FLUCTUATIONS IN RELATIVISTIC HEAVY-ION COLLISIONS FROM THE GLAUBER MODELS*

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In the first part of the paper we discuss the role of the two-body nucleon–nucleon correlations on signatures of the heavy-ion collisions which are *a priori* expected to be sensitive to these effects. We find that while the fluctuations of the number of produced particles are indeed affected, other quantities (v_2 fluctuations, size fluctuations) are insensitive to the presence of the NN correlations in the nucleon distributions. In the second part we show that the fluctuations of the transverse size of the initial source cause, after a suitable hydrodynamic evolution, fluctuations of the transverse flow velocity at hadronic freeze-out. This in turn yields the event-by-event fluctuations of the transverse momentum of the produced particles, $\langle p_{\rm T} \rangle$. Our results demonstrate that practically all of the observed event-by-event $\langle p_{\rm T} \rangle$ fluctuations may be explained this way.

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1. Introduction

This paper consists of two independent parts, both related to novel aspects of the correlations and event-by-event fluctuations present in the commonly used Glauber treatment of the early phase of the ultra-relativistic heavy-ion collisions.

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2. NN correlations

The standard way of generating the nucleon distribution in a nucleus for studies of ultra-relativistic heavy-ion collisions is to *independently* populate the nucleus according to a one-body density in the form of the Woods–Saxon function. Thus, the typically used procedure completely neglects the NN correlations (apart for the poor man's implementation of the hard-core repulsion, precluding the centers of the nucleons to be closer than a certain distance), known to be crucial in a nuclear system. Recently, however, Alvioli, Drescher, and Strikman [1] published nuclear distributions for several nuclei which fully account for the central two-body NN correlations, thus accomplishing the long-awaited task [2, 3].

In this work we use the distributions from $[1]^1$ in GLISSANDO [4] to investigate within the Glauber treatment the role of the NN correlations on several signatures of heavy-ion collisions. We use the wounded-nucleon model [5], however we have checked that the results in other variants [6] of the Glauber approach are qualitatively similar. Such more detailed studies will be published elsewhere. We recall that the nucleons from the two nuclei get wounded when their centers pass closer to each other than the distance $d = \sqrt{\sigma_{NN}/\pi}$, where σ_{NN} is the inelastic nucleon–nucleon cross-section. For the highest SPS, RHIC, and LHC energies it is equal to 32, 42, and 63 mb, respectively.

Our results for the ²⁰⁸Pb–²⁰⁸Pb collisions are shown in Fig. 1. The notation is as follows: $N_{\rm W}$ — the total (projectile+target) number of wounded nucleons, $N_{\rm P}^{\rm PROJ}$ — the number of wounded nucleons in the projectile, and $N = N_{\rm W}/2$ denotes the number of the wounded-nucleon pairs. We note that except for the fluctuations of the total number of the wounded-nucleon pairs plotted as a function of the wounded nucleons in the projectile (the NA49 setup [7], where the VETO calorimeter essentially measures $N_{\rm P}^{\rm PROJ}$), other investigated quantities are not affected by the inclusion of the NN correlations. While for the one-body observables from the left-side panels this is expected, as two-body correlations by definition do not affect one-body observables, the weakness of the effect in the fluctuation of the eccentricities ε and ε^* [6] is somewhat surprising. Our conclusions for ε agree with similar observations drawn in [8].

We conclude that apart for the multiplicity fluctuations, the neglect of the NN correlations in numerous previous studies of relativistic heavy-ion collisions was innocuous. In particular, the previous studies of v_2 and its fluctuations [9–16] are not affected by the NN correlations in the nucleon distributions.

¹ Publicly available at http://www.phys.psu.edu/~malvioli/eventgenerator



Fig. 1. Various quantities computed for the ²⁰⁸Pb–²⁰⁸Pb collisions without (solid line) and with the NN correlations (dashed line) in the wounded-nucleon model for $\sigma_{NN} = 32$ mb. The top panels show the mean total (projectile+target) number of the wounded-nucleon pairs and its scaled variance as a function of the number of the wounded nucleons in the projectile. The middle and bottom panels show the fixed-axes (standard) and variable-axes (participant) eccentricities [6] and their scaled standard deviation as functions of the total number of wounded nucleons. The horizontal line in the bottom-right panel indicates the limit $\sqrt{4/\pi - 1} \simeq 0.52$ derived in [6].

3. $p_{\rm T}$ fluctuations

The $p_{\rm T}$ fluctuations in relativistic heavy-ion collisions have been a subject of intense studies [17–32]. Despite numerous theoretical efforts, up to now the magnitude and centrality dependence of these correlations has not been convincingly understood. In this part of the paper we present a very simple mechanism, capable of describing very well the data. The approach is described in a greater detail in [33]. It generates the event-by-event $p_{\rm T}$ -fluctuations based on the fluctuations of the initial size of the formed system and its subsequent hydrodynamic evolution followed by statistical hadronization (we use the single-freeze-out variant from [34–37]). Due to its statistical nature, the Glauber approach leads to an initial configuration of the wounded nucleons (or binary collisions) which are randomly distributed. This promptly yields the fluctuations of the initial transverse size. In short, this is the scheme: smaller initial size has more compression, leading to faster hydrodynamic expansion, larger flow at freeze-out, and, finally, larger transverse momenta, and vice versa. We note that the effects of inhomogeneities in the initial condition for some observables have been studied in [38]. The event-by-event fluctuations of the initial shape have been studied in detail for its elliptic component, where they cause enhancement of the elliptic flow [6, 9-14, 39-42].

We define the average transverse size (in the wounded nucleon model for the simplicity of notation) as

$$\langle r \rangle = \sum_{i=1}^{N_{\rm W}} \sqrt{x_i^2 + y_i^2} \,, \tag{1}$$

where x_i and y_i are coordinates of a wounded nucleon in the transverse plane. The original positions of nucleons in each nucleus are randomly generated from an appropriate Woods–Saxon distribution. The notation $\langle \langle . \rangle \rangle$ indicates averaging over the events. In order to focus on the relative size of the effect we use the scaled standard deviation, defined for a fixed value of $N_{\rm W}$ as $\sigma(\langle r \rangle)/\langle \langle r \rangle \rangle$.

The results of our Monte Carlo simulations for ¹⁹⁷Au–¹⁹⁷Au performed with GLISSANDO [4] in the wounded nucleon model are shown in Fig. 2. The three curves overlap, showing insensitivity to the value of σ_{NN} in the considered range. We note that the scaled standard deviation of $\langle r \rangle$ is about 2–3% for central collisions, and grows towards the peripheral collisions approximately as $1/\sqrt{N_W}$.

Qualitatively very similar results are obtained for other variants, in particular for the mixed model and models with superimposed distribution of particles produced by each wounded nucleon [4]. We have also checked that using a Gaussian wounding profile $\sigma_{NN}(b)$ [43] for the NN collision,



Fig. 2. Event-by-event scaled standard deviation of the size parameter $\langle r \rangle$, evaluated at fixed values of the number of wounded nucleons, $N_{\rm W}$, for several values of σ_{NN} in ¹⁹⁷Au–¹⁹⁷Au collisions.

rather than the sharp wounding distance criterion applied here, leads to indistinguishable curves. Also, the use of the nucleon distributions including realistically the NN correlations, as described in the first part of this paper, leads to practically no difference. In other words, the behavior displayed in Fig. 2 is robust, reflecting the random nature of the Glauber approach.

The next step, crucial in converting the size fluctuation into the observable momentum fluctuations, is hydrodynamics. We use the hydrodynamic approach of [44], followed with statistical hadronization as implemented in THERMINATOR [45]. The goal is to find how exactly the size fluctuations get converted into the $p_{\rm T}$ fluctuations. Rather than doing tedious event-by-event hydrodynamic calculations, it is enough to see how much the results change when the size of the initial profile is scaled. The procedure presented below works, since the studied fluctuations follow from the initial conditions, while the differential equations of hydrodynamics are deterministic.

The event-by-event distribution of $\langle r \rangle$ is approximately Gaussian,

$$f(\langle r \rangle) \sim \exp\left(-\frac{(\langle r \rangle - \langle \langle r \rangle \rangle)^2}{2\sigma^2(\langle r \rangle)}\right).$$
 (2)

Suppose first that we run the simulations at a fixed value of $\langle r \rangle$ and as the result obtain a certain average transverse momentum, $\bar{p}_{\rm T}$. Because of the deterministic nature of hydrodynamics, $\bar{p}_{\rm T}$ is a (very complicated) function of $\langle r \rangle$. Its value fluctuates because of the fluctuations of the initial size. Now, we can expand near the central value:

$$\bar{p}_{\rm T} - \langle \langle p_{\rm T} \rangle \rangle = \left. \frac{d\bar{p}_{\rm T}}{d\langle r \rangle} \right|_{\langle r \rangle = \langle \langle r \rangle \rangle} \left(\langle r \rangle - \langle \langle r \rangle \rangle \right) + \dots \tag{3}$$

Substituting (3) into (2) and comparing to (6) we get the key formula

$$\sigma_{\rm dyn}^2 = \sigma^2(\langle r \rangle) \left(\left. \frac{d\bar{p}_{\rm T}}{d\langle r \rangle} \right|_{\langle r \rangle = \langle \langle r \rangle \rangle} \right)^2 \,, \tag{4}$$

or, for the scaled standard deviation,

$$\frac{\sigma_{\rm dyn}}{\langle \langle p_{\rm T} \rangle \rangle} = -\frac{\sigma(\langle r \rangle)}{\langle \langle r \rangle \rangle} \frac{\langle \langle r \rangle \rangle}{\langle \langle p_{\rm T} \rangle \rangle} \left. \frac{d\bar{p}_{\rm T}}{d\langle r \rangle} \right|_{\langle r \rangle = \langle \langle r \rangle \rangle} \,. \tag{5}$$

The full statistical distribution $f(\langle p_{\rm T} \rangle)$ is a folding of the statistical distribution of $\langle p_{\rm T} \rangle$ at a fixed initial size, centered around $\bar{p}_{\rm T}$, with the distribution of $\bar{p}_{\rm T}$ centered around $\langle \langle p_{\rm T} \rangle \rangle$. In the central regions both are close to Gaussian distributions, hence we have, to a very good approximation,

$$f(\langle p_{\rm T} \rangle) \sim \int d^2 \bar{p}_{\rm T} \exp\left(-\frac{(\langle p_{\rm T} \rangle - \bar{p}_{\rm T})^2}{2\sigma_{\rm stat}^2}\right) \exp\left(-\frac{(\bar{p}_{\rm T} - \langle \langle p_{\rm T} \rangle \rangle)^2}{2\sigma_{\rm dyn}^2}\right).$$
(6)

Carrying out the $\bar{p}_{\rm T}$ integration yields the Gaussian event-by-event distribution of $\langle p_{\rm T} \rangle$ centered around $\langle \langle p_{\rm T} \rangle \rangle$ with the width parameter satisfying $\sigma^2 = \sigma_{\rm stat}^2 + \sigma_{\rm dyn}^2$. Statistical procedures used in experimental analyses of fluctuations are designed in such a way that the dynamical component of the fluctuations is extracted. In our case, where the source of fluctuations is in the initial condition, we find the simple formula (4), which may be compared to the experimental $\sigma_{\rm dyn}$.

The derivative in Eqs. (4), (5) can be computed numerically without difficulty by running just two simulations at each centrality. We perform calculations for initial profiles which are squeezed or stretched by 5%. In addition to squeezing or stretching, we also simultaneously adjust the central temperature in such a way, that the energy contained in the profile is preserved. This is natural, as the total energy deposited in the transverse plane should be the same (up to possible additional fluctuations neglected here) for a given number of elementary collisions. Thus, we include in some sense also the temperature fluctuations discussed, *e.g.*, in [46].

Our final result is shown in Fig. 3, where we compare the model points to the data from the STAR [30] and PHENIX [29] Collaborations. The experimental cuts of the STAR detector have been used in our model simulations, 0.2 GeV $< p_T < 2$ GeV. Looking at Fig. 3, we note a strikingly good agreement between our calculation and the experiment, in particular for the wounded nucleon model. The mixed model, admittedly more realistic than the wounded-nucleon model, overshoots the data by about 20%. This suggests that the coefficient in (5) is somewhat too large. Its value incorporates all the dynamics of the system (the choice of the initial profile, details



Fig. 3. Scaled dynamical transverse-momentum fluctuations, $\sigma_{\rm dyn}/\langle\langle p_{\rm T}\rangle\rangle$ (for $\sigma_{NN} = 42$ mb in ¹⁹⁷Au⁻¹⁹⁷Au collisions) compared to the experimental data from STAR [30] and PHENIX [29]. The lower (upper) crosses indicate our results for the wounded nucleon model (mixed model). The STAR experimental data range from $\sqrt{s_{NN}} = 20$ GeV (triangles), through 130 GeV (squares), 62 GeV (diamonds), to 200 GeV (dots). The PHENIX data (dots with large systematic error bars) are for 200 GeV.

of hydrodynamics, the statistical hadronization), hence modifying any of these components, *e.g.* including viscosity effects, will lead to changes. We note a proper dependence on centrality, with an approximate dependence $\sigma_{\rm dyn}(\langle p_{\rm T} \rangle)/\langle \langle p_{\rm T} \rangle \rangle \sim 1/\sqrt{N_{\rm W}}$. Since the results of Fig. 2 very weakly depend on σ_{NN} , to the extent that the hydrodynamic "pushing" is similar at various energies, our results should also weakly depend on the incident energy, which is a desired experimental feature in view of the STAR results.

Finally, we note that the result (5) bears similarity to the formula derived by Ollitrault [47], where

$$\frac{\sigma_{\rm dyn}}{\langle\langle p_{\rm T}\rangle\rangle} = \frac{P}{\epsilon} \frac{\sigma(\langle s\rangle)}{\langle\langle s\rangle\rangle} = \frac{2P}{\epsilon} \frac{\sigma(\langle r\rangle)}{\langle\langle r\rangle\rangle},\tag{7}$$

with s denoting the entropy density, P — the pressure, and ϵ — the energy density. The second equality follows from the assumption that the total entropy deposited in the transverse plane depends only on the number of collisions and not on the size, hence $\langle s \rangle \sim 1/\langle r \rangle^2$ [48]. The coefficient P/ϵ in Eq. (7) is to be understood in the averaged sense over space and time. Numerically, we find

$$\langle \langle r \rangle \rangle / \langle \langle p_{\rm T} \rangle \rangle \, d\bar{p}_{\rm T} / d \langle r \rangle |_{\langle r \rangle = \langle \langle r \rangle \rangle} \sim -0.4 \,,$$
(8)

independent of centrality. This yields the average P/ϵ about 0.2, which is in the expected ball park of realistic equations of state [49, 50], see Fig. 4².



Fig. 4. Estimated *average* value of P/ϵ based on Eq. (8) (band), compared to the curve from the equation of state from [49, 50].

4. Conclusion

Here are our main conclusions:

- 1. The inclusion of the two-body NN correlations is important for the fluctuations of the number of produced particles. Other observables, such as the shape eccentricity fluctuations, carried over to the v_2 fluctuations, are not affected.
- 2. We reproduce the dynamical event-by-event $p_{\rm T}$ fluctuations, as measured at RHIC by STAR and PHENIX, with the simple mechanism based on fluctuations of the initial size, which are then carried over by hydrodynamics to the fluctuations of the transverse flow velocity, and consequently to the transverse momentum of the produced particles. The hydrodynamic "push" is crucial in this scheme. Other possible sources of fluctuations, such as the formation of clusters at freeze-out [51–54], minijets [29, 55], or correlations originating from the elementary NN collisions in the corona in the core-corona picture [56–59], should all be considered at the "background" of the fluctuations described in this paper.
- 3. The hydrodynamic push is related to the equation of state, in particular to the ratio P/ϵ averaged over space and time. Thus, interestingly, the $p_{\rm T}$ correlations carry, via their relation to the size fluctuations, information on the stiffness of the medium.

² Presented by M. Chojnacki at the V Workshop on Particle Correlations and Femtoscopy, CERN, October 14–17, 2009.

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