

SOFT PHOTONS IN SEMILEPTONIC $B \rightarrow D$ DECAYS*

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Determination of V_{cb} in exclusive semileptonic decays is crucial consistency check against the V_{cb} determined inclusively. Anticipated precision of V_{cb} at the Super Flavor factory is $\sim 1\%$, with most of the theoretical error due to hadronic form factor uncertainties, but at this level of precision treating electromagnetic effects becomes inevitable. In addition to virtual photon corrections there are also emissions of real photons which are soft enough to avoid detection. The bremsstrahlung part is completely universal and is accounted for in the experimental analyses. However, the so-called *structure dependent* contribution, which probes the hadronic content of the process and is infrared finite, has been neglected so far. To this end, we estimated fraction of radiative events which are seen as ordinary semileptonic by experiment.

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1. Introduction

Many efforts have been devoted to experimentally check the validity of Kobayashi–Maskawa mechanism which predicts that all quark flavor observables agree with the unitary CKM matrix and single CP violating phase. If correct, the KM mechanism states that either measuring sides or angles of the unitarity triangle, the apex $(\bar{\rho}, \bar{\eta})$ comes out unique. Value of V_{cb} determines lengths of the sides adjacent to the apex, among them also the side opposite to angle β which is precisely known from time-dependent CP

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asymmetry in $B \rightarrow J/\psi K$. Current average of inclusive and exclusive determinations is [1]

$$|V_{cb}| = (41.2 \pm 1.1) \times 10^{-3}, \quad (1.1)$$

where $|V_{cb}|_{\text{excl}} = (38.6 \pm 1.3) \times 10^{-3}$ is significantly lower than $|V_{cb}|_{\text{incl}} = (41.6 \pm 0.6) \times 10^{-3}$ (p -value of the fit is 0.03). Common lore is that most important unknown in the exclusive method stems from the $B \rightarrow D$ form factors uncertainties and detection efficiencies. Although inclusive analyses are under better control theoretically and consequently result in more precise result, exclusive method provides a crucial cross-check, since errors are believed to be largely independent for both methods. In future expectations for the exclusive precision is of order 1% which could be obtained at Super Flavor factory [2].

2. V_{cb} from $B \rightarrow D\ell\nu$

Differential rate of exclusive decay to pseudoscalar D is

$$\frac{d\Gamma}{dw}(B \rightarrow D\ell\nu) = \frac{G_F^2 V_{cb}^2}{48\pi^3} (m_B + m_D)^2 m_D^3 (w^2 - 1)^{3/2} \mathcal{G}(w)^2, \quad (2.1)$$

where $w = v \cdot v'$ is the scalar product of meson four-velocities. Heavy quark symmetry normalizes the form factor $\mathcal{G}(w)$ at the kinematical point $w = 1$ where final state D meson is at rest in the B rest frame. Symmetry breaking corrections, perturbative α_s , α_{em} and nonperturbative $(\Lambda_{\text{QCD}}/m_b)^n$ were also computed and are under control at the maximum recoil point. However, further theoretical insight is required to isolate the value of V_{cb} . Close to the zero-recoil point, allowed phase space becomes scarce and there are practically no recorded events there. So to infer the experimental value of $V_{cb} \times \mathcal{G}(1)$ one has to rely on a particular shape (parameterization) of the form factor to guide the extrapolation down to $w = 1$. In experimental literature it has become customary to use so-called CLN shapes of the form factors [3] which rely on analyticity and unitarity. Measured differential decay rate is then fitted with $V_{cb} \times \mathcal{G}(1)$ and slope ρ^2 of the form factor at $w = 1$. In the end theoretical prediction of $\mathcal{G}(1)$ is used to determine V_{cb} .

Measuring semileptonic $B \rightarrow D\ell\nu$ in e^+e^- collider operating at $\Upsilon(4s)$ resonance one can focus only on events where the tag side momentum is completely reconstructed and ensure that missing invariant mass is peaking at zero, as anticipated for a single neutrino in final state. In order not to sacrifice statistics too much kinematical constraints are applied with some tolerance (invariant mass of the tag side is 5.27–5.29 GeV for decay of B^- , see [4]) which allows the soft photon events to be included among the semileptonic events. In this study we set out to study radiative corrections

of semileptonic decay $B \rightarrow D\ell\nu$ and in particular what is the effect of the structure dependent (SD) terms depending on photon energy resolution of the experiment. We will keep only the lowest pole contributions in our treatment as they turn out to contribute dominantly due to kinematics. Similar studies were carried out for K meson semileptonic decays using chiral perturbation theory and as it had turned out, SD part was negligible for a typical experimental setup [7]. On the contrary, SD amplitude of $B \rightarrow \mu\nu\gamma$ leads to 20% systematic error in a typical experiment measuring $\text{Br}(B \rightarrow \mu\nu)$ [5].

3. Infrared electromagnetic corrections

Electromagnetic effects render all experimentally measured widths a sum of rate of specific process plus rates of radiative events with final or initial state photons which cannot be resolved by the experiment. Such inclusive and infrared (IR) safe quantity is *e.g.*

$$d\Gamma_{\text{exp}}(i \rightarrow f) = d\Gamma(i \rightarrow f) + d\Gamma(i \rightarrow f\gamma)_{E_\gamma < E_{\text{cut}}} + \dots \quad (3.1)$$

The above inclusive width solves the IR problem of electrodynamics by canceling soft divergences due to virtual photon corrections against real emission. The amplitude of the so-called inner bremsstrahlung (IB) diverges as an IB photon energy approaches zero and residue of the pole is fixed by the charge of the external leg where the photon is coming from. In the IR limit emitted photons can only resolve total charge of the emitting particle. Accordingly, Low's theorem states that leading two terms in momentum expansion of the radiative decay width are given by the value and derivative of the nonradiative decay [6]. These IR divergences are compensated by the corresponding virtual corrections at the same order of α_{EM} at the level of decay width.

However, there are also subleading, IR finite, terms in the $d\Gamma_{\text{exp}}$ which are usually neglected in experimental analyses. These SD photon emissions can resolve structure of charged particles. Consequently, prediction of SD terms require knowledge of additional form factors.

3.1. Amplitude

Framework established in [7] for semileptonic K decays turns out very handy. Decay amplitude of $B^- \rightarrow D^0\ell\nu\gamma$ is

$$\mathcal{A}_\mu = \frac{eG_F V_{cb}}{\sqrt{2}} \bar{u}(p_\ell) \left(-\frac{F_\nu(t)}{2p_\ell \cdot q} \gamma_\mu (\not{p}_\ell + \not{q} + m_\ell) + V_{\mu\nu} - A_{\mu\nu} \right) \gamma^\nu (1 - \gamma_5) v(k) \quad (3.2)$$

which is then contracted with photon polarization. Here q , k , and p_ℓ are the photon, neutrino, and lepton momenta, respectively. First term in brackets which is proportional to

$$F_\nu(t) = i \langle D(p') | H_\nu | B(p) \rangle, \quad t = (p - p')^2 \quad (3.3)$$

represents photon emitted from lepton leg, while the $V_{\mu\nu}$ and $A_{\mu\nu}$ are the hadronic vector and axial form factors of $B \rightarrow D\gamma$ transition, namely when photon is emitted from a hadronic line

$$V_{\mu\nu} - A_{\mu\nu} = \int d^4y e^{iq \cdot y} \langle D(p') | T [J_\mu(y) H_\nu(0)] | B(p) \rangle, \quad (3.4)$$

$$H^\nu = \bar{c} \gamma^\nu (1 - \gamma_5) b.$$

Here J_μ is the electromagnetic current. These form factors obey the electromagnetic Ward identities

$$q^\mu V_{\mu\nu} = F_\nu(t), \quad (3.5)$$

$$q^\mu A_{\mu\nu} = 0, \quad (3.6)$$

which ensure total amplitude is gauge invariant. Intermediate 1-particle resonances give rise to poles due to excited beauty and charm states. The soft photon part of phase space should be well approximated by lowest pole contributions due to B , B^* and D^* , where the B -pole satisfies the inhomogeneous Ward identity above. It is IR divergent and thus qualifies as IB. We split the vector form factor into SD and IB pieces

$$V_{\mu\nu}^{\text{IB}} = \frac{p_\mu}{p \cdot q} F_\nu(t),$$

$$V_{\mu\nu}^{\text{SD}} = V_{\mu\nu} - V_{\mu\nu}^{\text{IB}}, \quad q^\mu V_{\mu\nu}^{\text{SD}} = 0.$$

Lorentz covariance and Ward identity allow the form factors to be split down into eight scalar functions

$$V_{\mu\nu}^{\text{SD}} = V_1 (p'_\mu q_\nu - p' \cdot q g_{\mu\nu}) + V_2 (p_\mu q_\nu - p \cdot q g_{\mu\nu})$$

$$+ (p \cdot q p'_\mu - p' \cdot q p_\mu) (V_3 p_\nu + V_4 p'_\nu), \quad (3.7)$$

$$A_{\mu\nu} = A_1 \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta + A_2 \epsilon_{\mu\nu\alpha\beta} p'^\alpha q^\beta + (A_3 p_\nu + A_4 p'_\nu) \epsilon_{\mu\alpha\beta\gamma} p^\alpha q^\beta p'^\gamma. \quad (3.8)$$

We saturate the SD part of the amplitude with D^* and B^* resonances, which contribute to $V_{\mu\nu} - A_{\mu\nu}$ as

$$\frac{i \langle D | J_\mu | D^*(\mathbf{p}' + \mathbf{q}) \rangle \langle D^*(\mathbf{p}' + \mathbf{q}) | V_\nu - A_\nu | B \rangle}{(p' + q)^2 - m_{D^*}^2 + i\epsilon}, \quad (3.9a)$$

$$\frac{i \langle D | V_\nu - A_\nu | B^*(\mathbf{p} - \mathbf{q}) \rangle \langle B^*(\mathbf{p} - \mathbf{q}) | J_\mu | B \rangle}{(p - q)^2 - m_{B^*}^2 + i\epsilon}. \quad (3.9b)$$

The B^* pole is not far in the unphysical region and thus for $E_\gamma \rightarrow 0$ its contribution gets dangerously large due to small splitting: $1/(m_B^2 - m_{B^*}^2)$. The D^* pole, on the other hand, can be on-shell and we model its contribution by Breit–Wigner shape

$$\frac{i \langle D | J_\mu | D^* \rangle \langle D^* | V_\nu - A_\nu | B \rangle}{(p' + q)^2 - m_{D^*}^2 + im_{D^*} \Gamma_{D^*}}. \quad (3.10)$$

The D^* contribution is expected to be dominant over the B^* one. Form factors $V_{\mu\nu}, A_{\mu\nu}$ contain nonperturbative matrix elements, as evident from (3.9), for which we take quenched lattice results for $B \rightarrow D^*$ form factors [8,9]. Value of $g_{D^*D\gamma}$ was computed on the lattice with dynamical light quarks [10].

The intermediate D^* is kinematically allowed to be on-shell only for photon energies in the range of $\sim [50, 350]$ MeV (see Fig. 1). This resonant enhancement of the soft photon kinematical region originates from relatively small difference of m_{D^*} and m_D . Next higher excited charm state lies already above 2.4 GeV in mass and the photon needs to have larger energy to allow the resonance to get on-shell. Moreover, if we take into account larger width of higher charm states we make a general statement that the higher the charm resonant state is the more energetic is the photon it produces, making it visible to the experiment. This contribution is clearly seen in the E_γ spectrum, Fig. 2, with approximately half of the width hidden in the $E_\gamma < 200$ MeV region.

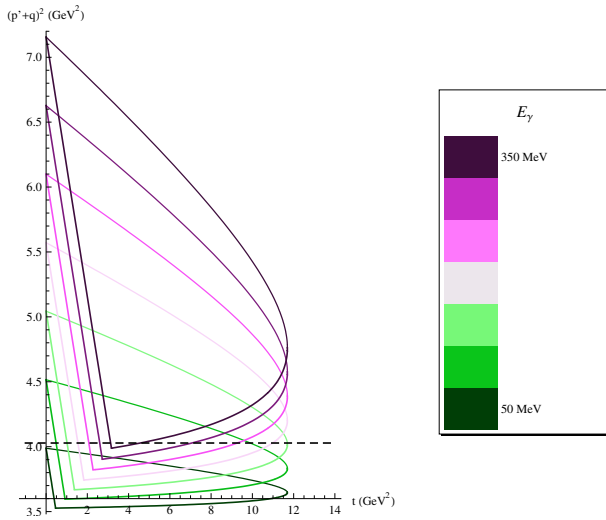


Fig. 1. Slices of phase space in the D^* invariant mass *versus* momentum transfer t for range different photon energies. Horizontal line represents on-shell D^* , which is reachable only in the range $50 \text{ MeV} < E_\gamma < 350 \text{ MeV}$.

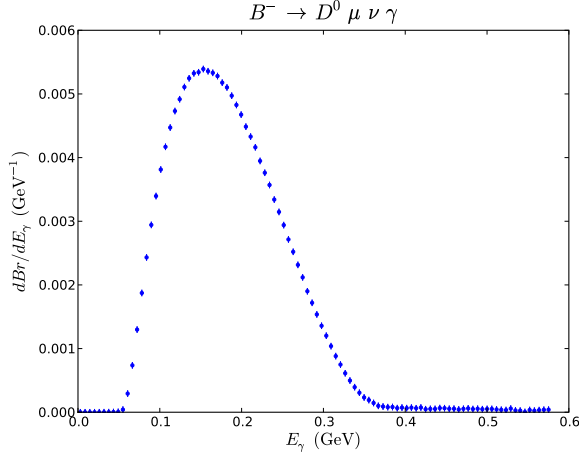


Fig. 2. Resonant D^* spectrum of $B^- \rightarrow D^0 \mu \nu \gamma$. Mainly produces experimentally invisible photons.

4. Conclusion

The importance of pushing experimental resolution to 100 MeV or account for the missed photons is clearly seen in Fig. 3, which shows the fraction of misidentified radiative events as a function of photon energy resolution. For example, an energy resolution of 300 MeV results in $\sim 4\%$ of the recorded events to be fake, which affects the extraction of V_{cb} , which comes out 2% too large.

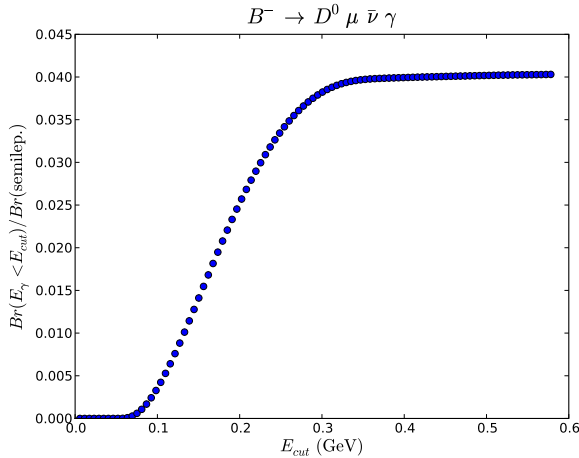


Fig. 3. Fraction of miss-identified radiative events plotted against the experimental resolution of photon energy E_{cut} .

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