Vol. 3 (2010) *Acta Physica Polonica B Proceedings Supplement* No 1

THREE-LOOP VERTEX CORRECTIONS: MASSLESS FERMION AND GLUON FORM FACTORS AND MASS EFFECTS IN THE HIGGS BOSON PRODUCTION AT LHC*

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(Received February 9, 2010)

In this contribution three-loop corrections to the quark and gluon form factor are discussed in massless QCD. They constitute building blocks to the third-order corrections of a number for physical processes. Furthermore, we discuss the Higgs–boson–gluon vertex to three-loop order including finite top-quark mass effects.

PACS numbers: 12.38.–t, 12.38.Bx, 14.65.Bt, 14.80.Bn

1. Introduction

One of the main tasks of the Large Hadron Collider (LHC) at CERN is the search for the Higgs boson which is responsible for the particle masses in the Standard Model and most of its extensions. The theoretical predictions are currently known up to next-to-next-to-leading order (NNLO), however, only in the heavy-top quark limit which is based the Lagrange density

$$\mathcal{L} \;=\; -\frac{H}{v} C_1 \left(G_{\mu\nu}\right)^2 \;, \tag{1}$$

where H is the Higgs boson field, v the vacuum expectation value, $G_{\mu\nu}$ the gluon field strength tensor and C_1 the coefficient function incorporating the residual top-quark mass dependence. Knowing the perturbative expansion of C_1 one can compute the Higgs production cross-section (including real and virtual corrections) with the help of Eq. (1) which — as compared to the calculation in the full theory — reduces the number of loops by one unit.

* Presented at the FLAVIAnet Topical Workshop, “Low energy constraints on extensions of the Standard Model”, Kazimierz, Poland, July 23–27, 2009.

In Sec. 2 we provide the virtual corrections of the Higgs–gluon coupling within the effective theory to the three-loop level, which constitutes together with the four-loop calculation of C_1 [1–3], the first step to the NNNLO corrections to the Higgs boson production in gluon fusion. Besides the gluon form factor, F_g, we discuss in Sec. 2 also the fermion form factor, F_q, *i.e.* three-loop corrections to the photon–quark vertex in massless QCD. The NNNLO results for both F_g and F_q have been obtained in Ref. [4].

In Sec. 3 we also consider the Higgs boson production in gluon–gluon fusion. This time we go beyond the effective theory of Eq. (1) and consider the Higgs–boson–gluon coupling in the full theory. In order to obtain NNLO corrections the evaluation of three-loop diagrams is required, which we evaluate in a series expansion for large top-quark mass. This calculation has been performed by two independent groups [5,6] and constitutes a first step to study the finite top quark mass effects at NNLO.

2. Three-loop fermion and gluon form factors

In this section we consider massless QCD (accompanied by the effective Lagrangian of Eq. (1)) and discuss the virtual corrections to the photon quark and Higgs–boson–gluon vertex (see also Ref. [7] and references therein). It is convenient to extract the tensor structure and define the form factors F_q and F_g

$$
\begin{aligned}
\Gamma_q^{\mu} &= \gamma^{\mu} F_q\left(q^2\right) , \\
\Gamma_g^{\mu\nu} &= \left(q_1 \cdot q_2\, g^{\mu\nu} - q_1^{\nu}\, q_2^{\mu}\right) F_g\left(q^2\right) ,
\end{aligned}
\tag{2}
$$

where $q = q_1 + q_2$ and q_1 (q_2) is the incoming (anti-)quark momentum in the case of F_q, and F_g depends on the gluon momenta q_1 and q_2 with polarization vectors $\varepsilon^{\mu}(q_1)$ and $\varepsilon^{\nu}(q_2)$. Note that $q_1^2 = q_2^2 = 0$. Some sample of Feynman diagrams contributing to F_q and F_g are shown in Fig. 1. Starting from three-loop level a new class of diagrams occurs, the so-called singlet diagrams, where the external photon is not connected to the fermion line involving the final-state quarks (see Fig. 1(b)). Since at three-loop level there are no counter term contributions to the singlet diagrams and furthermore there is no corresponding real emission contribution the sum of all diagrams has to be finite.

In a first step projectors are applied in order to obtain scalar expressions for the functions F_q and F_g. After the decomposition of the numerator in order to arrive at a minimal number of scalar products a reduction procedure is applied in order to express each occurring integral as a linear combination of master integrals. For our calculation we applied two different

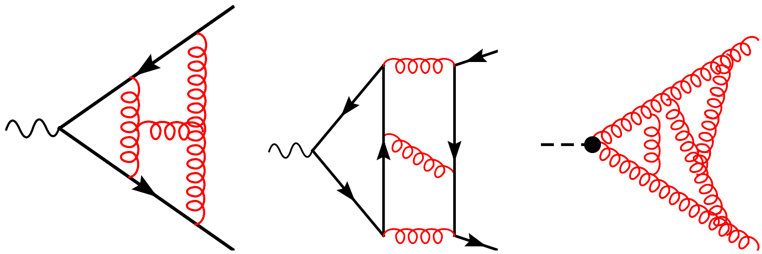

Fig. 1. Sample Feynman diagrams contributing to the F_q ((a) and (b)) and F_g (c) at three-loop level. Straight and curly lines denote quarks and gluons, respectively.

procedures. The first one has been described in Refs. [8–10], where an integral representation for the coefficient is provided. The integrals depend on the exponents of the denominators of the Feynman diagram and the space-time dimension d. In the recent years a procedure has been developed to evaluate the resulting parameter integrals in the limit of large d (see, *e.g.*, Ref. [11]). Knowing sufficiently many expansion terms the coefficient function can be reconstructed since (for fixed exponents) it is a rational function in d. The evaluation of the three-loop vertex corrections profited quite a lot from the experience gained in the context of the evaluation of the four-loop two-point functions [12] and the findings of Ref. [13]. In the latter paper it has been shown that the recurrence relations of n-loop three-point functions are equivalent to $(n+1)$-loop two-point functions.

The second method has only been applied to the singlet diagrams contributing to F_q. It relies on the idea to combine the Laporta method [14] with the Gröbner bases technique [15] which has been published in the computer code `FIRE` [16]. Needless to say, that for the contributions where both methods have been used complete agreement has been obtained.

We parameterize the results in terms of the bare coupling which allows us to factorize all occurring logarithms of the form $\ln(Q^2/\mu^2)$ where $Q^2 = -q^2 > 0$. Furthermore, we cast the results in the form $(x = q, g)$

$$F_x \;=\; 1 + \sum_n \left(\frac{\alpha_s}{4\pi}\right)^n \left(\frac{\mu^2}{Q^2}\right)^{n\epsilon} F_x^{(n)} \,. \tag{3}$$

We refrain from listing the results in term of general SU(3) colour factors, which can be found in Ref. [4], however, we present for illustration the finite[1] part of F_q in the case of QCD where it takes the form:

[1] We refer to Refs. [17, 18] for the divergent contribution.

$$
\begin{aligned}
F_q^{(3)}\Big|_{\rm finite} =& -\frac{15214694}{729} - \frac{1008569\zeta_2}{243} + \frac{910616\zeta_3}{81} + \frac{467815\zeta_4}{216} - \frac{69343\zeta_2\zeta_3}{81} \\
&+ \frac{160186\zeta_5}{135} - \frac{17851\zeta_3^2}{81} - \frac{43040081\zeta_6}{15552} + \frac{20X_{9,1}}{9} - \frac{8X_{9,2}}{9} + \frac{4X_{9,4}}{9} \\
&+ n_f\left(\frac{23311516}{6561} + \frac{524042\zeta_2}{729} - \frac{806584\zeta_3}{729} - \frac{57625\zeta_4}{243} + \frac{7760\zeta_2\zeta_3}{81}\right. \\
&\left. - \frac{107872\zeta_5}{405}\right) + n_f^2\left(- \frac{2710864}{19683} - \frac{248\zeta_2}{9} + \frac{12784\zeta_3}{729} - \frac{166\zeta_4}{81}\right) \\
&+ \frac{80}{9} + \frac{200\zeta_2}{9} + \frac{280\zeta_3}{27} - \frac{20\zeta_4}{9} - \frac{1600\zeta_5}{27}\,. \qquad (4)
\end{aligned}
$$

The term in the last line corresponds to the singlet contribution. The three constants $X_{9,i}$ take the numerical values $X_{9,1} \approx 1428.9963678666183591$, $X_{9,2} \approx 528.0583 \pm 0.0326$, and $X_{9,4} \approx -2085.380547 \pm 0.000025$, where $X_{9,1}$ is available analytically [19] while $X_{9,2}$ and $X_{9,4}$ are known numerically [4, 19], with the indicated precision.

As already mentioned above the new NNNLO results for the form factors constitute building blocks for a number of applications. Among them are the Higgs boson production in gluon fusion, the Drell–Yan process and the two-jet cross-section in e^+e^- collisions.

3. Finite top quark mass effects of the Higgs boson production

Also in this section we discuss corrections to the Higgs boson production. Again three-loop corrections are considered, however, not in the effective theory but in the full Standard Model including finite top quark mass effects at NNLO. Sample diagrams are shown in Fig. 2.

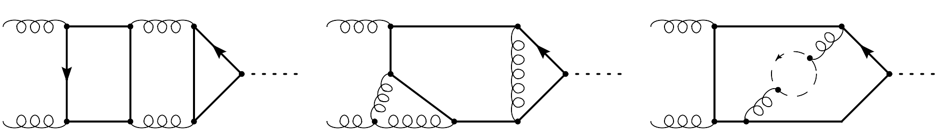

Fig. 2. Sample diagrams contributing to the NNLO virtual corrections to $gg \to h$.

The virtual contribution to the partonic cross-section can be cast in the form

$$
\hat\sigma_{ggh}^{\rm virt} = \hat\sigma_{\rm LO}\left(1 + \frac{\alpha_{\rm s}}{\pi}\,\delta^{(1)} + \left(\frac{\alpha_{\rm s}}{\pi}\right)^2\delta^{(2)} + \ldots\right)\,, \qquad (5)
$$

where the LO cross-section is given by

$$\hat{\sigma}_{\rm LO} \;=\; \frac{G_{\rm F}\;\alpha_{\rm s}^2}{288\sqrt{2}\pi}\,\frac{f_0(\rho,\epsilon)}{(1-\epsilon)}\,\delta(1-x)\,, \tag{6}$$

with $x = M_H^2/\hat{s}$, where $\sqrt{\hat{s}}$ is the partonic center-of-mass energy. The function f_0 and the analytical results of the first five terms in the $\rho = M_H^2/M_t^2 \to 0$ expansion for $\delta^{(1)}$ and $\delta^{(2)}$ can be found in Ref. [5] (see also Ref. [6]). We refrain from listing explicit results in this contribution but discuss the convergence properties in Fig. 3, where the finite part of $\delta^{(1)}$ and $\delta^{(2)}$ is shown as a function of ρ. One observes good convergence up to $\rho \approx 3$ which corresponds to $M_H \approx 1.7 M_t$.

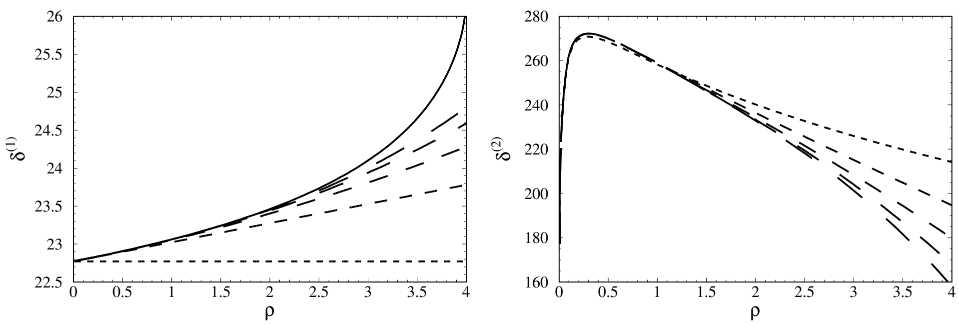

Fig. 3. Finite part of $\delta^{(1)}$ (left) and $\delta^{(2)}$ (right) as a function of ρ. The longer-dashed lines include successively higher orders in ρ and the solid line corresponds to the exact result for $\delta^{(1)}$.

The real corrections, which are necessary to obtain physical cross-sections, involve in addition to M_H^2/M_t^2 also the ratio $\hat{s}/M_t^2$. Whereas for Higgs boson masses in the intermediate-mass range the former ratio is small and an expansion is possible this is in general not true for the latter. An obvious way out is a clever combination of the expansion for $M_t \to \infty$ and for $\hat{s} \to \infty$ (the latter limit has been considered in Ref. [20]). Very recently the real corrections have been evaluated in Ref. [21] in an expansion around the soft limit and the "matching" of the $M_t \to \infty$ and $\hat{s} \to \infty$ results has been performed. Further work is in progress [22].

I would like to thank the organizers for the kind invitation and the pleasant atmosphere at the workshop. Furthermore, I thank P.A. Baikov, K.G. Chetyrkin, A. Pak, M. Rogal, A.V. Smirnov and V.A. Smirnov for the fruitful collaboration on the subjects presented in this contribution.
This work was supported by DFG through the Sonderforschungsbereich/Transregio 9 "Computational Particle Physics".

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