LANGEVIN EQUATIONS FOR MODELING EVACUATION PROCESSES*

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In the paper the processes of evacuating rooms and buildings of different types are modeled using Langevin equations with the social force term describing the mental component in pedestrian motion. The level of panic during an evacuation is connected with the desired velocity — a parameter in the social force term. Introducing an additional vertical force exerted on pedestrians on the staircases makes it possible to extend the application of the model to multistorey buildings. Numerical simulations make it possible to observe trajectories of pedestrians and to calculate the time it takes to evacuate different buildings. Factors influencing the effectiveness of evacuation are discussed.

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1. Introduction

During an evacuation in great panic, the flow of pedestrians is disturbed, which may result in injuries or even death. As a result of terrorist attacks, the number of victims in such events has increased in recent years. The 9/11 attack on the World Trade Center in New York in 2001 is well-known. It was carefully analyzed using individual interviews with 3000 witnesses of this catastrophe [1]. It was concluded *e.g.* that in large buildings of this type, in the case of not localized fire or damage whose range exceeds one storey, only approximately 30% of the total number of persons present in the building have a chance of escaping from it [1]. After this catastrophe the number of scientific papers on modeling evacuation processes increased significantly (see *e.g.* [2–7]).

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Different approaches are used for mathematical modeling of pedestrian motion in hazardous situations, which is a case of collective dynamics of intelligent agents [8, 9]. An early approach involved the application of 2-dimensional probabilistic cellular automata, in which the spatial distribution of cells corresponded to the internal geometry of rooms in a building. Cellular automaton for modeling the flow of pedestrians in a room has transition rules defining the shift of a pedestrian to neighboring cells. They can take into consideration specific dynamic phenomena in pedestrian motion (see *e.g.* [2, 3, 7, 9, 10]). Another type of model is based on Navier–Stokes equations, in which pedestrian motion is treated as the flow of a fluid [8].

The application of Langevin equations with additional terms describing the mental component in pedestrian motion is successful in modeling. Initially, Langevin equations were proposed for describing the Brownian motion of particles in a liquid medium [8]. However, in pedestrian motion conservation laws for energy and momentum are not fulfilled because pedestrians — as active particles — are able to suddenly change their velocity and their direction of motion, without interacting with other particles or obstacles. This is so because of the mental processes of each pedestrians, which result from their complex interactions with their surrounding and their general estimation of the situation (*e.g.* the level of hazard in the building). The mental component can be introduced into the Langevin equation in the form of an additional term called social force [6, 11–13].

In the present work modified Langevin equations with the social force term are used for mathematical modeling of pedestrian motion. Numerical simulations of evacuation using this model were investigated for different types of rooms and buildings and for different levels of panic.

2. Langevin equation with the social force term

The stochastic force acting on the particle (which has substantial meaning in the description of Brownian motion [8]) is the most characteristic feature of the Langevin equation. This equation can be also adapted to the modeling of pedestrian motion, provided the social force term, describing the mental component in the pedestrian motion, is added. Such modification of the Langevin equation was proposed by Helbing and Molnar [12]. In our paper we use the following form of the modified Langevin equation [8]:

$$\begin{aligned} \frac{d\vec{v}_i(t)}{dt} &= -\frac{1}{\tau_i}\vec{v}_i(t) + \vec{f}_i(t) + \sqrt{\frac{2\epsilon_i}{\tau_i}}\vec{\xi}_i(t)\,, \\ \frac{d\vec{r}_i}{dt} &= \vec{v}_i(t)\,, \end{aligned} \tag{1}$$

where $\vec{r}_i(t)$ and $\vec{v}_i(t)$ — respectively, position and velocity of i-th particle (pedestrian), $\frac{1}{\tau_i}$ — a damping coefficient; $\vec{f}_i(t)$ — the social force; $\vec{\xi}_i(t)$ — the stochastic force with the amplitude proportional to ϵ_i.

The social force term is a result of psychological and mental processes *e.g.* perception of the current situation (including the recognition of an external hazard), and personal aims and interests. After information processing pedestrians define the parameters of their further motion (velocity, direction, *etc.*) [12]. The form of the social force is [6]:

$$\vec{f}_i(t) = \vec{f}_i^{\,0}(v_{\mathrm{D}}) + \sum_{j \neq i} \vec{f}_{ij}(\vec{r}_i - \vec{r}_j) + \sum_{B \neq i} \vec{f}_{iB}(\vec{r}_i - \vec{r}_B)\,, \tag{2}$$

where the first term $\vec{f}_i^{\,0}(v_{\mathrm{D}}) = \frac{v_{\mathrm{D}}\vec{e}_{i\mathrm{D}}}{\tau_i}$ defines the tendency of the pedestrian to move in a desired direction $\vec{e}_{i\mathrm{D}}$, with a *desired velocity* v_{D}; the second term is the *territorial effect*, which is connected with the tendency of pedestrians to avoid other pedestrians and obstacles. The territorial effect is described using the repulsion force:

$$\vec{f}_{ij}(\vec{r}_i - \vec{r}_j) = A \exp\left(\frac{-\epsilon_{ij}}{C}\right) \vec{e}_{ij}^{\,n}\,, \tag{3}$$

where $\epsilon_{ij} = (r_i - r_j) - (R_i + R_j)$, A and C — scaling constants; R_i and R_j — the width of the shoulders of i-th and j-th pedestrians; r_i and r_j — the positions of i-th and j-th pedestrians [8].

The third term is the sum of forces coming from granular interactions with other pedestrians and obstacles (in location $\vec{r}_B$) in the surrounding of the i-th pedestrian; and has the following form [6]:

$$\vec{f}_{iB} = \left[(-\epsilon_{iB}k_n - \gamma v_{iB}^n)\vec{e}_{iB}^{\,n} + (v_{iB}^t \epsilon_{iB} k_t)\vec{e}_{iB}^{\,t}\right] g(\epsilon_{iB})\,, \tag{4}$$

where ϵ_{iB} — distance of i-th pedestrian from the surface of obstacle B minus the width R_i of the shoulder of this pedestrian (in the case of other pedestrians $\epsilon_{iB} = \epsilon_{ij}$); k_n and k_t — respectively, normal and tangent components of quasielastic force; v_{iB}^n and v_{iB}^t — respectively, normal and tangent components of relative velocity of i-the pedestrian; $\vec{e}_{iB}^{\,n}$ and $\vec{e}_{iB}^{\,t}$ — respectively, normal and tangent versors of the segment connecting i-th pedestrian with the obstacle B; $g(\epsilon_{iB}) = 1$, if $\epsilon_{iB} < 0$, otherwise $g(\epsilon_{iB}) = 0$ [6].

In the modeling of pedestrians motion in the staircase, gravity force, describing pedestrians' vertical motion, was added to the three types of forces in Eq. (2): $f_g = -mg$, where $m = 80$ kg. This makes describing pedestrian motion in multistorey buildings possible and extends the applicability of the presented model.

Another important feature of this model is that it considers different levels of panic. This is a complex phenomenon mainly studied from the perspective of social psychology. Investigations indicate that the movements of pedestrians in great panic are uncoordinated, which results in jamming and life-threatening overcrowding [11]. In the present paper we use the desired velocity v_{D} as a measure of the level of panic (see Eq. 2). The value of this parameter equals the velocity the pedestrian wants to reach. It is higher than the pedestrian's real velocity v_{r}, because of its interactions with other pedestrians and obstacles and the greater the value of v_{D} the greater the level of panic. Investigations of evacuation processes indicate that optimal velocity of evacuation is about 1.3 m/s [13,14]. In our paper we assume that pedestrians do not perceive any hazard in their surrounding if v_{D} does not exceed 2 m/s. Then, only social repulsion (Eq. 3) is present and no granular interactions (Eq. 4) between pedestrians take place [6,13,14].

The numerical program for solving the set of N Langevin equations (1) also contains a part for designing the internal architecture of the rooms and the building in which the evacuation process of N pedestrians will be investigated. The values of the parameters used in the computations were: $\tau_i = 0.5$ sec; $k_n = 1.2 \times 10^5 N$; $k_t = 2.4 \times 10^5 N$; $\gamma = 100$ kg/s; $A = 1000\,N$; $C = 0.08$ m. The grid edge in the figures is 1 m long.

3. Pedestrian motion in selected types of rooms

If there is no hazard *i.e.* for the values of pedestrian motion with $v_{\mathrm{D}} < 2$ m/s laminar motion is observed. Evacuation from office rooms, the warehouse and the classroom are examples of such motion. The pedestrians' positions in these rooms are shown in Fig. 1. Their trajectories for

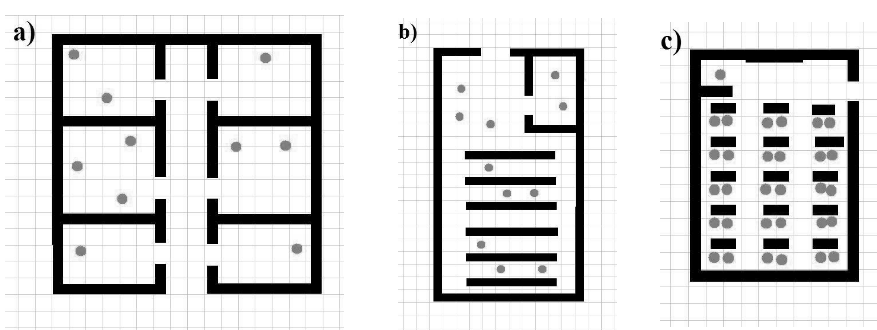

Fig. 1. Geometry and initial positions of N pedestrians in the rooms under investigation: (a) — office rooms ($N = 10$), (b) — a warehouse ($N = 11$), (c) — a classroom ($N = 31$).

$v_{\rm D} = 1$ m/s are shown in Fig. 2. It can be seen that for the cases of office rooms and the warehouse (Fig. 2 (a),(b)) each pedestrian's trajectory is smooth and not perturbed, *i.e.* pedestrian motion in these rooms and in these conditions is laminar. In the case of the classroom (Fig. 2 (c)), however, the trajectories near the door are denser. This is so because of the relatively large number of pedestrians ($N = 31$) in the classroom and the presence of a number of desks. Also an increase in the number of pedestrians in the room can change the character of the trajectories. Fig. 2 (d) shows the trajectories of 50 pedestrians initially present in the office rooms. A comparison of the course of trajectories in this case *i.e.* in overcrowded rooms with the previous cases (see Fig. 2 (a),(b)) reveals significant perturbations of motion and crowding (clogging) of pedestrians near the door.

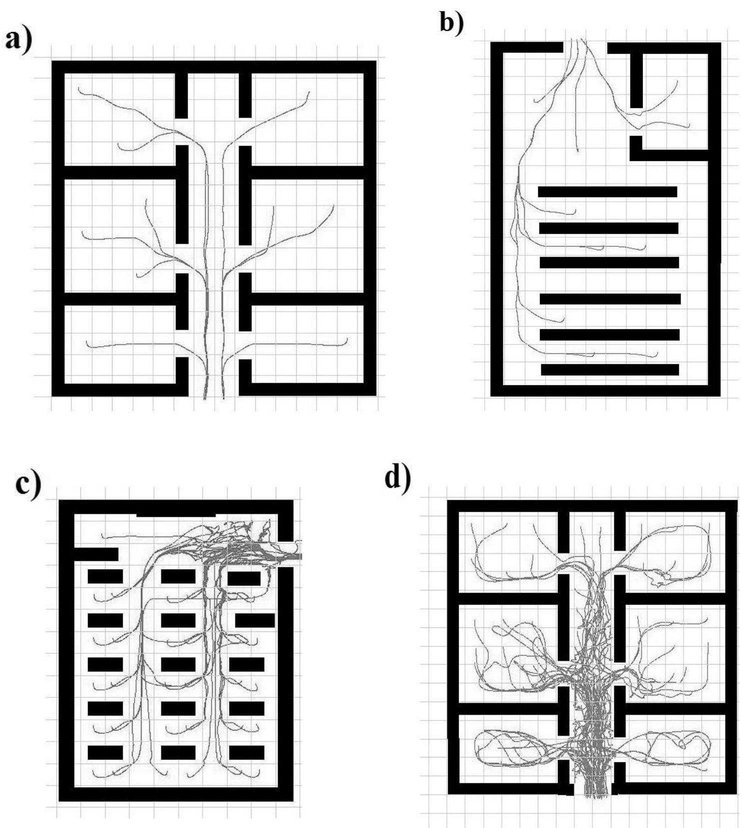

Fig. 2. Trajectories of pedestrians obtained in numerical simulations for $v_{\rm D} = 1$ m/s; (a) — office rooms, (b) — a warehouse, (c) — a classroom, (d) — overcrowded office rooms, $N = 50$ (*cf.* Fig. 1 (a)).

The relations between the evacuation time T and the desired velocity v_{D} are shown in Fig. 3. It can be seen that the in the cases of office rooms with a normal number of pedestrians inside (Fig. 3 (a) and the warehouse (Fig. 3 (b), an increase in the values of desired velocities causes a decrease in evacuation times. This is an evidence of the laminar character of pedestrian motion.

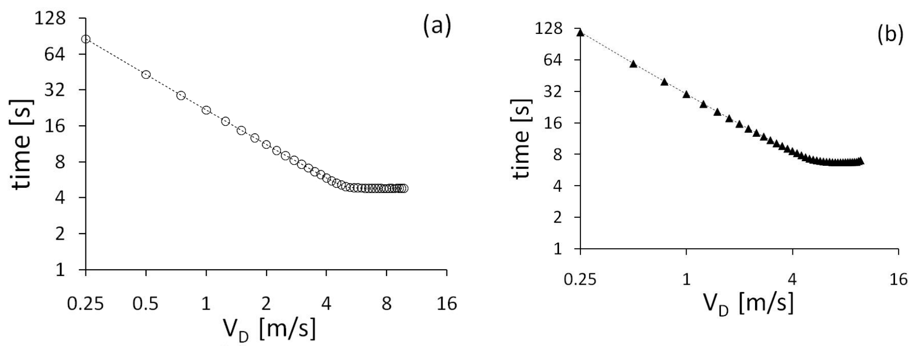

Fig. 3. Times of evacuation T as a function of desired velocity v_{D}; (a) — office rooms; (b) — a warehouse.

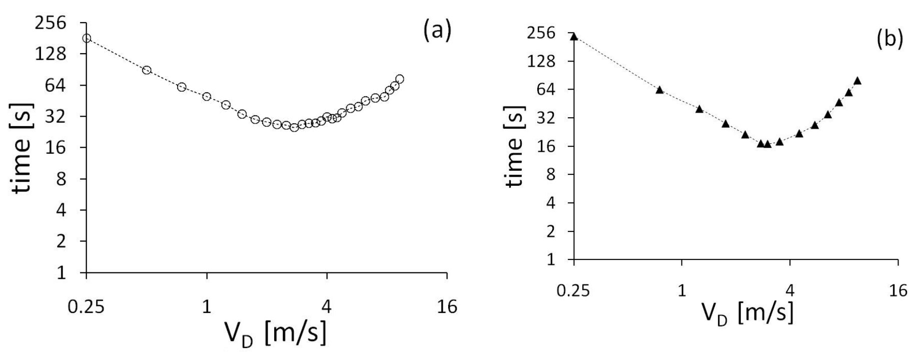

Fig. 4. Times of evacuation T as a function of desired velocity v_{D}; (a) — a classroom; (b) — overcrowded office rooms.

In a special conditions and for high values of desired velocity (*i.e.* an increase in the level of panic) laminar motion of pedestrians disappears and turbulent motion can emerge in an evacuation process. Such type of pedestrian motion is observed in the case of the classroom. Due to the dense crowding of pedestrians near the door, the relation $T(v_{\mathrm{D}})$ starts to increase for $v_{\mathrm{D}} > 3.8$ m/s, *i.e.* the time of evacuation increases with an increase in v_D (Fig. 4 (a)) [13]. The situation is similar for the case of overcrowded office rooms — the relation $T(v_D)$ for this case is shown in Fig. 4 (b) (*cf.* this figure with Fig. 3 (a)).

It is interesting to observe the relation between the reduced real velocity $\frac{v_i}{v_D}$ as a function of time. This relation can reveal the details of the dynamics of an individual pedestrian. This is shown in Fig. 5 for the case of a simple rectangular room, $v_D = 3$ m/s and a pedestrian who is marked in black. Rapid acceleration of the pedestrian in the lower, almost empty, part of the room can be seen: real velocity quickly approaches desired velocity. In the next part of the motion the marked pedestrian is located close to a number of neighbors. They all have very similar velocity, therefore the velocity of the marked pedestrian saturates. In the nearest neighborhood of the door dense crowding leads to a sudden decrease in real pedestrian velocity at the moment of passing through the door it is almost zero. After passing though the door pedestrians rapidly accelerate, but the fluctuations of their velocity are visible. They come from the random interactions with other pedestrians running in the same direction.

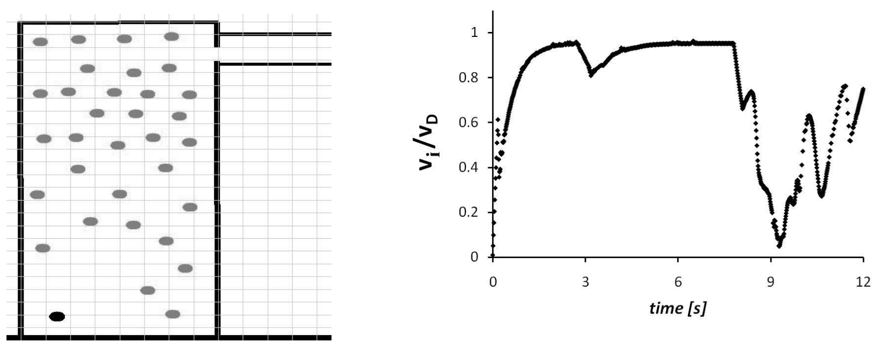

Fig. 5. The relation between the reduced real velocity of the pedestrian $\frac{v_i}{v_D}$ and the time for the case of evacuation from the rectangular room shown in the left part of the figure. The observed pedestrian's initial position is marked in black; $v_D = 3$ m/s.

If there is great panic, turbulent motion and a surge of pedestrians, they can fall. Thus they become obstacles for other pedestrians and their trajectories have to be suddenly modified (see Fig. 6). In great panic the trajectories of pedestrians can lead over the lying ones and they can be trampled. Such random falls have an important influence on the motion of other pedestrians and significantly increase the time of evacuation.

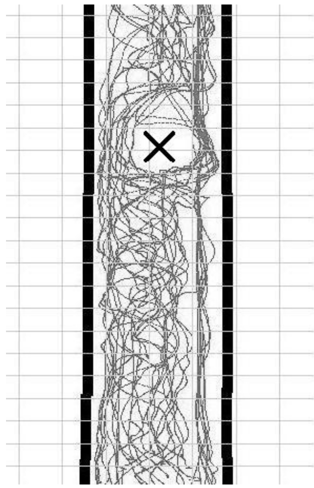

Fig. 6. A falling pedestrian causes a rapid modification of the trajectories of other pedestrians ($v_{\mathrm{D}} = 2.5$ m/s).

4. Pedestrian motion in buildings

As an example of evacuation processes from buildings we chose a 3-storey office building and a movie theater auditorium. The geometry and initial positions of the pedestrians in both buildings are shown in Fig. 7.

The pedestrians' trajectories for $v_{\mathrm{D}} = 2.5$ m/s, resulting from a simulation of an evacuation process from a movie theater, is shown in Fig. 8 (a). Because of the large number of pedestrians, the trajectories are significantly denser especially in the corridors on both sides of the building leading to the door. The relation $T(v_{\mathrm{D}})$ (see Fig. 9 (a)) increases for higher values of v_{D}, which shows that pedestrian motion is turbulent. An observation of individual pedestrian's trajectories shows that an increase in the times of evacuation for $v_{\mathrm{D}} > 4$ m/s is connected with the crowding of pedestrians near the door. In this case, the width D of the door has a great influence on the time of evacuation. This relation was calculated for different values of D (Fig. 10). It can be seen that relatively small changes of D can drastically decrease the time of evacuation T. For $D = 1$ m and a high level of panic — $v_{\mathrm{D}} = 5$ the time of evacuation equals 110 s, while for a door twice as wide ($D = 2$ m/s) it decreases to 35 s.

The case of multiple-storey building is shown in Fig. 7 (b), where the geometry of the first, second and third storeys and a 3-dimensional graphic of rooms (Fig. 7 (c)), are shown. Pedestrians from the upper storeys enter the staircase (marked in gray), thus the lower the level of the staircase, the

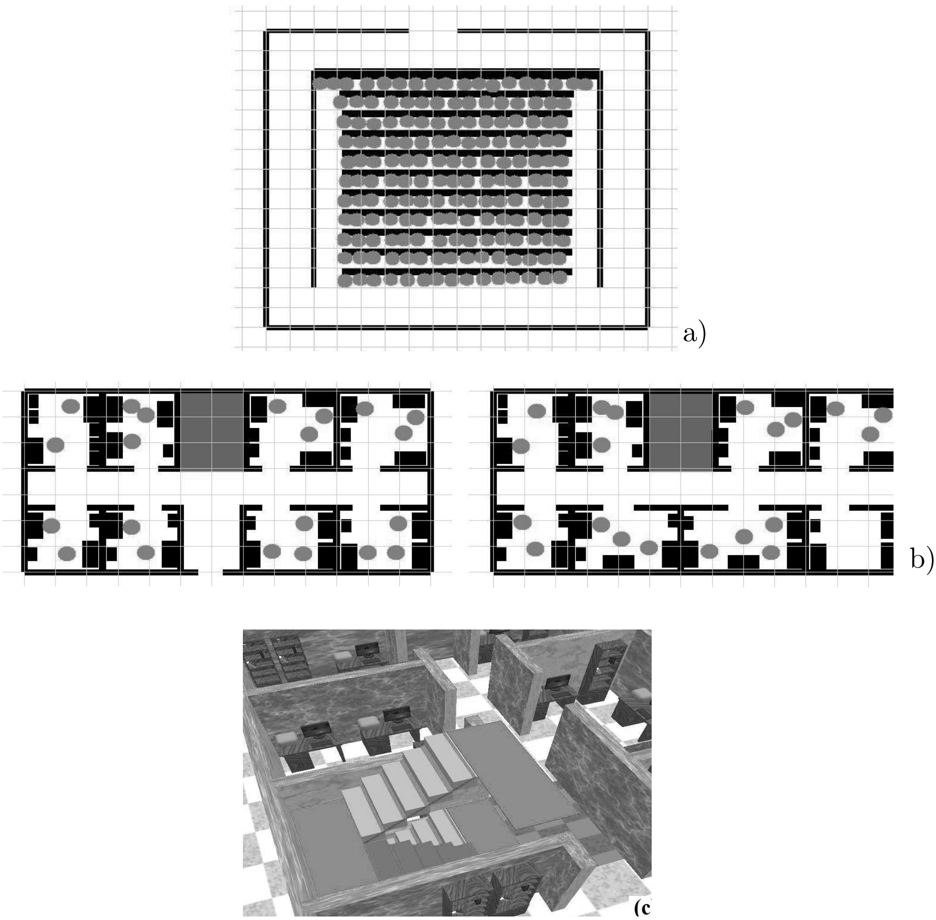

Fig. 7. The geometry and the pedestrians' initial positions in two types of buildings under investigation: a 3-storey office building and a movie theater. (a) — the movie theater; (b) — the office building, left — first (ground) storey, right — second and third storeys; (c) — a 3-dimensional graphic of a storey in the office building.

larger the number of pedestrians present in the staircase. The trajectories of pedestrians illustrate this (Fig. 8 (b)): denser crowding near the entrance to the staircase is greater in the second storey than in the third. This phenomenon is exacerbated when high buildings are evacuated as in emergencies elevators stop working. Then dynamical phenomena in the staircases and their geometry have fundamental meaning in the evacuation process. Unfortunately, stairs in modern high buildings are not designed to handle a full evacuation [1].

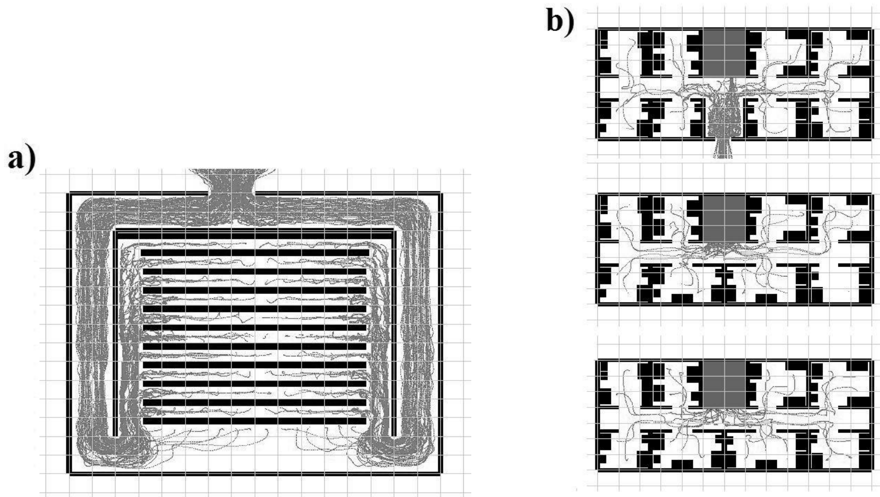

Fig. 8. Pedestrians' trajectories during the evacuation of the movie theater auditorium — (a), and the office building — (b); $v_{\rm D} = 2.5$ m/s.

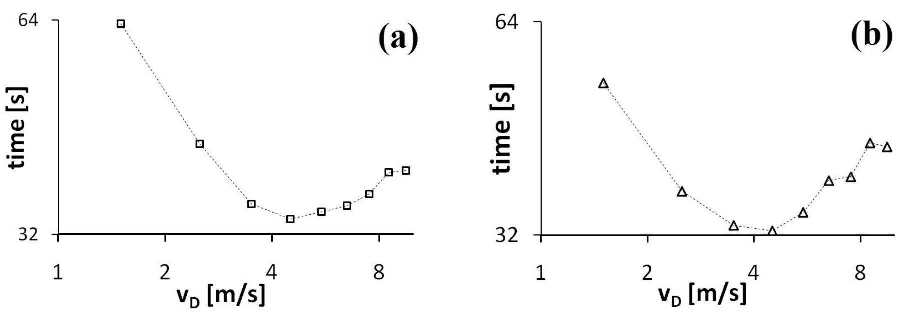

Fig. 9. Times of evacuation T as a function of desired velocity $v_{\rm D}$ in double logarithmic scale; (a) — a movie theater auditorium; (b) — office rooms.

In conclusion, it can be said that mathematical modeling of pedestrian motion in evacuation processes using Langevin equations with the social force and vertical force on staircases makes it possible to investigate this process in the different buildings, including multistorey buildings, and to compare the effectiveness of evacuation for different internal geometry. It should be stressed that numerical simulations based on this mathematical model make it possible to observe dynamical phenomena which have an important influence on the evacuation, *e.g.* crowding and falling down of

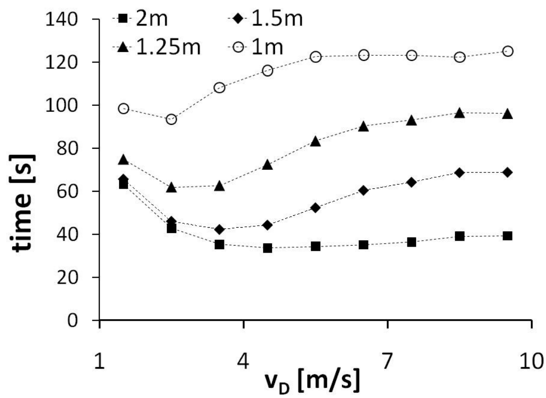

Fig. 10. Influence of the door width D on the relation between time of evacuation and desired velocity for the movie theater auditorium.

pedestrians. It is also possible to compare the effectiveness of an evacuation process for different internal geometry of buildings (*e.g.* a different number and width of doors in rooms, or different location and geometry of staircases). Such observations can be useful in planning evacuation strategies in buildings as well as in training of emergency services.

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