RELEVANCE OF ULTRA-SOFT GLUONS AND k_t RESUMMATION FOR TOTAL CROSS-SECTIONS*

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Inclusion of down to zero-momentum gluons and their k_t resummation is shown to quench the too fast rise of the mini jet cross-section and thereby obtain realistic total cross-sections.

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1. Introduction

We present details of the very high energy behavior of hadron–hadron and hadron–photon total cross-sections in a model which incorporates soft gluon k_t-resummation in the infrared (IR) region. The effects of this resummation are discussed to highlight the mechanisms responsible for the rate of the rise of the total cross-section with energy.

The Froissart–Martin (FM) bound [1, 2] says that the hadronic cross-sections cannot rise faster than $\ln^2 s$. The most common parametrizations of $\sigma_{\rm total}$ for pp processes, based on unitarity and constraints from analyticity, impose the FM bound. By contrast, in our QCD based model [3–5], we find an interesting relationship between the IR behavior of $\alpha_{\rm s}$ and the rise of $\sigma_{\rm total}$.

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We first present a short introduction to the soft gluon k_t-resummation technique, then the main features of our eikonal model based on QCD-minijet cross-section and soft gluon resummation, after which we compare our results with data from pp and γp processes. Finally, we show how k_t-resummation affects the asymptotic rise which, while respecting the FM bound, does not necessarily saturate it.

2. k_t-resummation and the IR limit

To compute the total cross-section one has to integrate over large values of the impact parameters b, which gets linked to the IR regions of the underlying partonic processes. Here we are considering (within a semi-classical approach) soft gluon emissions from the colliding partons and their k_t-resummation, whose Fourier transforms we assume to describe matter distribution within the colliding hadrons. The expression for the transverse momentum distribution resulting from soft gluon radiation emitted by the colliding quarks is

$$d^2P(\boldsymbol{K}_\perp) = d^2\boldsymbol{K}_\perp \frac{1}{(2\pi)^2} \int d^2\boldsymbol{b}\; e^{-i\boldsymbol{K}_\perp\cdot\boldsymbol{b} - h(b,M)} \tag{1}$$

with

$$h(b,M) = \frac{16}{3} \int_0^M \frac{\alpha_{\rm s}(k_t^2)}{\pi} \frac{dk_t}{k_t} \ln\frac{2M}{k_t} \left[1 - J_0(k_t b)\right], \tag{2}$$

where M gives the energy scale of the process, in our case it is the maximum allowed transverse momentum for a single gluon emission [6]. In various QCD problems [7, 8] one splits this expression in two terms

$$h(b,M) = c_0(\mu, b, M) + \Delta h(b,M), \tag{3}$$

with the first term parameterizing the infrared behavior and the second term, which is calculable in the perturbative regime, defined as

$$\Delta h(b,M) = \frac{16}{3} \int_\mu^M \frac{\alpha_{\rm s}(k_t^2)}{\pi} \left[1 - J_o(bk_t)\right] \frac{dk_t}{k_t} \ln\frac{2M}{k_t}. \tag{4}$$

Since this integral only runs down to a scale $\mu > \Lambda_{\rm QCD}$, one uses the asymptotic freedom expression for $\alpha_{\rm s}$ and one assumes that $J_o(bk_t)$ oscillates to zero. In the range of $1/M < b < 1/\Lambda$ an effective h is obtained fixing the scale $\mu = 1/b$. Then one finds [9]

$$e^{-h_{\rm eff}(b,M)} = \left[\frac{\ln(1/b^2\Lambda^2)}{\ln(M^2/\Lambda^2)}\right]^{(16/25)\ln(M^2/\Lambda^2)}. \tag{5}$$

In our model, the integral (4) includes zero momentum values and we investigate their contribution in the very large impact parameter region. Since in the IR limit, the perturbative expression for $\alpha_{\rm s}$ is not valid, we employ a phenomenological expression [3], which is singular but integrable in the IR limit of the soft gluon integral

$$\alpha_{\rm s}(k_t^2) = \frac{12\pi}{(33-2N_f)} \frac{p}{\ln\left[1+p\left(\frac{k_t^2}{\Lambda^2}\right)^p\right]}, \tag{6}$$

with the parameter $1/2 < p < 1$. For $p = 1$ it coincides with the Richardson potential [10]. In the large-b region ($b > \frac{1}{N_p \Lambda} > \frac{1}{M}$), we can obtain an approximate analytic expression:

$$\begin{aligned} h(b,M,\Lambda) \;=\; & \frac{8}{3\pi}\left\{ \frac{\bar{b}}{8(1-p)}(b^2\Lambda^2)^p \left[2\ln(2Mb)+\frac{1}{1-p}\right] \right. \\ & + \frac{\bar{b}}{2p}(b^2\Lambda^2)^p \left[2\ln(Mb)-\frac{1}{p}\right] + \frac{\bar{b}}{2pN_p^{2p}}\left[-2\ln\frac{M}{\Lambda N_p}+\frac{1}{p}\right] \\ & \left. + \bar{b}\ln\frac{M}{\Lambda}\left[\ln\frac{\ln\frac{M}{\Lambda}}{\ln N_p} - 1 + \frac{\ln N_p}{\ln\frac{M}{\Lambda}}\right]\right\}, \end{aligned} \tag{7}$$

where $N_p = (1/p)^{1/2p} > 1$ for $p < 1$, and $\bar{b} = 12\pi/(33-2N_f)$.

Our soft gluon resummation approach allows us to obtain a value for the "intrinsic" transverse momentum [11], through

$$h_{\rm intrinsic} = \frac{16}{3}\int_0^{\Lambda} \frac{\alpha_{\rm s}(k_t^2)}{\pi}\frac{dk_t}{k_t}\ln\frac{2M}{k_t}[1-J_0(k_t b)] \approx \frac{b^2}{4}\left\langle k_t^2\right\rangle_{\rm int} \tag{8}$$

with

$$\left\langle k_t^2\right\rangle_{\rm int} = \frac{8}{3\pi}\frac{\bar{b}}{(1-p)}\left(\frac{1}{2(1-p)}+\ln\frac{2M}{\Lambda}\right)\Lambda^2, \tag{9}$$

which corresponds to an intrinsic transverse momentum of a few hundred MeV for Λ in the 100 MeV range and $M \leq 1$ GeV.

3. The Bloch–Nordsieck model for total cross-sections

In our model for the total hadronic cross-section we use the eikonal formalism which implies multiple scattering and requires impact parameter distributions of the scattering particles [3]. In the high energy limit, neglecting the real part of the eikonal, the total cross-section can be approximated as

$$\sigma_{\rm total} = 2\int d^2\boldsymbol{b}\left[1-e^{-\mathcal{I}{\rm m}\chi(b,s)}\right]. \tag{10}$$

The imaginary part of the eikonal is related to the average number of inelastic collisions $n(b,s)$ as

$$2\mathcal{I}\mathrm{m}\chi(b,s) = \bar{n}(b,s) \tag{11}$$

for which we use the expression

$$\bar{n}(b,s) = n_{\mathrm{NP}}(b,s) + n_{\mathrm{hard}}(b,s)\,, \tag{12}$$

with the non perturbative (NP) term relevant only for the low-energy description of σ_{tot} and for its normalization, and the hard one (hard) responsible for the high-energy rise. This term is given by

$$n_{\mathrm{hard}}(b,s) = A(b,s)\sigma_{\mathrm{jet}}(s)\,, \tag{13}$$

where σ_{jet} is the mini-jet cross-section describing high-energy partonic collisions. It drives the rise of the total cross-section and depends on the parameter p_{tmin}, the minimum transverse momentum of the scattered partons which separates hard parton–parton scattering from all other low-p_t processes, and on currently used, DGLAP evoluted, parametrizations for the Partonic Density Functions.

$A(b,s)$ is the overlap function given by the Fourier transform of the transverse momentum distribution resulting from initial state soft gluon radiation, which breaks the collinearity of the colliding partons, making the

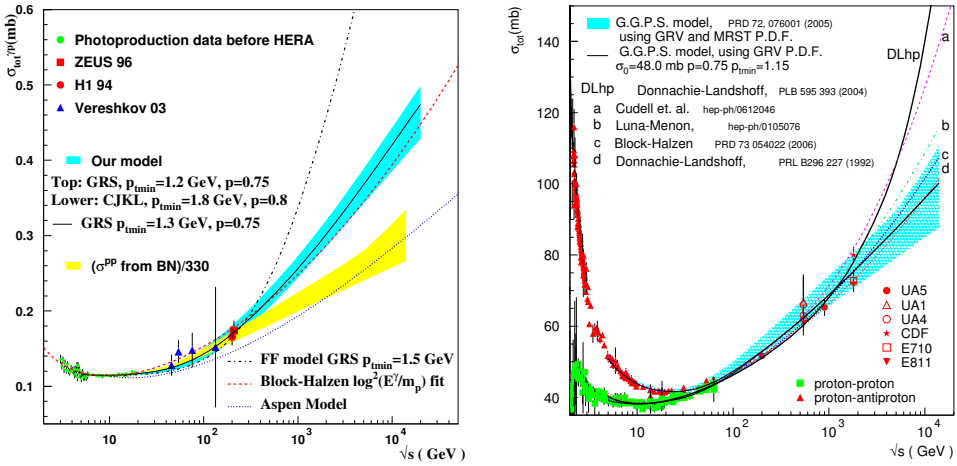


Fig. 1. γp total cross-section (left) from [5] and pp total cross-section (right) from [4] are shown compared with data and with other phenomenological models [12, 13].

scattering process less efficient. We put

$$A_{\rm BN}(b,s) \;=\; N\int d^2\boldsymbol{K}_\perp\, e^{-i\boldsymbol{K}_\perp\cdot b}\;\frac{d^2P(\boldsymbol{K}_\perp)}{d^2\boldsymbol{K}_\perp} = \frac{e^{-h(b,\,q_{\rm max})}}{\int d^2\boldsymbol{b}\; e^{-h(b,\,q_{\rm max})}}$$
$$= \; A_0(s)e^{-h(b,\,q_{\rm max})}\,, \tag{14}$$

where the parameter $q_{\rm max}$ is linked to the maximum transverse momentum allowed by the kinematics for single gluon emission [6]. It should be calculated for every partonic subprocess but we make the simplifying assumption of assigning it a value averaged over all the possible subprocesses [3] and use it in Eq. (13) to calculate the rising part of $\sigma_{\rm total}$.

Fig. 1 shows our results for proton–proton (band) compared with other models [3, 12]. In this case, our central prediction (black line in the band) has been obtained with $p_{t\rm min} = 1.15\,{\rm GeV}$, $p = 0.75$ and GRV densities. Application of the model to γp [5] is shown on left panel.

4. Asymptotic limit of the total cross-section

We can now estimate the very large s-limit when the minijet cross-section rises asymptotically like a power s^ε ($\varepsilon \approx 0.3$ [14]). We find

$$n_{\rm hard}(b,s) = A_{\rm BN}(b,s)\sigma_{\rm jet}(s,\,p_t{\rm min}) \sim A_0(s)e^{-h(b,\,q_{\rm max})}\sigma_1\left(\frac{s}{s_0}\right)^\varepsilon. \tag{15}$$

From it, we can deduce the very large b-limit to be

$$n_{\rm hard}(b,s) \sim A_0(s)\sigma_1 e^{-(b\bar{\Lambda})^{2p}}\left(\frac{s}{s_0}\right)^\varepsilon, \tag{16}$$

with [9]

$$\bar{\Lambda} \equiv \bar{\Lambda}(b,s) = \Lambda\left\{\frac{\bar{b}}{3\pi(1-p)}\left[\ln(2q_{\rm max}(s)b) + \frac{1}{1-p}\right]\right\}^{1/2p}, \tag{17}$$

where $A_0(s) \propto \Lambda^2$, and $q_{\rm max}$ a very slowly varying function of s. In this limit, the total cross-section becomes

$$\sigma_{\rm T}(s) \approx 2\pi\int\limits_0^\infty db^2\left[1 - e^{-C(s)e^{-(b\bar{\Lambda})^{2p}}}\right] \tag{18}$$

with $2C(s) = A_0(s)\sigma_1(s/s_0)^\varepsilon$. We thus obtain

$$\sigma_{\rm T} \approx ({\rm const.})\left[\ln\frac{s}{s_0}\right]^{1/p}. \tag{19}$$

The constant in Eq. (19) depends on Λ and $q_{\max}$ [9] and is about one order of magnitude smaller than the constant which appears in the Martin bound. We thus find that the introduction of soft gluon resummation introduces a new scale which imposes a more stringent limit on the high energy behavior of σ_{total}.

5. Conclusions

In an eikonal mini-jet model, soft gluon k_t-resummation down to zero gluon momenta does reduce the strong power-like rise of the minijet cross-section. We find a link between the infrared region in soft k_t-resummation and the very high energy behaviour of total cross-sections. Introduction of a dynamical scale related to the very soft gluon emission, reduces the constant occurring in the Martin bound.

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