THE CASIMIR EFFECT*

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The Casimir effect is usually interpreted as due to the modification of the zero point energy of QED when two perfectly conducting plates are put very close to each other, and as a proof of the "reality" of this zero point energy. The Dark Energy, necessary to explain the acceleration of the expansion of the Universe is sometimes viewed as another proof of the same reality. The usual interpretation of the Casimir effect is however challenged by some authors who rather consider it as a "giant" van der Waals effect. All these aspects are shortly reviewed.

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1. Introduction

The organizers of the XXVI Max Born Symposium have kindly invited me to present a lecture on an out-of-the-main-stream topic of my choice. I decided to make a short review on the Casimir effect (CE). The reason is that I discovered recently, while preparing a course on "Astroparticles" in my University, that the conventional wisdom about the CE is seriously challenged by many authors. In short (more details are given below), the CE is a quantum force between two uncharged conducting plates. It is usually considered as originating from the differences of the QED ground state energies when boundary conditions are modified. The ground state energy is not directly measurable, but the CE, which corresponds to a mere change of this energy when boundary conditions are modified, is generally considered as a proof of the physical reality of this ground state energy. In the recent years, this interpretation has been revisited and some authors even claim that the CE cannot really test the ground state energy. On the other hand, the field is booming. The CE is an everyday feature in nanotechnology.

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Furthermore, some theorists are trying to calculate the effect of modifying boundary conditions on the ground state energy of quantum field theories and some others are interested in the formulation of quantum field theory without reference to the ground state energy. From this point of view, the CE may not completely appear as an odd topic in this Symposium since many contributors spoke about the properties of the QCD vacuua. I will shortly review some of these aspects, occasionally putting them into an historical perspective.

2. The vacuum energy

Whether the vacuum is gifted with physical properties is an issue that has occupied physicists since the ancient Greeks. Einstein himself [1], after having deprived the Lorentz ether of its physical properties, argued for the existence of an ether endowed with physical qualities. Later, in the canonical formulation of quantum field theory, the vacuum, *i.e.* the ground state of a free quantum field, acquired physical properties. For instance, it is a polarisable medium. For our purpose here, it has also an infinite energy density, due to the so-called zero-point motion (or quantum vacuum fluctuations) of the field. Since this energy is infinite and cannot be tapped, it was rather considered as unphysical and forgotten. The theoretical prediction of the CE did not really modify the attitude of the physicists. But in the recent years, the situation has evolved, due to the realisation of precise measurements of the Casimir force and to the advent of the concept of dark energy in cosmology, on which I will elaborate a bit.

It is now more and more evident from the Hubble plot at large red shifts [2] and from fluctuations of the Cosmic Microwave Background [3] that the Universe is expanding at an accelerating rate, under the presence of a Dark Energy (DE) which pervades all the Universe and which seems to be a property of the vacuum (not of the matter nor of the radiation). This dark energy is characterized by a negative pressure and a constant and uniform energy density $\Omega_{\rm DE}$, whose value can be extracted from observation. It is equal to

$$\Omega_{\rm DE} \simeq 3/4\Omega_{\rm c} \simeq 4\,{\rm GeV/fm}^3\,, \tag{1}$$

where $\Omega_{\rm c}$ is the critical energy density, which is about the entire present energy density of the Universe. The dark energy has the same status as the cosmological constant Λ introduced for a while by Einstein in order to manage a static Universe. Indeed, $\Omega_{\rm DE}$ and Λ are interrelated. The introduction of DE may have appeared as an ad hoc procedure and some explanations of its origin have been searched for. The most promising was in fact the zero-point energy of quantum fields. Even if this energy cannot be tapped and measured by ordinary means, it gravitates as all other forms of energy. The first physicist to have made such a proposition seems to be Zeldovitch [4] in 1957. Many physicists have adopted this hypothesis, arguing that the CE is a proof of the existence of the vacuum energy. Just to quote Perkins [5]: "That this concept [the vacuum energy] is *not a figment* of the physicist's imagination was already *demonstrated* many years ago, when Casimir predicted that by modifying boundary conditions on the vacuum state, the change of the vacuum energy would lead to a measurable force, subsequently detected and measured by ... "

It should be mentioned right away that the identification of the zero-point energy of quantum fields with DE poses serious problems. Indeed for a free bosonic field, the zero-point energy density is the sum over the normal modes of half a quantum of energy. Since the number of modes is infinite, it is already necessary to introduce a cut. The energy density (for photon field) then writes:

$$\varepsilon = \frac{1}{2}\sum \hbar\omega = \frac{1}{(2\pi)^3} \int^{k=k_{\rm cut}} d^3\vec{k}\hbar k c = \frac{1}{8\pi^2}\hbar\, c k_{\rm cut}^4 \,. \tag{2}$$

Since one is dealing with gravitational properties, it seems natural to take $k_{\rm cut}$ roughly equal to the inverse of Planck length. A rapid calculation yields $\varepsilon \simeq 10^{121}$ GeV/fm^{-3}, absurdly too large. Of course, one may find smaller cut-offs, but the difference with $\Omega_{\rm DE}$ is so enormous that it looks hopeless. In addition, there are other fields (and also plenty of condensates) in the particle standard model. Presently, it is fair to say that there is no clear guide to make a meaningful identification of $\Omega_{\rm DE}$ with zero-point energies.

3. The Casimir effect as a manifestation of zero-point energy

Let us assume two parallel perfectly conducting plates, separated by a distance d (see Fig. 1). For simplicity, let us consider that they are perpendicular to the z direction and infinitely long in x and y directions. Between the plates the normal modes of the free electromagnetic field are not the same as in free space. In the latter case, the modes are characterized by a wave vector $\vec{k}$ of any value. In the former case, the tangential component of the electric field and the normal component of the magnetic field have to vanish on the plates. As a consequence, the wave vector takes discrete values. So, the zero-point energy is changed and a force is acting on the plates. The latter can also be understood as arising from the fact that the pressure is not the same between the plates and outside of the plates (where the field is the same as in free space; see Ref. [6] for a discussion of this point, as of the effects of finiteness of real plates). I will not describe here

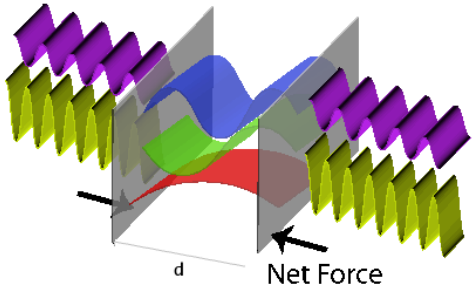

Fig. 1. Schematic representation of the normal modes of the electromagnetic field. Adapted from Ref. [8].

the normal modes (for details see Refs. [6, 7]), but it is more or less evident that the modes with a small perpendicular component of the wave vector (large wavelength) are meaningfully different from the free modes, whereas those with large wave number (small wavelength) are essentially the same in both configurations. The relevant separating parameter is $1/d$.

Not surprisingly the difference of zero-point energy (per unit surface) can be put in the form of the difference between the integral of a continuous function and the sum of the values of the same function at integer values of the suitably reduced wave number:

$$\frac{\Delta E}{S} = \frac{E_{\rm cav}}{S} - \frac{E_{\rm free}}{S}$$
$$= \frac{\hbar c \pi^2}{4d^3} \left\{ \frac{1}{2} \int_0^\infty du \sqrt{u} + \sum_{n=1}^{\infty} \int_0^\infty du \left(u + n^2\right)^{1/2} - \int_0^\infty dx \int_0^\infty du \left(u + x^2\right)^{1/2} \right\} . \quad (3)$$

Each term is infinite, but the final expression can be made meaningful by introducing a regularising factor (*e.g.* $\exp(-\beta\sqrt{u + x^2})$). The expression can thus be transformed by the Euler–McLaurin theorem and then depends only on the values of the function and its derivatives at the end of the integration domain [9]. As the regularizing parameter β goes to zero, only one term survives. One gets:

$$\boxed{\frac{\Delta E}{S} = -\frac{\hbar c \pi^2}{720\, d^3}\,.} \quad (4)$$

The Casimir force is obtained by differentiating with respect to d:

$$\frac{F}{S} = -\frac{\hbar c \pi^2}{240\, d^4}\,. \quad (5)$$

It is a remarkable result: it is a truly quantum attractive force (signalled by $\hbar$) and it is independent of the nature of the plates. No surprise that the CE is considered as a genuine property of the vacuum.

The Casimir force is a very tiny force for ordinary values of the separation distance. For $d = 1\mu\,\mathrm{m}$, $F/S \approx 4 \times 10^{-4}\,\mathrm{N/m^2}$. Of course, since it is inversely proportional to the fourth power of separation distance, it can be sizeable for much smaller values of d. It is not a surprise that it took a long time to verify Eq. (5) experimentally. After a few unconclusive or partially conclusive attempts [10–12], Eq. (5) was verified to a accuracy of a few percent by Lamoreaux [13] in 1997 and of one percent by Ederth [14] in 2000.

Casimir force is a reality that is encountered daily in nanotechnology and in particular in the so-called MEMS (Microelectromechanical Systems) technology. For a review, see Ref. [15].

4. Dependence upon the fine structure constant

Expression (4) looks universal, independent of the properties of matter. Yet it is surprising that it does not depend upon the fine structure constant as all other elementary electromagnetic effects. In fact, there is some dependence, which is hidden by the implicit hypothesis made above of a perfect conductor (through the boundary conditions on the plates). Simple considerations will help to understand the role of coupling of the field to the plates. Here I closely follow Ref. [16].

Real conductors are roughly characterized by two important parameters: the plasma frequency ω_{pl} and the skin depth δ (or the conductivity σ the two quantities being related by $\delta^{-2} = 2\pi\omega|\sigma|/c$). There is basically no propagation of electromagnetic waves inside the plasma for frequencies lower than ω_{pl} and the skin depth gives the penetration of incident waves into the conductor. Ideal conductors correspond to infinite ω_{pl} and δ. It is very convenient to use the Drude model to exhibit the effect of these two parameters. In spite of its simplicity, this model is able to produce the important features. In the Drude model, the conducting electrons are free except that they are subject to a friction force of the simplest type $\vec{f} = -\gamma\vec{v}$. It is a simple excercise to find the expressions for ω_{pl} and δ (n is the electron density):

$$\omega_{\mathrm{pl}} = \sqrt{\frac{4\pi n e^2}{m_e}}\,, \qquad \delta^{-2} = \frac{1}{2}\frac{\omega\omega_{\mathrm{pl}}^2}{\sqrt{\gamma^2+\omega^2}}\,. \tag{6}$$

The limit of a perfect conductor requires that the typical frequencies are much smaller than the plasma frequency. In the case of the CE, the typical frequencies are smaller than or of the order of c/d. Thus the approximation of a perfect conductor is satisfied when $c/d \ll \omega_{\rm pl}$, *i.e.* when

$$\alpha \gg \frac{m_c}{4\pi\, \hbar n d^2}\,. \tag{7}$$

For typical cases (Cu, $d = 1\mu$ m), the r.h.s. amounts to $\sim 10^{-6}$. Therefore, the above condition is comfortably satisfied by the actual value of the fine structure constant. In fact, Casimir's result is the $\alpha \to \infty$ limit of the effect. The exact result contains corrections involving negative powers of the fine structure constant. This can be viewed naively from the following considerations. Due to the meaning of the skin depth, the effective distance between the two plates is roughly $d + 2\delta$. Then expression (4) becomes

$$\frac{\Delta E}{S} \approx -\frac{\hbar c \pi^2}{720(d+2\delta)^3} = -\frac{\hbar c \pi^2}{720\, d^3}\left(1 - 6\frac{\delta}{d} + \dots\right)\,. \tag{8}$$

The correction appears indeed as negative powers of α.

It is also interesting to discuss the $\alpha \to 0$ limit. The latter is a bit tricky. One has to realize that, in this limit, the Bohr radius $a_{\rm B} = \hbar^2/(m_e e^2)$ becomes infinite. The atoms are much larger than in the real world. Actually, n scales as α^3, $\omega_{\rm pl}$ scales like α^2 and δ scales as $1/\alpha$. Thus δ becomes very large, the plates are transparent to the radiation and the CE goes away. As all other electromagnetic effects, the CE vanishes when $\alpha \to 0$. The only distinctive feature is that it has a finite contribution when $\alpha \to \infty$ and that the assymptotic behaviour is largely reached for the actual value of α.

5. The Casimir effect as a van der Waals force between macroscopic neutral objects

5.1. The London treatment of the van der Waals force

In Section 4, it is shown that the CE disappears when the fine structure constant, which measures the coupling between radiation and matter, vanishes. This strongly suggests that the CE is not a property of the vacuum and can be viewed as the interaction between the atoms of the two plates mediated by the electromagnetic field. There is another quantum force of this kind, the van der Waals (vdW) force between atoms or molecules. We argue below that the CE can be viewed as a vdW effect between two "gigantic molecules". It is of some interest to put the discussion in an historical perspective.

The first quantum calculation of the vdW interaction has been done by London [17] in 1937. Let us assume two spherical atoms separated by a distance r. The interaction energy between the two atoms, due to the mutual Coulomb interaction, is given in second order perturbation theory (in α and $R_{\rm at}/r$, $R_{\rm at}$ being the radius of the atoms) by:

$$\Delta E^{(2)} = -\frac{e^4}{r^6}\sum_{k\neq 0}\sum_{l\neq 0}\frac{\mid a^1_{k0}\mid^2\mid a^2_{l0}\mid^2}{E^1_k - E^1_0 + E^2_l - E^2_0}\,, \tag{9}$$

where the superscripts 1 and 2 refer to the respective atoms and where the quantities $a_{k0} = \langle k \mid \sum z_i \mid 0\rangle$ are the matrix elements of the dipole operator between the excited state $\mid k\rangle$ and the ground state $\mid 0\rangle$. Quantum mechanically, the atoms are spherical on the average only and the force arises from the fluctuations of the electric dipoles of the atoms around zero.

5.2. The long range behaviour of the van der Waals force

When he was working at the Philips company, Casimir was approached by two experimentalists, Verwey and Overbeek, who were studying colloidal suspensions of tiny particles. They had cooked up a theory, based on the vdW force, which accounted for their observations, but for very dilute suspensions [18–20]. In that case, the force seems weaker than predicted. Overbeek [19] hypothesized that this could be due to retardation effects (it takes a finite time for the fluctuation of one atom to influence the other). He asked Casimir whether he could calculate this effect. In a relatively short time, Casimir and Polder [21] generalized the London approach by incorporating the coupling of the atoms to the free radiation field, quantized in the ordinary manner. Compared to Eq. (9), this leads to intermediate states where the atom, and also the field, are excited. The calculation is technically difficult, since the system should be enclosed in a box with conducting walls and and the dimensions of the box should be raised to infinity. The results are remarkable. For small distances r, compared to the absolute value of all the elements a_{k0}, a_{l0}, the London result is recovered. For large distances, Eq. (9) becomes

$$\Delta E^{(2)} = -\frac{23\,\hbar c}{4\pi r^7}\,\alpha_1\alpha_2\,, \tag{10}$$

where the α_i's are the static polarisabilities of the atom; in second order, they are $\alpha = e^2\sum_{k\neq 0}\mid a_{k0}\mid^2\,(E_k - E_0)^{-1}$. The force indeed softens at large r due to the 7th power and factorizes, contrarily to London's result.

5.3. The van der Waals force between an atom and a conducting plane

For some unknown reason, in the very same paper, Casimir and Polder calculated, by the same method, the interaction of an atom with a conducting plane. They arrived at the following results:

$$\Delta E^{(2)}_{\text{atom}-\text{wall}} = -\frac{e^2}{4d^3}\sum_{k\neq 0} \mid a_{k0} \mid^2 , \qquad \Delta E^{(2)}_{\text{atom}-\text{wall}} = -\frac{3\hbar c}{8\pi d^4}\alpha_1 , \tag{11}$$

for small and large distances, respectively.

Casimir was particularly amazed by the simplicity of his results, especially for large distances and wondered whether they can be trusted (after all, the results are obtained in second order of standard perturbation theory). At some time, he discussed the matter with Niels Bohr, who is supposed to have simply said [22]: "Why do not you calculate the effect by evaluating the differences of zero point energies of the electromagnetic field?". Soon later, Casimir succeeded in recovering his results by the method proposed by Bohr [23]. Then he realised that the calculation is even simpler with two plates (the normal modes are indeed much simpler). And he published the calculation sketched in Section 2, in Ref. [24].

5.4. The Casimir force as a van der Waals force

The continuity between expressions (10), (11) (second part) and (4) is manifest from the fact that they have been obtained by Casimir with the same method as advocated by Bohr. But it is interesting to note that the continuity results from the fact that the polarisability of a conducting body is proportional to its geometrical extension and does not depend upon its nature (a conductor can be viewed as a dielectrics of infinite dielectric constant). Then it is rather easy to show that replacing a atom by a plane removes the polarisability and changes, up to numerical factor, the dependence upon the relevant distance by increasing the exponent by 3. The continuity between expressions (9), (11) (first part) and (4) can be obtained also by a similar limiting procedure. Replacing one of the two atoms by a plane amounts to take $e^2 \sum_{k\neq 0} \mid a_{k0} \mid^2$ divided by $E^1_k - E^1_0$ to be proportional to the cube of the size of the system. This limit increases the exponent by 3. Replacing the other atom by a plane yields, up to a constant, expression (4), if one takes into account that the difference in energy scales as the inverse of the typical distance, here d. See Ref. [7] for more details.

All these considerations point to the interpretation of the CE as a vdW effect between gigantic molecules with perfectly conducting properties.

6. The "reality" of the quantum fluctuations of the vacuum

The arguments given in Sections 4, 5 cast a serious doubt on the interpretation of the CE as a "proof" of the quantum fluctuations (zero-point energy) of the quantum vacuum. Once again, the zero-point energy of QED vacuum is a purely theoretical concept. It cannot be tapped by any electromagnetic process. If it exists, it can be tested through its gravitational properties only. As explained in Section 2, cosmological evidence is really poor. There are many arguments for and against its "reality". I quickly go through some of them:

1. In standard (Hamiltonian) field quantization, the zero-point energy comes from the normal ordering of fields in the classical Lagrangian. There is no a priori reason for doing so. It is usually justified by the link with quantization of 1D harmonic oscillators. It should be mentionned that for fermion fields, the zero-point energy is negative!
2. Interaction between neutral objects gives "no more and no less evidence of quantum fluctuations than any other one-loop effects" [16], as they vanish as $\alpha \to 0$.
3. The CE can be derived without reference to zero-point motion. For instance, Lifshitz got the Casimir results by using the general theory of fluctuations of the electromagnetic field [25–27]. Fluctuations of the field arise from the coupling to quantum fluctuations of the atoms, but there is no need to quantize the electromagnetic field.
4. Field theory can be formulated without reference to the zero-point motion, only through Green's functions and S-matrix elements. This has been done for the scalar field by Schwinger [28], and for QED by the authors of Ref. [29].
5. It should be mentioned that, alternatively, all features of QED can be reformulated from the point of view of zero point fluctuations [22].

7. Conclusion

I have tried to explain that the CE, whose reality is testified daily in the micro and the nano-worlds, and which is often advocated as a manifestation of the quantum fluctuations of the vacuum, often considered a candidate for DE, can, and perhaps, should be viewed rather as a vdW force between gigantic conducting molecules, which, as other electromagnetic effects, disappears in the weak coupling limit. It thus can hardly be taken as a property of the quantum vacuum. Furthermore, it can be formulated without reference to zero-point energy. The reality of the vacuum energy remains an open question. The fact that the CE can be derived with or without recourse to the zero-point fluctuations introduces a puzzling "duality", which may cover some deeper truth.

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