{q}_b u) + \text{h.c.} , \qquad\qquad O_{lq}^t = (\bar{l}_a \sigma^{\mu\nu} e)\epsilon^{ab}(\bar{q}_b \sigma_{\mu\nu} u) + \text{h.c.} , \tag{6}$$

$$O_{\varphi\varphi} = i(\varphi^T \epsilon D_\mu \varphi)(\bar{u}\gamma^\mu d) + \text{h.c.} , \tag{7}$$

where only the first five are U(3)$^5$-invariant.

---

[2] We refer to the U(3)$^5$ flavour symmetry of the SM gauge Lagrangian (the freedom to make U(3) rotations in family space for each of the five fermionic gauge multiplets).

## 3. Effective Lagrangian for $\mu$ and quark $\beta$ decays

Deriving the low-energy effective Lagrangian that describes the muon and beta decays (see Ref. [9] for details) we find

$$\mathcal{L}_\mu = \frac{-g^2}{2m_W^2}\left[(1+\tilde{v}_{\rm L})\cdot\bar{e}_{\rm L}\gamma_\mu\nu_{e{\rm L}}\bar{\nu}_{\mu{\rm L}}\gamma^\mu\mu_{\rm L}+\tilde{s}_{\rm R}\cdot\bar{e}_{\rm R}\nu_{e{\rm L}}\bar{\nu}_{\mu{\rm L}}\mu_{\rm R}\right]+{\rm h.c.}\,,\quad(8)$$

$$\tilde{v}_{\rm L} = 2\left[\hat{\alpha}_{\varphi l}^{(3)}\right]_{11+22^*}-\left[\hat{\alpha}_{ll}^{(1)}\right]_{1221}-2\left[\hat{\alpha}_{ll}^{(3)}\right]_{1122-\frac{1}{2}(1221)}\,,\quad(9)$$

$$\tilde{s}_{\rm R} = +2[\hat{\alpha}_{le}]_{2112}\,,\quad(10)$$

$$\mathcal{L}_{d_j} = \frac{-g^2}{2m_W^2}V_{ij}\left[\left(1+[v_{\rm L}]_{\ell\ell ij}\right)\bar{\ell}_{\rm L}\gamma_\mu\nu_{\ell L}\bar{u}_{\rm L}^i\gamma^\mu d_{\rm L}^j+[v_{\rm R}]_{\ell\ell ij}\bar{\ell}_{\rm L}\gamma_\mu\nu_{\ell L}\bar{u}_{\rm R}^i\gamma^\mu d_{\rm R}^j\right.$$

$$+[s_{\rm L}]_{\ell\ell ij}\bar{\ell}_{\rm R}\nu_{\ell L}\bar{u}_{\rm R}^i d_{\rm L}^j+[s_{\rm R}]_{\ell\ell ij}\bar{\ell}_{\rm R}\nu_{\ell L}\bar{u}_{\rm L}^i d_{\rm R}^j$$

$$\left.+[t_{\rm L}]_{\ell\ell ij}\bar{\ell}_{\rm R}\sigma_{\mu\nu}\nu_{\ell L}\bar{u}_{\rm R}^i\sigma^{\mu\nu}d_{\rm L}^j\right]+{\rm h.c.}\,,\quad(11)$$

$$V_{ij}\,[v_L]_{\ell\ell ij} = 2V_{ij}\left[\hat{\alpha}_{\varphi l}^{(3)}\right]_{\ell\ell}+2V_{im}\left[\hat{\alpha}_{\varphi q}^{(3)}\right]_{jm}^*-2V_{im}\left[\hat{\alpha}_{lq}^{(3)}\right]_{\ell\ell mj}\,,\quad(12)$$

$$V_{ij}\,[v_{\rm R}]_{\ell\ell ij} = -[\hat{\alpha}_{\varphi\varphi}]_{ij}\,,\quad(13)$$

$$V_{ij}\,[s_{\rm L}]_{\ell\ell ij} = -[\hat{\alpha}_{lq}]_{\ell\ell ji}^*\,,\quad(14)$$

$$V_{ij}\,[s_{\rm R}]_{\ell\ell ij} = -V_{im}[\hat{\alpha}_{qde}]_{\ell\ell jm}^*\,,\quad(15)$$

$$V_{ij}\,[t_{\rm L}]_{\ell\ell ij} = -\left[\hat{\alpha}_{lq}^t\right]_{\ell\ell ji}^*\,.\quad(16)$$

## 4. Flavour structure of the effective couplings

So far, we have not made any assumption about the flavour structures in the couplings $[\hat{\alpha}_X]_{abcd}$. However flavour changing neutral current (FCNC) processes forbid generic structures if $\Lambda\sim$ TeV and therefore we organise the discussion in terms of perturbations around the U(3)$^5$ flavour symmetry limit, where no problem arises with FCNC.

In the U(3)$^5$-limit the expressions greatly simplify and all the NP effects can be encoded into the following redefinitions

$$G_{\rm F}^\mu = (G_{\rm F})^{(0)}\left(1+4\,\hat{\alpha}_{\varphi l}^{(3)}-2\,\hat{\alpha}_{ll}^{(3)}\right)\,,\quad(17)$$

$$G_{\rm F}^{\rm SL} = (G_{\rm F})^{(0)}\left(1+2\left(\hat{\alpha}_{\varphi l}^{(3)}+\hat{\alpha}_{\varphi q}^{(3)}-\hat{\alpha}_{lq}^{(3)}\right)\right)\,,\quad(18)$$

where $G_{\rm F}^{(0)}=g^2/(4\sqrt{2}m_W^2)$. Consequently we will have

$$V_{ij}^{\rm (pheno)} = V_{ij}\left[1+2\left(\hat{\alpha}_{ll}^{(3)}-\hat{\alpha}_{lq}^{(3)}-\hat{\alpha}_{\varphi l}^{(3)}+\hat{\alpha}_{\varphi q}^{(3)}\right)\right]\,,\quad(19)$$

as phenomenological values of $V_{ud,us}$. This shift is independent of the channel used to extract $V_{ud,us}$ and the only way to expose NP contributions is to construct universality tests, in which the absolute normalisation of $V_{ij}$ matters. Therefore the NP effects are entirely captured by the quantity

$$\Delta_{\mathrm{CKM}} \equiv |V_{ud}^{(\mathrm{pheno})}|^2 + |V_{us}^{(\mathrm{pheno})}|^2 + |V_{ub}^{(\mathrm{pheno})}|^2 - 1 \,, \qquad (20)$$

that in our framework takes the value

$$\Delta_{\mathrm{CKM}} = 4\left(\hat{\alpha}_{ll}^{(3)} - \hat{\alpha}_{lq}^{(3)} - \hat{\alpha}_{\varphi l}^{(3)} + \hat{\alpha}_{\varphi q}^{(3)}\right) \,. \qquad (21)$$

The Minimal Flavour Violation (MFV) hypothesis requires that $U(3)^5$ symmetry is broken in the underlying model only by structures proportional to the SM Yukawa couplings [11], and structures generating neutrino masses [12]. But in MFV the coefficients parameterising deviations from the $U(3)^5$-limit are highly suppressed [9] and so we expect the conclusions of the previous subsection to hold. The elements $V_{ij}$ receive a common dominant shift plus suppressed channel-dependent corrections.

In a more general framework the situation can be different because the channel-dependent shifts to $V_{ij}$ could be appreciable and $\Delta_{\mathrm{CKM}}$ would depend on the channels used. Work in this direction is in progress.

## 5. $\Delta_{\mathrm{CKM}}$ versus precision EW measurements

In the limit of approximate $U(3)^5$ invariance, we have shown that $\Delta_{\mathrm{CKM}}$ constraints a specific combination of the coefficients, that also contribute to the EWPO [13], together with the remaining seventeen operators that make up the $U(3)^5$ invariant sector of our TeV scale effective Lagrangian.

The analysis of Han and Skiba [13], that studied the constraints on the same set of twenty-one $U(3)^5$ invariant operators via a global fit to the EWPO, allows us to compare the bound on $\Delta_{\mathrm{CKM}}$ that we get from them

$$-9.5 \times 10^{-3} \le \Delta_{\mathrm{CKM}} \le 0.1 \times 10^{-3} \qquad (90\%\mathrm{C.L.}) \,, \qquad (22)$$

with the direct experimental bound $|\Delta_{\mathrm{CKM}}| \le 1. \times 10^{-3}$ (90% C.L.) [8]. We see that EWPO leave room for a sizable non-zero $\Delta_{\mathrm{CKM}}$ and consequently we have to include the direct $\Delta_{\mathrm{CKM}}$ constraint in the global fit to improve the bounds on NP-couplings (see results in Fig. 1). We see that the main effect is to strengthen the constraints on $O_{lq}^{(3)}$.

In Fig. 2 we show the bounds if we assume a single operator dominance. For all the CKM-operators the direct $\Delta_{\mathrm{CKM}}$ measurement provides competitive constraints and in the case of $O_{lq}^{(3)}$ the improvement is remarkable. If a non-zero $\Delta_{\mathrm{CKM}}$ is observed, in the single-operator framework it would be correlated to deviations from the SM expectation in other observables as well. These correlations have been studied in Ref. [9].
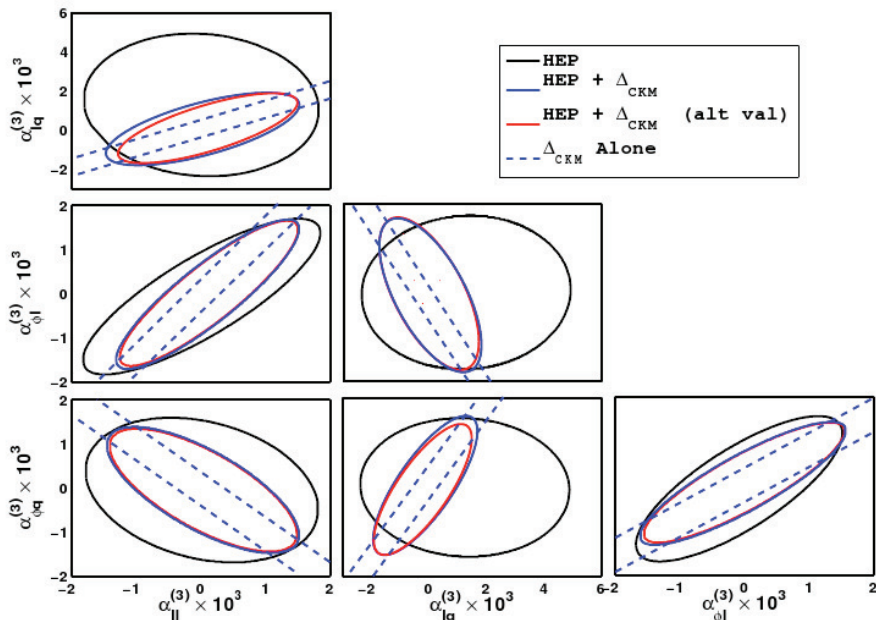
Fig. 1. 90% C.L. regions projections, using the high energy observables (HEP), the current $\Delta_{\mathrm{CKM}}$ constraint or an alternative value of $\Delta_{\mathrm{CKM}} = -0.0025 \pm 0.0006$.
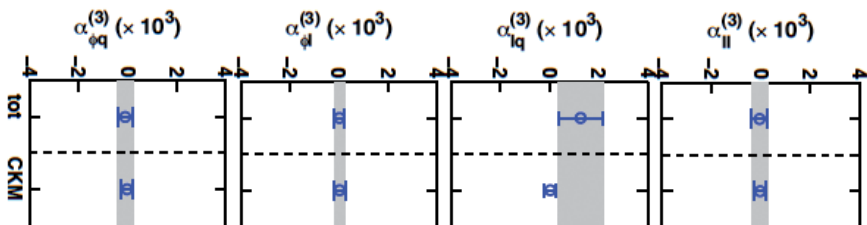


Fig. 2. 90 % C.L. regions in the single operator analysis. The first row displays the constraint from EWPO and the second row those coming only from $\Delta_{\mathrm{CKM}}$.

## 6. Conclusions

In a model-independent framework and assuming nearly flavor blind NP interactions, it has been shown that the extraction of $V_{ud,us}$ is channel independent and the only NP probe is $\Delta_{\mathrm{CKM}}$, which receives contributions from four short distance operators: $O^{(3)}_{ll,lq,\varphi l,\varphi q}$.

It has been shown that Cabibbo universality tests provide constraints on NP that currently cannot be obtained from other EW precision tests and collider measurements. The $\Delta_{\mathrm{CKM}}$ constraint bounds the effective NP scale

of all four CKM-operators to be $\Lambda > 11\,\text{TeV}$ (90 % C.L.), what for $O_{lq}^{(3)}$ is almost one order of magnitude stronger than EWPO-bound. Equivalently, should $V_{ud}$ and $V_{us}$ move from their current central values [4], EWPO data would leave room for sizable deviations from quark–lepton universality.

## REFERENCES

[1] [Particle Data Group] C. Amsler *et al., Phys. Lett.* **B667**, 1 (2008).

[2] W.J. Marciano, A. Sirlin, *Phys. Rev. Lett.* **96**, 032002 (2006); V. Cirigliano *et al., Eur. Phys. J.* **C23**, 121 (2002); V. Cirigliano, H. Neufeld, H. Pichl, *Eur. Phys. J.* **C35**, 53 (2004); V. Cirigliano, M. Giannotti, H. Neufeld, *J. High Energy Phys.* **0811**, 006 (2008); J. Bijnens, P. Talavera, *Nucl. Phys.* **B669**, 341 (2003); M. Jamin, J.A. Oller, A. Pich, *J. High Energy Phys.* **0402**, 047 (2004); V. Cirigliano *et al., J. High Energy Phys.* **0504**, 006 (2005); D. Becirevic *et al., Nucl. Phys.* **B705**, 339 (2005); C. Dawson *et al., Phys. Rev.* **D74**, 114502 (2006); P.A. Boyle *et al., Phys. Rev. Lett.* **100**, 141601 (2008); V. Lubicz *et al.*, `arXiv:0906.4728`.

[3] W.J. Marciano, *PoS* **KAON**, 003 (2008).

[4] [FlaviaNet WG on Kaon Decays] M. Antonelli *et al.*, `arXiv:0801.1817`.

[5] N. Cabibbo, *Phys. Rev. Lett.* **10**, 531 (1963); M. Kobayashi, T. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).

[6] R. Barbieri *et al., Phys. Lett.* **B156**, 348 (1985); W.J. Marciano, A. Sirlin, *Phys. Rev.* **D35**, 1672 (1987); K. Hagiwara, S. Matsumoto, Y. Yamada, *Phys. Rev. Lett.* **75**, 3605 (1995); A. Kurylov, M.J. Ramsey-Musolf, *Phys. Rev. Lett.* **88**, 071804 (2002).

[7] J.C. Hardy, I.S. Towner, *Phys. Rev.* **C79**, 055502 (2009)

[8] M. Antonelli *et al.*, `arXiv:0907.5386[hep-ph]`.

[9] V. Cirigliano, M. Gonzalez-Alonso, J. Jenkins, *Nucl. Phys.* **B830**, 95 (2010) [`arXiv:0908.1754`].

[10] W. Buchmuller, D. Wyler, *Nucl. Phys.* **B268**, 621 (1986).

[11] R.S. Chivukula, H. Georgi, *Phys. Lett.* **B188**, 99 (1987); L.J. Hall, L. Randall, *Phys. Rev. Lett.* **65**, 2939 (1990); A.J. Buras *et al., Phys. Lett.* **B500**, 161 (2001); G. D'Ambrosio *et al., Nucl. Phys.* **B645**, 155 (2002).

[12] V. Cirigliano *et al., Nucl. Phys.* **B728**, 121 (2005).

[13] Z. Han, W. Skiba, *Phys. Rev.* **D71**, 075009 (2005).