# LEPTON-FLAVOUR VIOLATION IN MINIMAL SEESAW MODELS\*

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In the presence of a low scale seesaw of type I + III, flavor violating effects in the leptonic sector are expected. Their presence in the charged sector is due to the mixing of the fermionic vector-like weak triplets with the chiral doublets, which cause non-universality of the tree-level Z coupling. We investigate the bounds on the Yukawa couplings which are responsible for the mixing and present the results for two minimal cases, a fermionic triplet with a singlet or two fermionic triplets. Different channels for these processes are considered and their current and future potential to probe these couplings is discussed.

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#### 1. Introduction

Available experimental data on neutrino oscillations indicates a small mass of left-handed neutrinos. This is in contrast with the Standard Model (SM) where the left-handed neutrinos are massless. The nature of neutrinos, whether they are Dirac or Majorana particles, is not known. The latter possibility is theoretically more compelling, since it introduces new physics at the scale  $\Lambda$ , where the neutrino mass operator

$$\mathcal{O}_{\nu}^{d=5} = y_{\nu}^{ij} \, \frac{L_i H L_j H}{\Lambda} \,, \tag{1}$$

is formed. There are only three different ways to realize this operator at the tree level when a single representation is added [1]. Adding a right-handed neutrino is referred to as the type I seesaw [2], while an extra bosonic triplet

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with hypercharge 1 results in type II seesaw [3]. The third option is to couple the leptonic and Higgs doublets to a fermionic weak triplet with zero hypercharge and this is the type III seesaw [4].

The Weinberg operator (1) can be probed through its contribution to the neutrinoless double beta decay (however, other new physics (NP) contributions are possible as well [5]). In addition, integrating out the heavy mediators produces dimension-6 operators, suppressed by the same scale  $\Lambda$ , mediating lepton–flavor violating (LFV) processes. Unfortunately, the scale  $\Lambda$  is not known since it depends on the size of the Yukawa couplings. If they are of order one, as in certain Grand Unified Theories (GUTs), Eq. (1) predicts  $\Lambda$  around  $10^{13}$  GeV. Such a high scale would make it very hard to probe the origin of the mass operator directly at colliders. Also, since dimension-6 operator contributions decouple faster than dimension-5 any LFV effects at low energies would be unmeasurably small. On the other hand, when Yukawa couplings in Eq. (1) are small, the seesaw scale may lie anywhere below  $10^{13}$  GeV.

# 2. Testable seesaw

Several recent studies have shown that TeV — scale seesaw can be probed via direct production of mediators at colliders [6,7] as well as through LFV effects at low energies [8,9]. As discussed above, light mediators require tiny Yukawa eigenvalues to accommodate the measured neutrino masses. Therefore other (gauge) couplings are needed for efficient production of the mediators at colliders. These are automatically present in type II or III seesaw models. On the other hand, observable LFV effects typically require some fine-tunning but are also possible. While the leptonic mixing matrix becomes non-unitary in both type I and III cases [10], the unique feature of the type III is the presence of charged lepton–flavor changing neutral currents (FCNCs) at the tree level.

Present data require two massive neutrinos in a normal (NH) or inverse (IH) mass hierarchy. In type I/III seesaw, this can be accomplished using a combination of two mediators. Since in these scenarios the atmospheric scale sets the largest neutrino mass, such models can be excluded by direct neutrino mass measurements [11]. At the same time, they can be tested via direct production and decays of mediators. Consider a type I+III example, where there is at least one triplet mediator. In non-minimal case, the  $\nu$ mass matrix has rank 3 and one can parametrize the Yukawa coupling of the lightest triplet mediator (T) in a convenient way using the low energy neutrino parameters [12]

$$v y_{\rm T}^i = \sqrt{m_{\rm T}} \sum_j U_{ij} \sqrt{m_{\nu}^j R_{ji}(z_1, z_2, z_3)},$$

where R is a complex orthogonal  $3 \times 3$  matrix while U is the unitary PMNS matrix defined by the standard parameterization. There are all in all 11 unknowns entering the above expression: complex  $z_{1,2,3}$ , one neutrino mass, and 4 unknown angles and phases  $\theta_{13}$ ,  $\delta$ ,  $\phi_{1,2}$  from U (PMNS). It is clearly difficult to disentangle useful information on all of these neutrino parameters using only 3 measurements constraining  $|y_{\rm T}^i|$  from the decays of the lightest mediator produced at a collider.

Consider instead a similar I+III scenario, but with a rank-2 neutrino mass matrix. The above parameterization now further simplifies to [13]

$$y_{\rm T}^{i} = -i\sqrt{2m_{\rm T}}/v \left( U_{i2}\sqrt{m_{2}^{\nu}}\cos z + U_{i3}\sqrt{m_{3}^{\nu}}\sin z \right)^{*},$$
  
$$y_{\rm S}^{i} = -i\sqrt{2m_{\rm S}}/v \left( -U_{i2}\sqrt{m_{2}^{\nu}}\sin z + U_{i3}\sqrt{m_{3}^{\nu}}\cos z \right)^{*}, \qquad (2)$$

for the NH and similarly for the IH. Again U is the PMNS matrix while z is a single complex number. In this scenario, measuring lightest mediator decays directly constrains z,  $\theta_{13}$  and the phases  $\delta$ ,  $\phi$  [7].

#### 3. Minimal seesaw generalities

We consider minimal pure type III (two triplets) and mixed I+III (singlet and triplet) seesaw scenarios where the neutrino masses recieve contributions from two terms, e.g.

$$(m^{\nu})^{ij} = -\frac{v^2}{2} \left( \frac{y_{\rm T}^i y_{\rm T}^j}{m_{\rm T}} + \frac{y_{\rm S}^i y_{\rm S}^j}{m_{\rm S}} \right) \,,$$

for I+III scenario and similarly for pure type III. The lightest neutrino is massless and there is only one physical Majorana phase. Parameterizations like (2) apply and the size of the Yukawa couplings is completely determined by a single complex parameter z. They increase exponentially with Im(z), with Re(z) becoming irrelevant as  $\text{Im}(z) \gg 1$ . Consequently, for large Im(z), LFV effects can become observable due to systematic cancellations in the Yukawas. The higher the seesaw scale however, the more severe fine-tuning is needed in order to produce observable effects.

The neutral and charged lepton mass matrices can be written in matrix notation

$$M_{\ell} = \begin{pmatrix} v/\sqrt{2} \ y_{\ell}^{ij} \delta^{ij} & 0 \\ v \ y_{T}^{j} & m_{T} \end{pmatrix} \quad \text{and} \quad M_{\nu} = \begin{pmatrix} 0_{3\times3} & v \ y_{T}^{i} & v \ y_{S}^{i} \\ v \ y_{T}^{j} & m_{T} & 0 \\ v \ y_{S}^{j} & 0 & m_{S} \end{pmatrix},$$

and brought to a diagonal form by a biunitary and congruent transformation for the charged and neutral fields

$$\hat{M}_{\ell} = U^{\dagger \dagger} M_{\ell} U^{-}, \qquad \hat{M}_{\nu} = U^{0T} M_{\nu} U^{0}.$$

It produces mixing of chiral and vector-like fermions and alters interactions with the W and Z bosons  $(f_i^- = (e, \mu, \tau, T^-), f_j^0 = (\nu_1, \nu_2, \nu_3, T^0, S))$ 

$$\mathcal{L}_{\text{int}}^{W,Z} = g \,\overline{f}_i' \mathcal{W}^+ \left( L^W P_L + R^W P_R \right)_{ij} f_j + \text{h.c.} + \frac{g}{c_w} \,\overline{f}_i \mathcal{Z} \left( L^Z P_L + R^Z P_R \right)_{ij} f_j \,.$$

## 4. Phenomenology

The main signature of I+III seesaw models at low energies is the appearance of tree-level Z-mediated LFV processes. These include LFV lepton decays,  $\mu$ -e conversion in nuclei, LFV semileptonic tau decays and LFV Z decay widths. Furthermore, there are tree-level lepton-flavour universality violations in charged currents. These affect  $G_{\rm F}$  determination from the muon lifetime, lepton-flavor universality (LFU) ratios both at low energies (mostly in pion, kaon and tau decays) as well as at colliders  $(W \to l\nu)/(W \to l'\nu)$ . Finally, there are loop-induced LFV processes such as the radiative lepton decays and the anomalous lepton magnetic moments.

In minimal models all effects are predicted and correlated in terms of  $m_{\rm T}$ ,  $m_{\rm S}$  and  ${\rm Im}(z)$ . Charged fermion LFV Z couplings scale as  $\exp[2{\rm Im}(z)]/m_{\rm T}$  while the associated fine-tuning can be measured in  $\exp[2{\rm Im}(z)]$ . The Yukawas can be considered natural for  ${\rm Im}(z) < 1$ . More importantly, the most stringent bound constrains all other low-energy phenomenology for all three lepton families. Presently, it is obtained from the search for  $\mu$ -e conversion in nuclei performed by the SINDRUM collaboration in experiments on titanium and gold targets [14]. The resulting bounds on the LFV couplings  $\mu eZ$  are

$$\left|L_{e\mu}^{Z}\right|^{2} + \left|R_{e\mu}^{Z}\right|^{2} < 10^{-14}, \ 10^{-15},$$

for Ti and Au, respectively. After allowing to vary the poorly known neutrino mass parameter  $\theta_{13}$  within the experimentally allowed range and the unknown phases  $\delta$  and  $\phi$  between 0 and  $2\pi$ , one obtains in the minimal models a bound on Im(z) < 7.5(7.1) for NH (IH) in case of one triplet and one singlet and Im(z) < 7.2(6.8) for two triplets, all at the reference mass of  $m_{\rm T} = 100$  GeV for the lightest triplet [9].

Among the other constraints, LFV leptonic tau decays constitute the most sensitive bounds on Im(z) coming from  $\tau - \ell$  transitions. On the other hand, radiative LFV decays are suppressed. Flavor conserving leptonic Z widths turn out to be more constraining than LFV ones, while charged

current LFU are more constrained at low energies than from direct W decay branching ratio measurements. Many other observables have been studied and found not relevant. The differences between NH and IH, and between III vs. I+III scenarios turn out not to be crucial in the minimal models. The results indicate a mild dependence on the Majorana phase while precise values of  $\theta_{13}$ ,  $\delta$  turn out to be irrelevant [9].

#### 5. Beyond minimal I+III models

Adding another heavy fermion increases the number of free parameters. However, correlations between different channels are generically preserved. This can easily be seen by considering the non-universal coupling

$$L_{e\mu}^{Z} \simeq \frac{v^{2}}{2} \sum_{\alpha=1}^{n_{\rm T}} y_{\alpha e}^{*} y_{\alpha \mu} / m_{\alpha}^{2} = \sum_{\alpha=1}^{n_{\rm T}} \sum_{i,j=1}^{3} \left( \sqrt{m_{i}^{\nu} m_{j}^{\nu}} / m_{\alpha} \right) R_{\alpha i} R_{\alpha j} U_{ei} U_{\mu j} + \frac{1}{2} \sum_{\alpha=1}^{n_{\rm T}} \left( \sqrt{m_{i}^{\nu} m_{j}^{\nu}} / m_{\alpha} \right) R_{\alpha i} R_{\alpha j} U_{ei} U_{\mu j} + \frac{1}{2} \sum_{\alpha=1}^{n_{\rm T}} \left( \sqrt{m_{i}^{\nu} m_{j}^{\nu}} / m_{\alpha} \right) R_{\alpha i} R_{\alpha j} U_{ei} U_{\mu j} + \frac{1}{2} \sum_{\alpha=1}^{n_{\rm T}} \left( \sqrt{m_{i}^{\nu} m_{j}^{\nu}} / m_{\alpha} \right) R_{\alpha i} R_{\alpha j} U_{ei} U_{\mu j} + \frac{1}{2} \sum_{\alpha=1}^{n_{\rm T}} \left( \sqrt{m_{i}^{\nu} m_{j}^{\nu}} / m_{\alpha} \right) R_{\alpha i} R_{\alpha j} U_{ei} U_{\mu j} + \frac{1}{2} \sum_{\alpha=1}^{n_{\rm T}} \left( \sqrt{m_{i}^{\nu} m_{j}^{\nu}} / m_{\alpha} \right) R_{\alpha i} R_{\alpha j} U_{ei} U_{\mu j} + \frac{1}{2} \sum_{\alpha=1}^{n_{\rm T}} \left( \sqrt{m_{i}^{\nu} m_{j}^{\nu}} / m_{\alpha} \right) R_{\alpha i} R_{\alpha j} U_{ei} U_{\mu j} + \frac{1}{2} \sum_{\alpha=1}^{n_{\rm T}} \left( \sqrt{m_{i}^{\nu} m_{j}^{\nu}} / m_{\alpha} \right) R_{\alpha i} R_{\alpha j} U_{ei} U_{\mu j} + \frac{1}{2} \sum_{\alpha=1}^{n_{\rm T}} \left( \sqrt{m_{i}^{\nu} m_{j}^{\nu}} / m_{\alpha} \right) R_{\alpha i} R_{\alpha j} U_{ei} U_{\mu j} + \frac{1}{2} \sum_{\alpha=1}^{n_{\rm T}} \left( \sqrt{m_{i}^{\nu} m_{j}^{\nu}} / m_{\alpha} \right) R_{\alpha i} R_{\alpha j} U_{ei} U_{\mu j} + \frac{1}{2} \sum_{\alpha=1}^{n_{\rm T}} \left( \sqrt{m_{i}^{\nu} m_{j}^{\nu}} / m_{\alpha} \right) R_{\alpha i} R_{\alpha j} U_{ei} U_{\mu j} + \frac{1}{2} \sum_{\alpha=1}^{n_{\rm T}} \left( \sqrt{m_{i}^{\nu} m_{j}^{\nu}} / m_{\alpha} \right) R_{\alpha i} R_{\alpha j} U_{ei} U_{\mu j} + \frac{1}{2} \sum_{\alpha=1}^{n_{\rm T}} \left( \sqrt{m_{i}^{\nu} m_{j}^{\nu}} / m_{\alpha} \right) R_{\alpha i} R_{\alpha j} U_{ei} U_{\mu j} + \frac{1}{2} \sum_{\alpha=1}^{n_{\rm T}} \left( \sqrt{m_{i}^{\nu} m_{j}^{\nu}} / m_{\alpha} \right) R_{\alpha i} R_{\alpha j} + \frac{1}{2} \sum_{\alpha=1}^{n_{\rm T}} \left( \sqrt{m_{i}^{\nu} m_{j}^{\nu}} / m_{\alpha} \right) R_{\alpha i} R_{\alpha j} + \frac{1}{2} \sum_{\alpha=1}^{n_{\rm T}} \left( \sqrt{m_{i}^{\nu} m_{j}^{\nu}} / m_{\alpha} \right) R_{\alpha i} R_{\alpha j} + \frac{1}{2} \sum_{\alpha=1}^{n_{\rm T}} \left( \sqrt{m_{i}^{\nu} m_{j}^{\nu}} / m_{\alpha} \right) R_{\alpha i} R_{\alpha j} + \frac{1}{2} \sum_{\alpha=1}^{n_{\rm T}} \left( \sqrt{m_{i}^{\nu} m_{j}^{\nu}} / m_{\alpha} \right) R_{\alpha i} + \frac{1}{2} \sum_{\alpha=1}^{n_{\rm T}} \left( \sqrt{m_{i}^{\nu} m_{j}^{\nu}} / m_{\alpha} \right) R_{\alpha i} + \frac{1}{2} \sum_{\alpha=1}^{n_{\rm T}} \left( \sqrt{m_{i}^{\nu} m_{j}^{\nu}} / m_{\alpha} \right) R_{\alpha i} + \frac{1}{2} \sum_{\alpha=1}^{n_{\rm T}} \left( \sqrt{m_{i}^{\nu} m_{j}^{\nu}} / m_{\alpha} \right) R_{\alpha i} + \frac{1}{2} \sum_{\alpha=1}^{n_{\rm T}} \left( \sqrt{m_{i}^{\nu} m_{j}^{\nu}} / m_{\alpha} \right) R_{\alpha i} + \frac{1}{2} \sum_{\alpha=1}^{n_{\rm T}$$

where we sum over *all* the elements of the orthogonal matrix R, regardless of the flavor. Therefore, one cannot easily enlarge the  $\tau \ell Z$  couplings by enhancing a single element of R without affecting the  $\mu e$  channel and running in contradiction with the  $\mu$ -e conversion experiments unless one aligns (finetunes) the available phases. This result holds for an arbitrary number of additional triplets and shows that the overall rate of the flavor processes is naturally dictated by the most constraining channel.

On the other hand, there is a potential gain in considering non-minimal models with three extra triplets. Namely, one can use the freedom of setting the overall scale of neutrinos at will and a positive signal is possible even for natural values of the Yukawas. For example, if light neutrinos are degenerate with the sum of their masses close to the upper limit from  $\beta$  decay and cosmology (say  $\sum m_{\nu} \leq \text{eV}$  [15]), present  $\mu$ -e conversion bounds already probe values of Im $(z_i) \simeq 2$ -4.

## 6. Conclusions

Minimal TeV-scale I+III see-saw models can be probed using low-energy observables, with the presently best limits coming from  $\mu$ -e conversion in nuclei. These make most other bounds irrelevant for the foreseeable future. Conversely, positive observation of any of the other processes would signal LFV beyond the minimal I+III seesaw. While the present bounds are still far from probing natural Yukawa values, non-minimal models could soon be probed in the interesting parameter space region. Planned  $\mu$ -e nuclear conversion sensitivity of  $10^{-16}$  or even  $10^{-18}$  on Br<sub> $\mu e$ </sub> [16] would constrain Im(z) to 4.1 (3.7) in case of the minimal I+III model and to 3.7(3.4) for the minimal type III. In non-minimal models even Im( $z_i$ ) < 1 could be probed. The author would like to thank M. Nemevšek for a fruitful collaboration and the organizers for the invitation and hospitality at this very stimulating and interesting workshop.

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