HADRONIC FORM FACTORS AND $V_{\rm CKM}$ DETERMINATION*

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I present a new determination of $|V_{cs}|$ and $|V_{cd}|$, using the latest CLEO data on semileptonic D decays and the $D \to \pi$ and $D \to K$ form factors obtained from QCD light-cone sum rules. This result emphasizes the universality of the method used before to calculate the $B \to \pi$ form factor and determine $|V_{ub}|$ from $B \to \pi l \nu$ decay.

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1. Introduction

The elements of the CKM matrix entering the flavour-changing quark weak current in the Standard Model:

$$j_\mu^W = (\bar{u}, \bar{c}, \bar{t})_{\rm L} V_{\rm CKM} \gamma_\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\rm L} , \tag{1}$$

can only be accessed via observable hadronic flavour-changing transitions.

Recent measurements of the semileptonic $D \to \pi \ell \nu_\ell$ and $D \to K \ell \nu_\ell$ decays by the CLEO Collaboration [1, 2] provide new accurate data on the decay rate distributions in bins of the variable q^2 (invariant mass squared of the lepton pair), yielding the products of transition form factors and CKM matrix elements, $|V_{cd} f^+_{D\pi}(q^2)|$ and $|V_{cs} f^+_{DK}(q^2)|$, where the form factors are defined in a standard way:

$$\langle \pi(K)(p) | j_\mu^W | D(p+q) \rangle = V_{cd(s)} f^+_{D\pi(K)}(q^2)(2p+q)_\mu + \dots . \tag{2}$$

Aiming at a more accurate determination of $|V_{cd}|$ and $|V_{cs}|$, we recently revisited and updated [3] the light-cone sum rule (LCSR) calculation of the

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$D \to \pi$ and $D \to K$ form factors. We used the same procedure and input as in our previous analysis of the $B \to \pi$ form factors [4] for the $|V_{ub}|$ determination, hence an independent check of the method is provided.

2. Outline of LCSR calculation

To apply the standard LCSR method [5, 6] to the calculation of the $D \to \pi$ form factor $f^+_{D\pi}$, one considers the correlation function of $j_\mu = \bar{d}\gamma_\mu c$ and $j_5 = m_c \bar{c} i\gamma_5 u$ currents:

$$F_\mu(p,q) = \int d^4x e^{iqx} \langle \pi(p)|T\{j_\mu(x) j_5(0)\}|0\rangle = F((p+q)^2, q^2)p_\mu + \dots \,, \quad (3)$$

isolating the appropriate Lorentz-structure. The invariant amplitude F is calculated in terms of the light-cone operator product expansion (OPE):

$$F_{\rm OPE}((p+q)^2,q^2) = \sum_{t=2,3,4} \int du \; C_t((p+q)^2, q^2, u) \otimes \varphi_\pi^{(t)}(u) \,, \quad (4)$$

valid at $(p+q)^2, q^2 \ll m_c^2$, that is, far from the physical thresholds. The universal elements of this OPE are the pion distribution amplitudes (DAs) $\varphi_\pi^{(t)}$ of twist t, accumulating nonperturbative effects, whereas the process-dependent coefficients C_t are perturbatively calculable. To obtain the sum rule, the result of (4) is matched to the hadronic dispersion relation at $(p+q)^2 \ll m_c^2$:

$$F_{\rm OPE}((p+q)^2,q^2) = \frac{2m_D^2 f_D f^+_{D\pi}(q^2)}{m_D^2 - (p+q)^2} + \frac{1}{\pi}\int\limits_{s_0^D}^{\infty} ds \frac{{\rm Im} F_{\rm OPE}(s,q^2)}{s-(p+q)^2} \,, \quad (5)$$

with the subsequent Borel transformation $(p+q)^2 \to M^2$. In the above, the form factor (2), multiplied by the D decay constant, $\langle 0|j_5|D\rangle = m_D^2 f_D$ enters the ground-state pole term, whereas quark–hadron duality is used to estimate the contribution of higher states. This approximation introduces a threshold parameter s_0^D which is estimated by fitting the differentiated LCSR to the measured D-meson mass value. Other important details of this calculation can be found in [3, 4]. Actually, we benefit from the universality of the method, employing the perturbative coefficients C_t from LCSR for the $B \to \pi$ form factor calculated earlier in [4, 6, 22], replacing $m_b \to m_c$ and adjusting the normalization and Borel scales correspondingly.

The analogous sum rule for the $D \to K$ form factor contains, in comparison to (5), nontrivial SU(3) breaking effects proportional to m_s, such as the asymmetry in the twist-2 kaon DA, calculated recently with an improved accuracy in [7].

The LCSR for $f^+_{D\pi}(q^2)$ and $f^+_{\rm DK}(q^2)$ are obtained at finite m_c, in the $\overline{MS}$ scheme, using recent very accurate estimates $\bar{m}_c(\bar{m}_c) = (1.29 \pm 0.03)$ GeV [8] from charmonium QCD sum rule [9]. The higher twists in LCSR are suppressed by the powers of $O(\Lambda_{\rm QCD}/m_b)$ or, at least of $O(\Lambda_{\rm QCD}/\tau)$, where $\tau \gg \Lambda_{\rm QCD}$ is an intermediate large scale originating from $M^2 \sim \tau m_c$. For the light quark masses we use $m_s(\mu = 2\,{\rm GeV}) = (98 \pm 16)\,{\rm MeV}$ from QCD sum rules in $O(\alpha_s^4)$ [10] and obtain $m_{u,d}$ from Leutwyler relations in ChPT [11]. This fixes the normalization parameters $\mu_\pi = m_\pi^2/(m_u + m_d)$ and $\mu_K = m_\pi^2/(m_u + m_s)$ of the twist-3 DAs. We also use the same pion twist-2 DA $\varphi_\pi^{(2)}(u)$ as in [4], where its main parameters were constrained by fitting the shape of $B \to \pi$ form factor predicted from LCSR to the experimentally measured q^2 distribution in $B \to \pi l \nu_l$. In $\varphi_K^{(2)}(u)$, the asymmetry parameter a_1 is taken from [7] and twist 3,4 pion and kaon DAs are taken from [12].

Importantly, LCSR is the only analytical approach which reproduces both "soft" and "hard" contributions to the form factor. The soft contribution naturally dominates, having no α_s suppression in the sum rule.

3. Short summary of the results

The $D \to \pi, K$ form factors at $q^2 = 0$ obtained in [3] are compared to the lattice QCD and previous LCSR results in the following table, demonstrating a good agreement between the two nonperturbative methods:

Method	[Ref.]	$f^+_{D\pi}(0)$	$f^+_{\rm DK}(0)$
Lattice QCD	[13]	$0.57 \pm 0.06 \pm 0.02$	$0.66 \pm 0.04 \pm 0.01$
	[14]	$0.64 \pm 0.03 \pm 0.06$	$0.73 \pm 0.03 \pm 0.07$
	[15]	$0.74 \pm 0.06 \pm 0.04$	$0.78 \pm 0.05 \pm 0.04$
LCSR	[16]	0.65 ± 0.11	$0.78^{+0.2}_{-0.15}$
	[17]	0.63 ± 0.11	0.75 ± 0.12
This work	[3]	$0.67^{+0.10}_{-0.07}$	$0.75^{+0.11}_{-0.08}$

The accuracy of our form factor calculation, estimated at the $\sim \pm 15\%$ level, is limited by the truncated twist expansion, uncertainties of quark masses, scales and pion and kaon DAs.

The region of accessible q^2 is restricted: $f^+_{D\pi,K}(q^2)$ can only be calculated at $q^2 \simeq 0$, as opposed to the LCSR calculation of $f^+_{B\pi}(q^2)$ which is valid in a wider region, at $q^2 \ll (m_B - m_\pi)^2$. To enlarge the region of available momentum transfers, we combined our calculation at $q^2 \leq 0$ with the conformal mapping parameterization and analytical continuation (see [3] for references and details). The results are in a good agreement with the measured shapes of both $D \to \pi$ and $D \to K$ form factors.

4. Determination of $|V_{cd}|$ and $|V_{cs}|$

The latest CLEO measurements of semileptonic charm decays [2] yield the product:

$$f_{D\pi}(0)|V_{cd}| = 0.150 \pm 0.004 \pm 0.001\,. \tag{6}$$

The CKM matrix element $|V_{cd}|$ can now be determined using our predictions for the form factor $f_{D\pi}(0)$. For the D-meson decay constant we use the CLEO result [18] from D leptonic decay (without the additional assumption $|V_{cd}| = |V_{us}|$):

$$f_D|V_{cd}| = 46.4 \pm 2.0 \text{ MeV}\,. \tag{7}$$

This input allows to extract $|V_{cd}|$ with less theory uncertainty than in previous analyses, where a QCD sum rule prediction for f_D was used. The product of the above two experimental numbers is then divided by the LCSR prediction, yielding $|V_{cd}|^2$, and our final result is:

$$|V_{cd}| = 0.225 \pm 0.005 \pm 0.003\ ^{+0.016}_{-0.012}\,, \tag{8}$$

where the first, second and third errors correspond to the experimental errors in (7), (6) and to the uncertainty of LCSR, respectively. Our result is in a good agreement with the value determined in [2] by using the lattice QCD value of $f^+_{D\pi}(0)$ from [14]. This agreement is not surprising because the form factor obtained from LCSR is close to the lattice result (see the table above).

Furthermore, we determine the ratio of $|V_{cd}|$ to $|V_{cs}|$, dividing (6) by

$$f_{DK}(0)|V_{cs}| = 0.719 \pm 0.006 \pm 0.005\,, \tag{9}$$

obtained from $D \to Ke\nu_e$ data fit [2]. Using the ratio $f_{DK}(0)/f_{D\pi}(0)$ calculated from LCSR, we obtain:

$$\frac{|V_{cd}|}{|V_{cs}|} = 0.236 \pm 0.006 \pm 0.003 \pm 0.013\,, \tag{10}$$

where the first and second uncertainties are due to the combined (in quadratures) errors in (6) and (9), respectively, and the third uncertainty stems from the LCSR calculation. Our determinations (8) and (10) are consistent with $|V_{cd}| = |V_{us}|$ and $|V_{cs}| = |V_{ud}|$.

5. Discussion

The calculation of $D \to \pi, K$ form factors described above ensures more confidence in the previous LCSR result for the $B \to \pi$ form factor [4]. In the following table the main outcome of the latter, that is, the determination of $|V_{ub}|$ from $B \to \pi l \nu_l$ data is presented, together with the previous LCSR and lattice QCD results.

[Ref.]	$f^+_{B\pi}(q^2)$ calculation	$f^+_{B\pi}(q^2)$ input	$\lvert V_{ub}\rvert \times 10^3$
[19]	lattice	—	3.38±0.36
[20]	lattice	—	3.55±0.25±0.50
[22]	LCSR	—	$3.5 \pm 0.4 \pm 0.1$
[21]	—	lattice $\oplus$ LCSR	$3.47 \pm 0.29 \pm 0.03$
[4]	LCSR	—	$3.5 \pm 0.4 \pm 0.2 \pm 0.1$
[23]	—	lattice $\oplus$ LCSR	3.54 ± 0.24

Furthermore, one can apply LCSR to calculate the spacelike pion e.m. form factor and $\gamma \to \pi$ form factor at large Q^2. Both form factors are quite sensitive to the twist-2 pion DA, hence comparison of the calculated and measured form factors provides a nontrivial check of this DA used in our heavy-light form factor calculations. As recently discussed in [24], the pion e.m. form factor from LCSR [25] is in a very good agreement with experimental data, whereas the $\gamma \to \pi$ form factor using the LCSR-like method [26], well reproduces data only at the photon virtuality $Q^2 < 15$ GeV^2. At larger Q^2 a common problem for all methods is to explain the very recent BaBar data [27].

QCD sum rules [9] and LCSR [5] remain reliable "non-lattice" tools for various heavy-light hadronic matrix elements, although the precision of both methods has probably already reached the limit. The duality approximation introduces a systematic uncertainty which can only be diminished by more input on the excited states in the hadronic sum. Note that OPE for the correlation functions provides not only sum rules but also useful bounds, which are independent of the duality ansatz and simply follow from the positivity of the spectral function. An example is provided by the upper bounds [28] for the decay constants f_D and f_{D_s}.

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REFERENCES

[1] J.Y. Ge *et al.* [CLEO Collaboration], *Phys. Rev.* **D79**, 052010 (2009) [`arXiv:0810.3878[hep-ex]`].

[2] D. Besson [CLEO Collaboration], *Phys. Rev.* **D80**, 032005 (2009) [`arXiv:0906.2983[hep-ex]`].

[3] A. Khodjamirian, C. Klein, T. Mannel, N. Offen, *Phys. Rev.* **D80**, 032005 (2009) [`arXiv:0907.2842[hep-ph]`].

[4] G. Duplancic, A. Khodjamirian, T. Mannel, B. Melic, N. Offen, *J. High Energy Phys.* **0804**, 014 (2008).

[5] I.I. Balitsky, V.M. Braun, A.V. Kolesnichenko, *Nucl. Phys.* **B312**, 509 (1989); V.M. Braun, I.E. Filyanov, *Z. Phys.* **C44**, 157 (1989); V.L. Chernyak, I.R. Zhitnitsky, *Nucl. Phys.* **B345**, 137 (1990).

[6] V.M. Belyaev, A. Khodjamirian, R. Rückl, *Z. Phys.* **C60**, 349 (1993); V.M. Belyaev, V.M. Braun, A. Khodjamirian, R. Rückl, *Phys. Rev.* **D51**, 6177 (1995); A. Khodjamirian, R. Rückl, S. Weinzierl, O.I. Yakovlev, *Phys. Lett.* **B410**, 275 (1997); E. Bagan, P. Ball, V.M. Braun, *Phys. Lett.* **B417**, 154 (1998).

[7] K.G. Chetyrkin, A. Khodjamirian, A.A. Pivovarov, *Phys. Lett.* **B661**, 250 (2008).

[8] J.H. Kuhn, M. Steinhauser, C. Sturm, *Nucl. Phys.* **B778**, 192 (2007); R. Boughezal, M. Czakon, T. Schutzmeier, *Phys. Rev.* **D74**, 074006 (2006); J.H. Kuhn, *Acta Phys. Pol. B, Proc. Suppl.* **3**, 171 (2010), this issue.

[9] M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, *Nucl. Phys.* **B147**, 385, 448 (1979).

[10] K.G. Chetyrkin, A. Khodjamirian, *Eur. Phys. J.* **C46**, 721 (2006); M. Jamin, J.A. Oller, A. Pich, *Phys. Rev.* **D74**, 074009 (2006).

[11] H. Leutwyler, *Phys. Lett.* **B378**, 313 (1996).

[12] P. Ball, V.M. Braun, A. Lenz, *J. High Energy Phys.* **0708**, 090 (2007).

[13] A. Abada *et al. Nucl. Phys.* **B619**, 565 (2001).

[14] C. Aubin *et al.* [Fermilab Lattice Collaboration, MILC Collaboration and HPQCD Collaboration], *Phys. Rev. Lett.* **94**, 011601 (2005); C. Bernard *et al.*, *Phys. Rev.* **D80**, 034026 (2009) [`arXiv:0906.2498[hep-lat]`].

[15] A. Al-Haydari *et al.* [QCDSF Collaboration], *Eur. Phys. J.* **A43**, 107 (2010) [`arXiv:0903.1664[hep-lat]`].

[16] A. Khodjamirian, R. Ruckl, S. Weinzierl, C.W. Winhart, O.I. Yakovlev, *Phys. Rev.* **D62**, 114002 (2000).

[17] P. Ball, *Phys. Lett.* **B641**, 50 (2006).

[18] B.I. Eisenstein *et al.* [CLEO Collaboration], *Phys. Rev.* **D78**, 052003 (2008).

[19] J.A. Bailey *et al.*, *Phys. Rev.* **D79**, 054507 (2009).

[20] E. Dalgic *et al.*, **D73**, 074502 (2006) [Erratum *Phys. Rev.* **D75**, 119906 (2007)].

[21] J.M. Flynn, J. Nieves, *Phys. Rev.* **D76**, 031302 (2007).

[22] P. Ball, R. Zwicky, *Phys. Rev.* **D71**, 014015 (2005).

[23] C. Bourrely, I. Caprini, L. Lellouch, *Phys. Rev.* **D79**, 013008 (2009).

[24] A. Khodjamirian, `arXiv:0909.2154[hep-ph]`.

[25] V.M. Braun, A. Khodjamirian, M. Maul, *Phys. Rev.* **D61**, 073004 (2000); J. Bijnens, A. Khodjamirian, *Eur. Phys. J.* **C26**, 67 (2002).

[26] A. Khodjamirian, *Eur. Phys. J.* **C6**, 477 (1999).

[27] B. Aubert [The BaBaR Collaboration], *Phys. Rev.* **D80**, 052002 (2009) [`arXiv:0905.4778[hep-ex]`].

[28] A. Khodjamirian, *Phys. Rev.* **D79**, 031503 (2009).