FCNC PROCESSES WAITING FOR THE NEXT DECADE*

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FCNC processes are expected to offer us a deep insight into the physics at very short distance scales. We present a list of 20 goals in quark and lepton flavour physics that could be reached already in the next decade. This list includes also flavour conserving observables like electric dipole moments of the neutron and leptons and $(g-2)_\mu$. Subsequently we will present some aspects of these goals by concentrating on supersymmetric flavour models. A much more extensive presentation of this material can be found in my recent EPS09 talk [1].

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1. Overture

Flavour-violating and CP-violating processes are very strongly suppressed and are governed by quantum fluctuations that allow us to probe energy scales far beyond the ones explored by the LHC and future colliders. Indeed, energy scales as high as 200 TeV corresponding to short distances in the ballpark of 10^{-21}m can be probed, albeit indirectly, in this manner. Consequently, frontiers in probing ultrashort distance scales belong to flavour physics, or more concretely to very rare processes like particle–antiparticle mixing, rare decays of mesons and of the top quark, CP violation and lepton flavour violation. Also electric dipole moments and $(g-2)_\mu$ belong to these frontiers even if they are flavour conserving. While such tests are not limited by the available energy, they are limited by the available precision. The latter has to be very high as the Standard Model (SM) has been until now very successful, and finding departures from its predictions has become a real challenge.

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Personally I expect that the coming decade will become the decade of discoveries not only at the LHC but, in particular, in high precision flavour experiments like the LHCb, Super-Belle, Super Flavour Facility (SFF) in Frascati and dedicated rare K experiments around the world. Also flavour conserving observables like electric dipole moments of the neutron and leptons and $(g-2)_\mu$ will play an important role in this progress.

Recently, I have given a talk at EPS09 in Cracow summarizing the present status of flavour theory and presenting flavour expectations for the coming decade [1]. In view of space limitations I can discuss here only few points made already in my EPS09 talk where further details can be found. In particular, very few references will be given here. This is, I hope, compensated by roughly 300 references given in the EPS09 writeup.

This presentation consists of three parts. First, I will make a list of twenty most important goals in this field for the coming decade. In the second part I will concentrate on a few topics that I find particularly important and interesting. The third part is dominated by a number of enthusiastic statements that close this report.

2. Twenty goals in Flavour Physics for the next decade

We will now list twenty goals in Flavour Physics for the coming decade. The order in which these goals will be listed does not represent by any means a ranking in importance.

Goal 1: The CKM matrix from tree level decays

This determination would give us the values of the elements of the CKM matrix without New Physics (NP) pollution. From the present perspective most important are the determinations of $|V_{ub}|$ and γ because they are presently not as well known as $|V_{cb}|$ and $|V_{us}|$. However, a precise determination of $|V_{cb}|$ is also important as ε_K, $\mathrm{Br}(K^+\to\pi^+\nu\bar\nu)$ and $\mathrm{Br}(K_L\to\pi^0\nu\bar\nu)$ are roughly proportional to $|V_{cb}|^4$. While Super-B facilities accompanied by improved theory should be able to determine $|V_{ub}|$ and $|V_{cb}|$ with precision of 1–2%, the best determination of the angle γ in the first half of the next decade will come from the LHCb. An error of a few degrees on γ should be achievable around 2015, and this measurement could be further improved at Super-B machines.

Goal 2: Improved lattice calculations of hadronic parameters

The knowledge of meson decay constants F_{B_s}, F_{B_d} and of various B_i parameters with high precision would allow in conjunction with Goal 1 to make precise calculations of the $B_{s,d}$ mixing mass differences $\Delta M_{s,d}$, ε_K, $\mathrm{Br}(B_{s,d}\to\mu^+\mu^-)$ and of other observables in the SM. We could then directly see whether the SM is capable of describing these observables or not. The most recent unquenched calculations allow for optimism and, in fact, a very significant progress in the calculation of $\hat B_K$, relevant for ε_K, has been made recently. We will discuss its implications in Sec. 3.

Goal 3: Is ε_K consistent with $S_{\psi K_S}$ within the SM?

The recent improved value of $\hat{B}_K$ from unquenched lattice QCD accompanied by a more careful look at ε_K [2] suggests that the size of CP violation measured in $B_d \to \psi K_S$, might be insufficient to describe ε_K within the SM. Clarification of this new tension is important as the $\sin 2\beta - \varepsilon_K$ correlation in the SM is presently the most important direct relation between CP violation in B_d and K systems that can be tested experimentally. The interplay of this relation with the one between the value of the angle α extracted from non-leptonic two-body B_d decays and ε_K is also an important test of the SM. We will return to this issue in Sec. 3.

Goal 4: Is $S_{\psi\phi}$ much larger than its tiny SM value?

Within the SM, CP violation in the B_s system is predicted to be very small. The best known representation of this fact is the value of the mixing induced CP asymmetry: $(S_{\psi\phi})_{\rm SM} \approx 0.04$. The present data from CDF and D0 indicate that CP violation in the B_s system could be much larger $S_{\psi\phi} = 0.81^{+0.12}_{-0.32}$. This is a very interesting deviation from the SM. Its clarification is of utmost importance, and I will return to this question in Sec. 3. Fortunately, we should know the answer to this question within the coming years as CDF, D0, LHCb, ATLAS and CMS will make big efforts to measure $S_{\psi\phi}$ precisely.

Goal 5: Non-leptonic two body B decays

The best information on CP violation in the B system to date comes from two body non-leptonic decays of B_d and $B^\pm$ mesons. The LHCb will extend these studies in an important manner to B_s and B_c decays. This is clearly a challenging field not only for experimentalists but in particular for theorists due to potential hadronic uncertainties. Yet, in the last ten years an impressive progress has been made in measuring many channels, in particular $B \to \pi\pi$ and $B \to \pi K$ decays, and developing a number of methods to analyze these data.

I think this field will continue to be important for the tests of the CKM framework in view of very many channels whose branching ratios should be measured in the next decade with a high precision. On the other hand, in view of potential hadronic uncertainties present in the branching ratios and direct CP asymmetries, these observables will not provide, in my opinion, definite answers about NP if the latter contributes to them only at the level of 20% or less. On the other hand, mixing induced CP-asymmetries like $S_{\psi K_S}$, $S_{\psi\phi}$ and alike being theoretically much cleaner will continue to be very important for the tests of NP.

Goal 6: $\mathrm{Br}(B_{s,d} \to \mu^+\mu^-)$

In the SM and in several of its extentions $\mathrm{Br}(B_s \to \mu^+\mu^-)$ is found in the ballpark of 3–5 $\times 10^{-9}$, which is by an order of magnitude lower than the present bounds from CDF and D0. A discovery of $\mathrm{Br}(B_{s,d} \to \mu^+\mu^-)$ at

$\mathcal{O}(10^{-8})$ would be a clear signal of NP, possibly related to Higgs penguins. The LHCb can reach the SM level for this branching ratio in the first years of operation. From my point of view, similarly to $S_{\psi\phi}$, precise measurements of $\mathrm{Br}(B_s \to \mu^+\mu^-)$ and $\mathrm{Br}(B_d \to \mu^+\mu^-)$ are among the most important goals in flavour physics in the coming years. We will discuss both decays in Sec. 3.

Goal 7: $B \to X_{s,d}\gamma$, $B \to K^*(\varrho)\gamma$ and $A_{\mathrm{CP}}^{\mathrm{dir}}(b \to s\gamma)$

The radiative decays in question, in particular $B \to X_s\gamma$, have played an important role in constraining NP in the last 15 years. Both the experimental data and also the theory have already been in a good shape for some time with the NNLO calculations of $\mathrm{Br}(B \to X_s\gamma)$ being at the forefront of perturbative QCD calculations in weak decays. Both theory and experiment reached roughly 10% precision, and the agreement of the SM with the data is good, implying not much room left for NP contributions. Still further progress both in theory and experiment should be made to further constrain NP models. Of particular interest is the direct CP asymmetry $A_{\mathrm{CP}}^{\mathrm{dir}}(b \to s\gamma)$ that similarly to $S_{\psi\phi}$ is predicted to be tiny (0.5%) in the SM but could be much larger in some of its extensions.

Goal 8: $B \to X_s l^+l^-$ and $B \to K^* l^+l^-$

While the branching ratios for $B \to X_s l^+l^-$ and $B \to K^* l^+l^-$ put already significant constraints on NP, the angular observables, CP-conserving ones (like the well known forward–backward asymmetry) and CP-violating ones will definitely be very useful for distinguishing various extensions of the SM. Recently, a number of detailed analyses of various CP averaged symmetries and CP asymmetries provided by the angular distributions in the exclusive decay $B \to K^*(\to K\pi)\, l^+l^-$ have been performed. In particular, zeroes in some of these observables can be accurately predicted. Belle and BaBar provided already interesting results for the best known forward–backward asymmetry, but the data have to be improved in order to see whether some sign of NP is seen in this asymmetry. Future studies by the LHCb and Super-B machines will be able to contribute here in a significant manner.

Goal 9: $B^+ \to \tau^+\nu$ and $B^+ \to D^0\tau^+\nu$

The SM expression for the branching ratio of the tree-level decay $B^+ \to \tau^+\nu$ is subject to parametric uncertainties induced by F_{B^+} and V_{ub}. In order to find the SM prediction for this branching ratio we can express it in terms of $\Delta M_{s,d}$ and $S_{\psi K_S}$, all to be taken from experiment. We then find [3]

$$\mathrm{Br}(B^+ \to \tau^+\nu)_{\mathrm{SM}} = (0.80 \pm 0.12) \times 10^{-4}\,. \tag{1}$$

This result agrees well with a recent result presented by the UTfit Collaboration [4].

On the other hand, the present experimental world average based on results by BaBar and Belle reads

$$\mathrm{Br}(B^+ \to \tau^+\nu)_{\rm exp} = (1.73 \pm 0.35) \times 10^{-4}\,, \tag{2}$$

and is roughly by a factor of 2 higher than the SM value. We can talk about a tension at the $2.5\,\sigma$ level. Interestingly, the tension between theory and experiment in the case of $\mathrm{Br}(B^+ \to \tau^+\nu)$ increases in the presence of a tree level $H^\pm$ exchange which interferes destructively with the $W^\pm$ contribution.

The full clarification of a possible discrepancy between the SM and the data will have to wait for the data from Super-B machines. Also improved values for F_B from lattice and $|V_{ub}|$ from tree level decays will be important if some NP like charged Higgs is at work here. The decay $B^+ \to D^0\tau^+\nu$ being sensitive to different couplings of $H^\pm$ can contribute significantly to this discussion but formfactor uncertainties make this decay less theoretically clean.

Goal 10: Rare kaon decays

Among the top highlights of flavour physics in the next decade will be measurements of the branching ratios of two *golden modes* $K^+ \to \pi^+\nu\bar\nu$ and $K_L \to \pi^0\nu\bar\nu$. $K^+ \to \pi^+\nu\bar\nu$ is CP conserving while $K_L \to \pi^0\nu\bar\nu$ is governed by CP violation. Both decays are dominated in the SM and many of its extensions by Z penguin contributions. It is well known that these decays are theoretically very clean and are known in the SM including NNLO QCD corrections and electroweak corrections. Moreover, extensive calculations of isospin breaking effects and non-perturbative effects have been done. The present theoretical uncertainties in $\mathrm{Br}(K^+ \to \pi^+\nu\bar\nu)$ and $\mathrm{Br}(K_L \to \pi^0\nu\bar\nu)$ are at the level of 2–3% and 1–2%, respectively.

Let me stress that measurements of the branching ratios in question with an accuracy of 10% will give us a very important insight into the physics at short distance scales. NA62 at CERN in the case of $K^+ \to \pi^+\nu\bar\nu$ and KOTO at J-PARC in the case of $K_L \to \pi^0\nu\bar\nu$ will tell us how these two decays are affected by NP.

The decays $K_L \to \pi^0 l^+ l^-$ are not as theoretically clean as the $K \to \pi\nu\bar\nu$ channels. They are less sensitive to NP contributions but probe different operators beyond the SM. Having accurate branching ratios for them would certainly be very useful.

Goal 11: $B \to X_s\nu\bar\nu$, $B \to K^*\nu\bar\nu$ and $B \to K\nu\bar\nu$

Also B decays with $\nu\bar\nu$ in the final state provide a very good test of modified Z penguin contributions, but their measurements appear to be even harder than those of the rare K decays just discussed.

The inclusive decay $B \to X_s\nu\bar\nu$ is theoretically as clean as $K \to \pi\nu\bar\nu$ decays but the parametric uncertainties are a bit larger. The two exclusive channels are affected by formfactor uncertainties but recently, in the case of

$B \to K^* \nu\bar{\nu}$ and $B \to K \nu\bar{\nu}$, significant progress has been made. The interesting feature of these three $b \to s\nu\bar{\nu}$ transitions, in particular when taken together, is their sensitivity to right-handed currents. Super-B machines should be able to measure them at a satisfactory level.

Goal 12: Calculations of hadronic matrix elements in ε'/ε

One of the important actors of the previous decade in flavour physics was the ratio ε'/ε that measures the size of direct CP violation in $K_L \to \pi\pi$ relative to the indirect CP violation described by ε_K. In the SM ε' is governed by QCD penguins but receives also an important destructively interfering contribution from electroweak penguins that is generally much more sensitive to NP than the QCD penguin contribution.

Here the problem is the strong cancellation of QCD penguin contributions and electroweak penguin contributions to ε'/ε. To obtain useful predictions, precision of the corresponding hadronic parameters B_6 and B_8 should be at least 10%. Lattice theorists hope to make progress on B_6, B_8 and other ε'/ε related hadronic matrix elements in the coming decade. This would really be good, as the calculations of short distance contributions to this ratio (Wilson coefficients of QCD and electroweak penguin operators) have been known already for 16 years at the NLO. The experimental world average from NA48 and KTeV $\varepsilon'/\varepsilon = (16.8 \pm 1.4) \times 10^{-4}$ could have an important impact on several extentions of the SM discussed in the literature if B_6 and B_8 were known.

Goal 13: CP Violation in charm decays and $D^+(D_s^+) \to l^+\nu$

Charm physics has been for many years shadowed by the successes of K decays and B decays, although a number of experimental groups and selected theorists have made a considerable effort to study them. This is due to the GIM mechanism being very effective in suppressing the FCNC transitions in this sector, long distance contributions plaguing the evaluation of ΔM_D, and insensitivity to top physics in the loops. However, large $D^0 - \bar{D}^0$ mixing discovered in 2007 and good prospects for the study of CP violation in these decays at Super Belle and SFF in Frascati gave a new impetus to this field. Also leptonic decays of D mesons remain to be important.

Goal 14: CP violation in the lepton sector and θ_{13}

The neutrino mixing angles θ_{12} and θ_{23} are already known with respectable precision. The obvious next targets in this field are θ_{13} and the CP phase $\delta_{\rm PMNS}$. Clearly the discovery of CP violation in the lepton sector would be a very important mile stone in particle physics for many reasons. In particular the most efficient explanations of the baryon–antibaryon asymmetry of the Universe (BAU) these days follow from leptogenesis. While in the past the necessary size of CP violation was obtained from new sources of CP violation at very high see-saw scales, the inclusion of flavour effects, in particular in resonant leptogenesis, gave hopes for the explanation of the

BAU using only the phases in the PMNS matrix. This implies certain conditions for the parameters of this matrix, that is the relevant $\delta_{\rm PMNS}$, two Majorana phases and θ_{13}.

Goal 15: Tests of $\mu - e$ and $\mu - \tau$ universalities

Lepton flavour violation (LFV) and the related breakdown of universality can be tested in meson decays by studying the ratios

$$R_{\mu e} = \frac{{\rm Br}(K^+ \to \mu^+ \nu)}{{\rm Br}(K^+ \to e^+ \nu)}\,, \qquad R_{\mu\tau} = \frac{{\rm Br}(B^+ \to \mu^+ \nu)}{{\rm Br}(B^+ \to \tau^+ \nu)}\,, \tag{3}$$

where the sum over different neutrino flavours is understood. The first case is a high precision affair both for experimentalists and theorists, as both groups decreased the uncertainties in $R_{\mu e}$ well below 1% with a precision of 0.5% recently achieved at CERN. It will continue to constitute an important test of the $\mu - e$ universality. The ratio $R_{\mu\tau}$ is even more sensitive to NP contributions but it will still take some time until it is known with good precision.

Goal: 16 flavour violation in charged lepton decays (LFV)

The search for LFV clearly belongs to the most important goals in flavour physics. In the SM with right-handed Dirac neutrinos, the smallness of neutrino masses implies tiny branching ratios for LFV processes. For instance, ${\rm Br}(\mu \to e\gamma)_{\rm SM} \approx 10^{-54}$ is more than 40 orders of magnitude below the 90% C.L. upper bound from the MEGA Collaboration

$${\rm Br}(\mu \to e\gamma) < 1.2 \times 10^{-11}\,. \tag{4}$$

Therefore, any observation of LFV would be a clear sign of NP. While we hope that new flavoured leptons will be observed at the LHC, even if it does not turn out to be the case, LFV has the virtue of sensitivity to high energy scales as high as 10^{10}–10^{14} GeV, in particular when the see-saw mechanism is at work.

In order to distinguish various NP scenarios that come close to the bound in (4), it will be essential to study a large set of decays to three leptons in the final state. There exist also interesting correlations between leptogenesis and LFV. Additional correlations relevant for LFV will be discussed in Sec 3.

Goal 17: Electric dipole moments

So far CP violation has only been observed in flavour violating processes. Non-vanishing electric dipole moments (EDMs) signal CP violation in flavour conserving transitions. In the SM, CP violation in flavour conserving processes is very strongly suppressed, as best expressed by the SM values of electric dipole moments of the neutron and electron that amount to

$$d_n \approx 10^{-32}\ e\,{\rm cm}\,, \qquad d_e \approx 10^{-38}\ e\,{\rm cm}\,. \tag{5}$$

This should be compared with the present experimental bounds

$$d_n \leq 2.9 \times 10^{-26}\ e\,\text{cm}\,, \qquad d_e \leq 1.6 \times 10^{-27}\ e\,\text{cm}\,. \tag{6}$$

They should be improved in the coming years by 1–2 orders of magnitude.

Similarly to LFV, an observation of non-vanishing EDM would imply necessarily NP at work. Consequently, correlations between LFV and EDMs in specific NP scenarios are to be expected, in particular in supersymmetric models, as both types of observables are governed in SUSY by dipole operators. We will encounter some examples in Sec. 3.

Goal 18: Clarification of the $(g-2)_\mu$ anomaly

The measured anomalous magnetic moment of the electron, $(g-2)_e$, is in excellent agreement with SM expectations. On the other hand, the measured anomalous magnetic moment of the muon, $(g-2)_\mu$, is roughly by 3σ larger than its SM value. Hadronic contributions to $(g-2)_\mu$ make the comparison of data and theory a bit problematic. Yet, as this anomaly has been with us already for a decade, and tremendous effort by a number of theorists has been made to clarify this issue, this anomaly could indeed come from NP.

The MSSM with large $\tan\beta$ and sleptons with masses below 400 GeV is capable of reproducing the experimental value of $(g-2)_\mu$ provided the μ parameter in the Higgs Lagrangian has a specific sign. At SFF also $(g-2)_\tau$ can be measured, and it is also sensitive to NP contributions.

Goal 19: Flavour violation at high energy

Our presentation deals mainly with tests of flavour and CP violation in low energy processes. However, at the LHC it will be possible to investigate these phenomena also in high energy processes, in particular in top quark decays.

Goal 20: Construction of a New Standard Model (NSM)

Finally, in view of so many parameters present in basically all extensions of the SM like the MSSM, the littlest Higgs model with T-parity (LHT) and Randall–Sundrum (RS) models, it is unlikely from my point of view that any of the models studied presently in the literature will turn out to be the new model of elementary particle physics. On the other hand, various structures, concepts and ideas explored these days in the context of specific models may well turn out to be included in the NSM that is predictive, consistent with all the data, and giving explanation of observed hierarchies in fermion masses and mixing matrices. While these statements may appear to be very naive, it is a fact that the construction of the NSM is the main goal of elementary particle physics, and every theorist, even as old as I am, has a dream that the future NSM will carry her (his) name.

3. Waiting for signals of New Physics in FCNC processes

3.1. Strategies for the New Physics search in the next decade

Let us first emphasize that the FCNC processes that have had the most important impact on the UT fits so far are the $\Delta F = 2$ ones. The measured $B \to X_s\gamma$ and $B \to X_s l^+l^-$ decays and their exclusive counterparts are sensitive to $|V_{ts}|$ that has nothing to do with the usual UT plots. The same applies to the observables in the B_s system that are becoming central for flavour physics, with the $S_{\psi\phi}$ anomaly observed by CDF and D0, and the studies of rare B_s decays at the Tevatron and later at the LHC. Obviously these comments also apply to all the lepton flavour violating processes.

In this context a special role is played by $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$, as their values allow a theoretically clean construction of the UT in a manner complementary to its present determinations: the height of the UT is determined from $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$, and the side R_t from $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$. Thus projecting the results of future experimental results for these two branching ratios on the $(\bar{\varrho}, \bar{\eta})$ plane could be a very good test of the SM.

Yet, generally I do not think that in the context of the search for the NSM (see Goal 20) it is a good strategy to project the results of all future measurements of rare decays on the $(\bar{\varrho}, \bar{\eta})$ plane or any other of five planes related to the remaining unitarity triangles. This would only teach us about possible inconsistencies within the SM but would not point towards a particular NP model.

In view of this, here comes a proposal for the strategy for searching for NP in the next decade, in which hopefully the side R_b and the angle γ in the UT will be precisely measured, CP violation in the B_s system explored, and many goals listed in the previous section reached. This strategy proceeds in three steps:

Step 1 In order to study transparently possible tensions between ε_K, $\sin 2\beta$, $|V_{ub}|$, γ and R_t, let us leave the $(\bar{\varrho}, \bar{\eta})$ plane and go to the R_b–γ plane [5] suggested already several years ago, and recently strongly supported by the analysis in [3,6]. The R_b–γ plane is shown in Fig. 1. We will explain this figure in the next subsection.

Step 2 In order to search for NP in rare K, B_d, B_s, D decays, in CP violation in B_s and charm decays, in LFV decays, in EDMs and $(g-2)_\mu$, let us go to specific plots that exhibit correlations between various observables. As we will see below, such correlations will be crucial to distinguish various NP scenarios. Of particular importance are the correlations between the CP asymmetry $S_{\psi\phi}$ and $B_s \to \mu^+\mu^-$, between the anomalies in $S_{\phi K_s}$ and $S_{\psi\phi}$, between $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$, between $K^+ \to \pi^+\nu\bar{\nu}$ and $S_{\psi\phi}$, between $S_{\phi K_s}$ and d_e, between $S_{\psi\phi}$ and $(g-2)_\mu$, and also those involving lepton flavour violating decays.

Step 3

In order to monitor the progress made in the next decade when additional data on flavour changing processes becomes available, it is useful to construct a "DNA-Flavour Test" of NP models [3] including Supersymmetry, the LHT model, the RS models, various supersymmetric flavour models, and other models, with the aim to distinguish between these NP scenarios in a global manner.

Having this strategy in mind, we will illustrate Steps 1 and 2 on several examples below. The full table representing "DNA-Flavour Test" can be found in [3]. Here we will reduce the illustration of Step 3 to few observables.

3.2. Tension in the R_b–γ plane

Recently, in connection with Goal 3 some tensions in the UT fits have been identified in [2, 7].

In order to see them transparently, let us have now a look at the R_b–γ plane in Fig. 1 taken from [3]. The upper-left plot shows the SM 1σ ranges allowed by the measurements of $S_{\psi K_S}$ (blue), $\Delta M_d/\Delta M_s$ (green) and $|\epsilon_K|$ (red). The solid black line corresponds to $\alpha = 90°$ that is close to the value favoured by UT fits and other analyses.

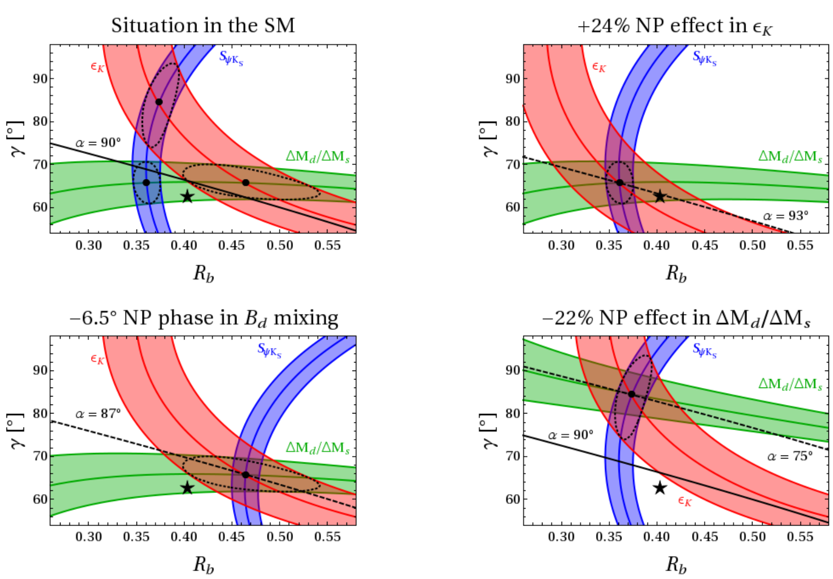

Fig. 1. The R_b–γ plane as discussed in the text. For further explanations see [3].

It is evident that there is a tension between various regions, as there are three different values of (R_b, γ) depending on which pair of constraints is simultaneously applied. The four immediate solutions to this tension are as follows:

1. There is a positive NP effect in ϵ_K while $\sin 2\beta$ and $\Delta M_d/\Delta M_s$ are SM-like [2], as shown by the upper right plot of Fig. 1. The required effect in ϵ_K could be, for instance, achieved within models with constrained minimal flavour violation (CMFV) by a positive shift in the relevant one-loop box diagram function. Alternatively, new non-minimal sources of flavour violation relevant only for the K system could solve the problem. Note that this solution corresponds to $\gamma \simeq 66^\circ$, $R_b \simeq 0.36$ and $\alpha \simeq 93^\circ$, in accordance with the usual UT analysis.

2. ϵ_K and $\Delta M_d/\Delta M_s$ are NP free while $S_{\psi K_S}$ is affected by a NP phase ϕ_{B_d} in B_d mixing of approximately -7°. This is shown in the lower left plot of Fig. 1. The predicted value for $\sin 2\beta$ is now shifted to $\sin 2\beta \approx 0.85$ [2,7]. This value is significantly larger than the measured $S_{\psi K_S}$, which allows to fit the experimental value of ϵ_K. Note that this solution is characterized by a large value of $R_b \simeq 0.47$ that is significantly larger than its exclusive determinations but still compatible with the inclusive determinations. The angles $\gamma \simeq 66^\circ$ and $\alpha \simeq 87^\circ$ agree with the usual UT analysis.

3. ϵ_K and $S_{\psi K_S}$ are NP free while the determination of R_t through $\Delta M_d/\Delta M_s$ is affected by NP. This is shown in the lower right plot of Fig. 1. In that scenario one finds $\Delta M_d^{\rm SM}/\Delta M_s^{\rm SM}$ to be much higher than the actual measurement. In order to agree exactly with the experimental central value, one needs a NP contribution to $\Delta M_d/\Delta M_s$ at the level of -22% leading to an increased value of R_t that compensates the negative effect of NP in $\Delta M_d/\Delta M_s$. This, in turn, allows to fit the experimental value of ϵ_K. This solution is characterized by a large value of $\gamma \simeq 84^\circ$ and α much below 90°. The latter fact could become problematic for this solution when the determination of α further improves.

4. The value of $|V_{cb}|$ is significantly increased to roughly 43.5×10^{-3}, which seems rather unlikely.

The first three NP scenarios characterized by black points in Fig. 1 will be clearly distinguished from each other once the values of γ and R_b from tree level decays will be precisely known. Moreover, if future measurements of (R_b, γ) select a point in the R_b–γ plane that differs from the black points in Fig. 1, it is likely that NP will simultaneously enter ϵ_K, $S_{\psi K_S}$ and $\Delta M_d/\Delta M_s$. It will be interesting to monitor future progress in the R_b–γ plane.

3.3. Correlations in supersymmetric flavour models

The correlations between various observables in the LHT model [8] and in the RS model with custodial protection [9, 10] have been already discussed at this workshop by Recksiegel and Duling, respectively. Therefore I will confine my discussion to supersymmetric flavour models (SF) with flavour symmetries that allow a simultaneous understanding of the flavour structures in the Yukawa couplings and in SUSY soft-breaking terms, adequately suppressing FCNC and CP-violating phenomena and solving SUSY flavour and CP problems. A recent detailed study of various SF models has been performed in [3], and I will summarize the results of this work here.

We have analysed the following representative scenarios in which NP contributions are characterized by:

(i) The dominance of right-handed (RH) currents (Abelian model by Agashe and Carone [11]),

(ii) Comparable left- and right-handed currents with CKM-like mixing angles represented by the special version (RVV2) of the non-Abelian SU(3) model by Ross, Velasco and Vives [12] as discussed recently by Calibbi *et al.*, and the model by Antusch, King and Malinsky (AKM) [13],

(iii) The dominance of left-handed (LH) currents in non-Abelian models (δLL).

We find [3]:

1. The ratio $\mathrm{Br}(B_d \to \mu^+\mu^-)/\mathrm{Br}(B_s \to \mu^+\mu^-)$ in the AC and RVV2 models is dominantly below its CMFV prediction, and can be much smaller than the latter. In the AKM model this ratio stays much closer to the minimal flavour violation (MFV) value of roughly $1/33$, and can be smaller or larger than this value with equal probability. Still, values of $\mathrm{Br}(B_d \to \mu^+\mu^-)$ as high as 1×10^{-9} are possible in all these models, as $\mathrm{Br}(B_s \to \mu^+\mu^-)$ can be strongly enhanced. We show this in the case of the RVV2 model in the left plot of Fig. 2.

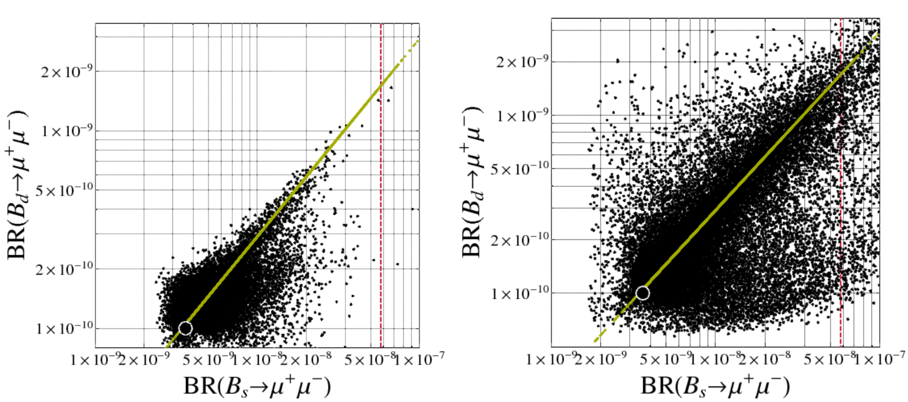

Fig. 2. $B_{d,s} \to \mu^+\mu^-$ branching ratios in the RVV2 model (left) and the δLL model (right) as obtained in [3].

2. Interestingly, in the δLL-models, the ratio $\mathrm{Br}(B_d \to \mu^+\mu^-)/\mathrm{Br}(B_s \to \mu^+\mu^-)$ cannot only deviate significantly from its CMFV value but, in contrast to the models with right-handed currents considered by us, it can also be larger that the MFV value. Consequently, $\mathrm{Br}(B_d \to \mu^+\mu^-)$ as high as $(1\text{–}2) \times 10^{-9}$ is possible while being consistent with the bounds on all other observables, in particular the one on $\mathrm{Br}(B_s \to \mu^+\mu^-)$. We show this in the right plot of Fig. 2.

3. The $S_{\psi\phi}$ anomaly within the supersymmetric flavour models with right-handed currents implies, in the case of the AC and AKM models, values of $\mathrm{Br}(B_s \to \mu^+\mu^-)$ as high as several 10^{-8}. These are very exciting news for the CDF, D0 and LHCb experiments! In the RVV2 model such values are also possible but not necessarily implied by the large value of $S_{\psi\phi}$. We show one example of this spectacular correlation for the case of the AC model in the left plot of Fig. 3.

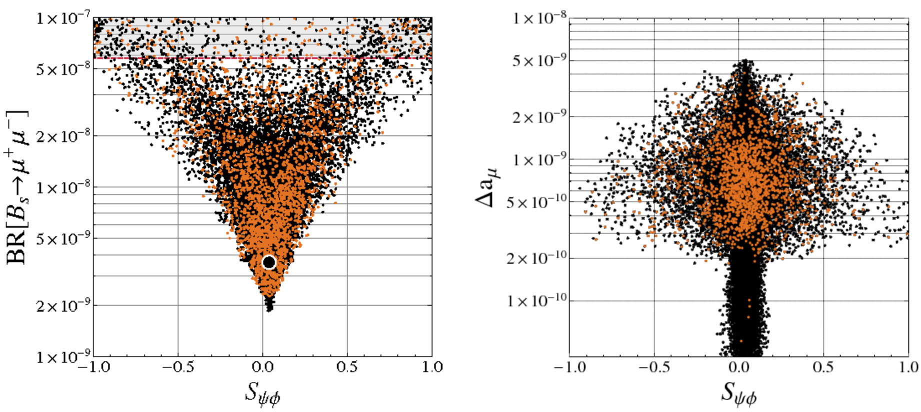

Fig. 3. $\mathrm{Br}(B_s \to \mu^+\mu^-)$ *versus* $S_{\psi\phi}$ (left) and Δa_μ *versus* $S_{\psi\phi}$ (right) in the AC model as obtained in [3]. We denote $\Delta a_\mu = a_\mu - a_\mu^{\mathrm{SM}}$, where $a_\mu = (g-2)_\mu/2$.

4. In the AC model, a large value of $S_{\psi\phi}$ implies a solution to the $(g-2)_\mu$ anomaly, as seen in the right plot of Fig. 3. In the RVV2 and the AKM models, additionally $\mathrm{Br}(\mu \to e\gamma)$ in the reach of the MEG experiment is implied. In the case of the RVV2 model, $d_e \geq 10^{-29} e\,\mathrm{cm}$. is predicted, while in the AKM model it is typically smaller. Moreover, in the case of the RVV2 model, $\mathrm{Br}(\tau \to \mu\gamma) \geq 10^{-9}$ is then in the reach of Super-B machines, while this is not the case in the AKM model. Some of these results are illustrated in Fig. 4.

5. In the supersymmetric models with exclusively left-handed currents (δLL), the desire to explain the $S_{\phi K_S}$ anomaly implies automatically a solution to the $(g-2)_\mu$ anomaly, and the direct CP asymmetry in $b \to s\gamma$

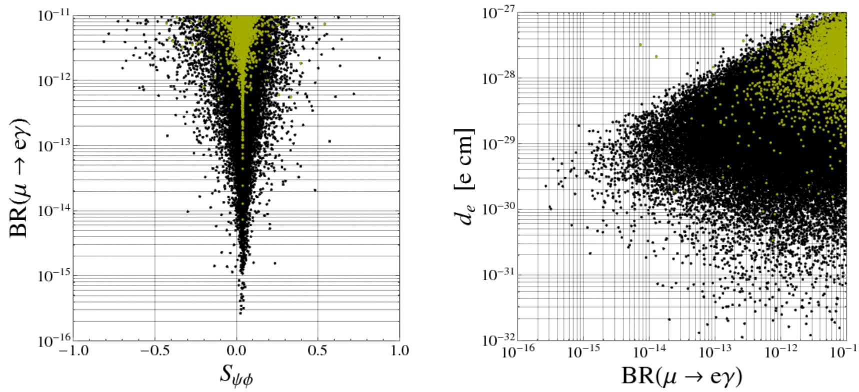

Fig. 4. $\mathrm{Br}(\mu\to e\gamma)$ *versus* $S_{\psi\phi}$ (left) and d_e *versus* $\mathrm{Br}(\mu\to e\gamma)$ (right) in the RVV2 model as obtained in [3]. The grey (*green*) points explain the $(g-2)_\mu$ anomaly at 95% C.L., *i.e.* $\Delta a_\mu \geq 1\times 10^{-9}$.

is much larger than its SM value. We illustrate this in Fig. 5. This is in contrast to the models with right-handed currents where the $A_{\rm CP}^{bs\gamma}$ remains SM-like.

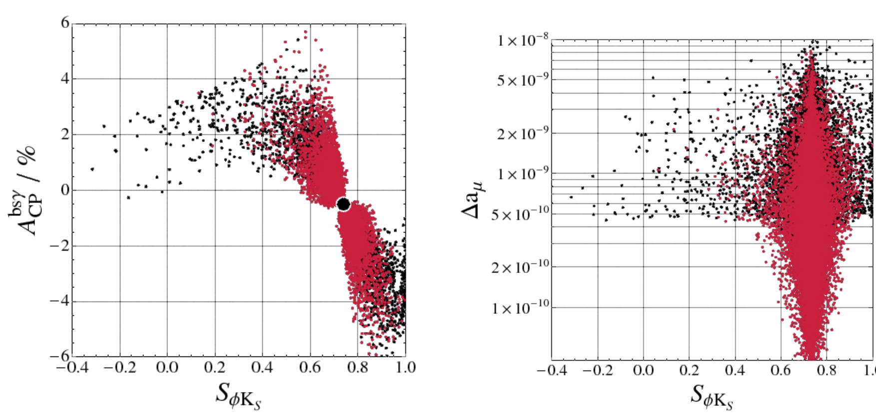

Fig. 5. $A_{\rm CP}^{bs\gamma}$ *versus* $S_{\phi K_S}$ (left) and Δa_μ *versus* $S_{\phi K_S}$ (right) in the δLL model as obtained in [3]. The grey (*red*) points satisfy $\mathrm{Br}(B_s\to\mu^+\mu^-)\leq 6\times 10^{-9}$.

3.4. Maximal enhancements in various models

Finally, we present in Table I approximate maximal enhancements for $\mathrm{Br}(K^+\to\pi^+\nu\bar\nu)$, $\mathrm{Br}(K_L\to\pi^0\nu\bar\nu)$, $\mathrm{Br}(B_s\to\mu^+\mu^-)$ and $S_{\psi\phi}$ in various models. Brief description of all these models can be found in [1].

TABLE I

Approximate maximal enhancements for various observables in different models of NP. In the case of $S_{\psi\phi}$ we give the maximal positive values. The NP models have been defined in [3].

Model	$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$	$\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})$	$\mathrm{Br}(B_s \to \mu^+\mu^-)$	$S_{\psi\phi}$
CMFV	20%	20%	20%	0.04
MFV	30%	30%	1000%	0.04
AC	2%	2%	1000%	1.0
RVV2	10%	10%	1000%	0.50
AKM	10%	10%	1000%	0.30
δLL	2%	2%	1000%	0.04
LHT	150%	200%	30%	0.30
RSc	60%	150%	10%	0.75

4. Final messages and five big questions

In our search for a more fundamental theory we need to improve our understanding of flavour physics. The study of flavour physics in conjuction with direct collider searches for New Physics, with electroweak precision tests and cosmological investigations will result one day in a NSM. When this will happen is not clear at present. After all, 35 years have passed since the completion of the present SM, and no fully convincing candidate for the NSM exists in the literature. On the other hand, in view of presently running and upcoming experiments, the next decade could be like 1970s when practically every year a new important discovery has been made. Even if by 2019 a NSM may not exist yet, it is conceivable that we will be able to answer the following crucial questions by then:

- Are there any fundamental scalars with masses $M_\mathrm{s} \leq 1\,\mathrm{TeV}$?
- Are there any new fundamental fermions like vector-like fermions or the 4th generation of quarks and leptons?
- Are there any new gauge bosons leading to new forces at very short distance scales and an extended gauge group?
- What are the precise patterns of interactions between the gauge bosons, fermions and scalars with respect to flavour and CP Violation?
- Can answers to these four questions help us in understanding the BAU and other fundamental cosmological questions?

There are, of course, many other profound questions in elementary particle physics and cosmology but, from my point of view, I would really be happy if in 2019 satisfactory answers to the five questions posed above were available.

In this presentation I wanted to emphasize that many observables in the quark and lepton flavour sectors have not been measured yet or only poorly measured, and that flavour physics only now enters the precision era. Indeed, spectacular deviations from the SM and MFV expectations are still possible in flavour physics. Interplay of the expected deviations with direct searches at Tevatron, LHC and later at ILC will be most interesting. Finally, correlations between various observables will pave the road to the NSM.

I would like to thank the organizers for inviting me to this very pleasant and interesting workshop. Special thanks go to Mikolaj Misiak for illuminating comments on the manuscript. I would also like to thank all my collaborators for a wonderful time we spent together exploring different avenues beyond the Standard Model. This research was supported by the Deutsche Forschungsgemeinschaft (DFG) under contract BU 706/2-1, the DFG Cluster of Excellence "Origin and Structure of the Universe" and by the German Bundesministerium für Bildung und Forschung under contract 05HT6WOA.

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