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DISSIPATIVE SUPERFLUIDS, FROM COLD ATOMS TO QUARK MATTER*

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Some results about dissipative processes in superfluids are presented. We focus on fermionic superfluidity and restrict our analysis to the contribution of phonons to bulk viscosity, shear viscosity and thermal conductivity. At sufficiently low temperatures phonons give the dominant contribution to the transport coefficients if all the other low energy excitations of the system are gapped. We first consider a system of cold fermionic atoms close to the unitarity limit. Then we turn to the superfluid phase of quark matter that may be realized at high baryonic density.

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1. Introduction

The condition for superfluidity and the hydrodynamic equations governing its normal and superfluid components have been derived by Landau in his pioneering work [1]. Superfluidity is a property of quantum fluids related with the existence of low energy excitations that satisfy the Landau's criterion for superfluidity [1–3]

$$\text{Min}\,\frac{\varepsilon(p)}{p} \neq 0\,, \tag{1}$$

where $\varepsilon(p)$ is the dispersion law of the excitation.

In general, superfluidity is due to the appearance of a condensate which spontaneously breaks a global symmetry of the system. As a consequence, in the low energy spectrum one has a Nambu–Goldstone boson, the phonon φ, with a linear dispersion law that satisfies Eq. (1).

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Superfluidity was first discovered in ^{4}He, which becomes a frictionless fluid when cooled at temperatures below 2.17 K [2, 4]. The superfluid property of ^{4}He is due to the Bose–Einstein condensation of the bosonic atoms in the lowest quantum state; thus quantum effects become macroscopically observable. Fermionic systems can become superfluid as well. According with the Cooper theorem, fermionic superfluidity takes place in quantum degenerate systems when the interaction between neutral fermions is attractive and the temperature is sufficiently low. In this case one has the formation of a di-fermion condensate that breaks a global continuous symmetry.

We discuss the hydrodynamic equations of superfluids focusing on two quite interesting fermionic systems. First, we consider trapped cold atomic gases [5] in the region of infinite scattering length (the so-called unitarity limit). Then, we turn to cold relativistic quark matter at extremely high baryonic densities in the color–flavor locked (CFL) phase [6]. These two phases of matter are quite different, however they share an important property: both systems are (approximately) scale invariant, and their properties do not depend on the detailed form of the interaction.

The hydrodynamic equations governing the fluctuations of a superfluid are essentially different from standard fluid equations. In a superfluid there are two independent motions, one normal and the other superfluid. The transport properties depend on the shear viscosity coefficient, η, on three independent bulk viscosity coefficients, $\zeta_1, \zeta_2, \zeta_3$, and on the thermal conductivity κ. These quantities can be understood as phenomenological coefficients which relate the rate of change of some quantity with the corresponding affinity [7]. The requirement that the dissipative processes lead to positive entropy production imposes that $\kappa, \eta, \zeta_2, \zeta_3$ are positive and that $\zeta_1^2 \leq \zeta_2\zeta_3$. The bulk viscosity coefficient ζ_2 plays the role of the standard bulk viscosity coefficient. On the other hand, ζ_1 and ζ_3 provide a coupling between the hydrodynamic equations of the two components. The friction forces due to bulk viscosities can be understood as drops, with respect to their equilibrium values, in the main driving forces acting on the normal and superfluid components. These forces are given by the gradients of the pressure P and of the chemical potential μ. One can write in the co-moving frame

$$P = P_{\rm eq} - \zeta_1 \,{\rm div}(V^2 \boldsymbol{w}) - \zeta_2 \,{\rm div}\, \boldsymbol{u} \,, \tag{2}$$

$$\mu = \mu_{\rm eq} - \zeta_3 \,{\rm div}(V^2 \boldsymbol{w}) - \zeta_1 \,{\rm div}\, \boldsymbol{u} \,, \tag{3}$$

where $P_{\rm eq}$ and $\mu_{\rm eq}$ are the equilibrium pressure and chemical potential, V is a quantity proportional to the quantum condensate, $\omega^\mu = -\left(\partial^\mu \varphi + \mu u^\mu\right)$ and u^μ is the velocity of the fluid.

In a conformally invariant system it has been shown in Ref. [8] that $\zeta_1 = \zeta_2 = 0$. However, ζ_3, κ and η cannot be determined by the same symmetry reasoning. Regarding the shear viscosity, it is worth mentioning that a lower bound for the shear viscosity to entropy ratio has been derived employing the AdS/CFT correspondence in the strong coupling limit within the $N = 4$ super-symmetry Yang Mills theory [9], obtaining $\eta/s = 1/4\pi$.

In the low temperature regime, $T \ll T_{\rm c}$, where $T_{\rm c}$ is the critical temperature for superfluidity, the transport properties of superfluids are determined by phonons. The contribution of other degrees of freedom is thermally suppressed. In this case one can show that $\zeta_1^2 = \zeta_2\zeta_3$, meaning that there are only two independent bulk viscosity coefficients and that one of the relation for positive entropy production is saturated; the system tends toward the state where the velocity of the superfluid component and the velocity of the normal component are parallel and bulk viscosity does not lead to dissipation.

For $T \ll T_{\rm c}$ the transport coefficients strongly depend on the phonon dispersion law. The shear viscosity is the only transport coefficient that does not vanish for phonons with a linear dispersion law. But, for the bulk viscosities and for the thermal conductivity one has to include the term cubic in momentum

$$\varepsilon(p) = c_{\rm s} p + Bp^3 + \mathcal{O}\left(p^5\right) \, . \tag{4}$$

Moreover, in the computation of the bulk viscosity one has to consider the processes that change the number of phonons. The parameter B determines whether some processes are or are not kinematically allowed. For $B > 0$ the leading contribution comes from the Beliaev process $\phi \to \phi\phi$. In the opposite case the Beliaev process is not kinematically allowed and one has to consider the processes $\phi\phi \to \phi\phi\phi$.

2. Cold atoms at unitarity

Experiments with trapped cold atomic gases have reached an extremely high level of accuracy. The system consists of fermionic atoms, like ^{6}Li or ^{40}K, in two different hyperfine states. The fermions in the two hyperfine states have opposite spin and the interaction between them can be tuned by means of a magnetic-field Feshbach resonance [10]. The strength of the interaction between atoms depends on the applied magnetic field and can be measured in terms of the s-wave scattering length. By varying the magnetic-field controlled interaction, fermionic pairing is observed to undergo the Bose–Einstein condensate (BEC) to Bardeen–Cooper–Schrieffer (BCS) crossover. In the weak coupling BCS region the system is characterized by the formation of Cooper pairs. In the strong coupling limit the system can be described as a BEC dilute gas. The unitary limit is reached when the magnetic field is tuned at the Feshbach resonance [11], where the two-body scattering length diverges.

Far from unitarity, the properties of the system are qualitatively and quantitatively well understood using mean field theory [12]. However, the mean field expansion is not reliable close to unitarity because the scattering length is much larger than the inter-particle distance and there is no small parameter in the Lagrangian to expand in. Therefore fluctuations may change the mean field results substantially.

Close to the unitarity region quantitative understanding of the phases comes from Monte Carlo simulations [13], or considering the expansion in a small parameter that comes from the generalization to an arbitrary number N of spins [14], or considering an $\varepsilon = 4 - d$ expansion and then extrapolating the results to $d = 3$ dimensions.

A different method comes from considering that at sufficiently low temperature the only active degrees of freedom are the phonons (see however [15]). The interesting point is that the effective Lagrangian for phonons can be determined from the pressure and by demanding non-relativistic general coordinate invariance and conformal invariance [16]

$$\mathcal{L}_{\rm eff} = c_0 m^{3/2} X^{5/2} + c_1 m^{1/2} \frac{(\nabla X)^2}{\sqrt{X}} + \frac{c_2}{\sqrt{m}} \left(\nabla^2 \phi\right)^2 \sqrt{X} \,, \tag{5}$$

where c_0, c_1 and c_2 are three dimensionless and universal constants. From the expression above one can determine the coefficients appearing in the dispersion law of phonons obtaining

$$c_{\rm s} = \sqrt{\frac{2\mu}{3}} \qquad \text{and} \qquad B = -\pi^2 c_{\rm s} \sqrt{2\xi} \left(c_1 + \frac{3}{2} c_2\right) \frac{1}{k_{\rm F}^2} \,. \tag{6}$$

The hydrodynamic equations describing the behavior of the system are

$$\begin{aligned} \frac{\partial j_i}{\partial t} + \partial_j (\Pi_{ij} + \tau_{ij}) &= 0 \,, \\ \frac{\partial \boldsymbol{v}_s}{\partial t} + \nabla \left(\mu + \frac{{\boldsymbol{v}_s}^2}{2} + h \right) &= 0 \,, \\ \frac{\partial E}{\partial t} + \nabla \cdot (\boldsymbol{Q} + \boldsymbol{Q}') &= 0 \,, \end{aligned} \tag{7}$$

where

$$\boldsymbol{Q}' = \boldsymbol{q} + h(\boldsymbol{j} - \rho \boldsymbol{v}_n) + \tau \cdot \boldsymbol{v}_n \tag{8}$$

and τ_{ij}, h and $\boldsymbol{q}$ are small dissipative terms that close to equilibrium are given by

$$\begin{aligned} \tau_{ij} &= -\eta \left(\partial_j v_{ni} + \partial_i v_{nj} - \tfrac{2}{3} \delta_{ij} \nabla \cdot \boldsymbol{v}_n\right) - \delta_{ij} (\zeta_1 \nabla \cdot (\rho_s (\boldsymbol{v}_s - \boldsymbol{v}_n)) + \zeta_2 \nabla \cdot \boldsymbol{v}_n) \,, \\ h &= -\zeta_3 \nabla \cdot (\rho_s (\boldsymbol{v}_s - \boldsymbol{v}_n)) - \zeta_1 \nabla \cdot \boldsymbol{v}_n \,, \\ \boldsymbol{q} &= -\kappa \nabla T \,. \end{aligned}$$

The various transport coefficients can be determined starting from the phonon Lagrangian in Eq. (5), considering the appropriate phonon scattering process. The shear viscosity of a unitary superfluid at low temperature has been computed in Ref. [17] obtaining that

$$\frac{\eta}{s} \simeq 7.7 \times 10^{-6}\, \xi^5 \frac{T_{\rm F}^8}{T^8}\,, \tag{9}$$

where $\xi = 0.2 - 0.3$ and $T_{\rm F}$ is the Fermi temperature.

The thermal conductivity from phonons of a unitary gas has not been determined, yet.

Regarding the bulk viscosity one has to consider phonon number changing processes. For cold atoms at unitarity $B > 0$ and the Beliaev process is kinematically allowed. In Ref. [18] it has been verified that in the conformal limit $\zeta_1 = \zeta_2 = 0$, while the remaining bulk viscosity coefficient evaluated within kinetic theory in the relaxation time approximation turns out to be

$$\zeta_3 \simeq 3695.4 \left(\frac{\xi}{\mu}\right)^{9/2} \frac{\left(c_1 + \frac{3}{2}c_2\right)^2}{m^8}\, T^3 + \mathcal{O}\left(T^5\right)\,. \tag{10}$$

The presence of non-vanishing transport coefficients leads to experimentally detectable effects. The damping of radial breathing mode depends on the bulk and shear viscosity coefficients. However, since ζ_2 vanishes, the bulk viscosity enters only in presence of a difference of velocity between the normal and the superfluid component and is in general negligible. In order to determine ζ_3 one should produce oscillations where the normal and superfluid component oscillate out of phase. The transport coefficients enter also into the damping rate for the propagation of first and second sound in a superfluid [2]. The damping of first sound, α_1, depends on the shear viscosity and on ζ_2, whereas the damping of second sound, α_2, depends on all the dissipative coefficients.

3. Quark matter at extremely high baryonic density

Relativistic superfluids might be realized in the interior of neutron stars where the temperature is low and the typical energy scale of particles is extremely high. In particular, in the inner crust of neutron stars the attractive interaction between neutrons can lead to the formation of a BCS condensate. Moreover, if deconfined quark matter is present in the core of neutron stars it will very likely be in a color superconducting phase [19]. Quantum Chromodynamics (QCD) predicts that at asymptotically high densities quark matter is in the color–flavor locked phase. In this phase up, down and strange quarks of all three colors pair forming a difermion condensate that is antisymmetric in color and flavor indices. CFL quark matter is a superfluid as well, because by picking a phase its order parameter breaks the quark-number $\mathrm{U}(1)_B$ symmetry spontaneously.

There are different formulations of the non-dissipative hydrodynamical equations of a relativistic superfluid [20, 21]. They were derived as relativistic generalizations of Landau's two-fluid model of non-relativistic superfluid dynamics. The dissipative terms which enter into the relativistic hydrodynamical equations were derived in [22]. As it occurs in the non-relativistic case, for a relativistic superfluid one can define the thermal conductivity, κ, the shear viscosity coefficient, η, and three independent bulk viscosity coefficients, $\zeta_1, \zeta_2, \zeta_3$.

The dispersion law of phonons in the CFL phase has been derived in [23], and one has

$$c_{\mathrm{s}} = \sqrt{\frac{1}{3}} \qquad \text{and} \qquad B = -\frac{11 c_{\mathrm{s}}}{540 \Delta^2} \,. \tag{11}$$

In the asymptotic high density limit, the gap, Δ, can be computed from QCD [24]

$$\Delta \simeq b_0 \mu g^{-5} \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right) , \tag{12}$$

where $b_0 = 512\,\pi^4 (2/3)^{5/2} \exp\left(-\frac{\pi^2+4}{8}\right)$ and g is the QCD gauge coupling constant. At very high chemical potentials, one can neglect quark masses, as far as $m_q \ll \mu$; moreover the coupling constant is small, $g(\mu) \ll 1$, and one can assume that CFL is approximately scale invariant.

A first study of the shear viscosity and of the contribution to the bulk viscosity coefficients due to phonons has been presented in Refs. [25, 26]. Beside phonons, kaons may give a sizable contribution to the transport coefficients. The contribution of kaons to ζ_2 has been studied in Ref. [27]. There is still no computation of the contribution of kaons to the remaining bulk viscosity coefficients.

Considering only the contribution of phonons one has that $\zeta_1 = \zeta_2 = 0$, while the third bulk viscosity coefficient does not vanish and depends parametrically on the physical scales as

$$\zeta_3 \sim \frac{1}{T} \frac{\mu^6}{\Delta^8} \,. \tag{13}$$

We remark that these are only approximated results that arises in the $g \ll 1$ limit, after neglecting the running of the QCD gauge coupling constant and the effect of the strange quark mass.

The contribution to thermal conductivity due to phonons and kaons has recently been studied in Ref. [28]. The thermal conductivity from phonons turns out to be dominant and given by:

$$\kappa \sim 6 \times 10^{-2} \frac{\mu^8}{\Delta^6}\,, \tag{14}$$

whereas the shear viscosity from phonons as determined in [25] is given by

$$\eta \simeq 1.3 \times 10^{-2} \frac{\mu^8}{T^5}\,. \tag{15}$$

If superfluidity occurs in the interior of compact stars, it should be possible to find signatures of its presence through a variety of astrophysical phenomena. For example, the most natural explanation for the sudden spin-up of pulsars [29], the so-called glitches, relies on the existence of a superfluid component in the interior of the star, rotating much faster than the outer solid crust. After the unpinning of the superfluid vortices, there is a transfer of angular momentum from the interior of the star to the outer crust, giving rise to the the pulsar glitch.

Another possibility to detect or discard the presence of relativistic superfluid phases consists in studying the evolution of the r-mode oscillations of compact stars [30]. R-modes are non-radial oscillations of the star with the Coriolis force acting as the restoring force. They provide a severe limitation on the rotation frequency of the star through coupling to gravitational radiation (GR) emission. When dissipative phenomena damp these r-modes the star can rotate without losing angular momentum to GR. If dissipative phenomena are not strong enough, these oscillations will grow exponentially and the star will keep slowing down until some dissipation mechanism is able to damp the r-mode oscillations. Therefore, the study of r-modes is useful in constraining the stellar structure and can be used to rule out some matter phases. For such studies it is necessary to consider in detail all the dissipative processes and to compute the corresponding transport coefficients.

4. Mutual friction

Beside shear and bulk viscosity in a rotating superfluid one has to consider one more dissipative process. This is due to the scattering of phonons off superfluid vortices and leads to the so-called mutual friction force between the normal and the superfluid components. This force can be evaluated from the differential cross-section per unit vortex length for the phonon–vortex scattering process. In case the phonons form a diluted gas, one has that

$$\frac{d\sigma}{d\theta} = \frac{c_{\rm s}}{2\pi E} \frac{\cos^2\theta}{\tan^2(\theta/2)} \sin^2 \frac{\pi E}{\Lambda}\,, \tag{16}$$

where θ is the scattering angle, E is the phonon energy and $1/\Lambda = \left(1 - c_{\rm s}^2\right)$ $[k/(2\pi c_{\rm s}^2)]$, with k is the quantized circulation.

5. Addendum: Gravity analogs

The evaluation of the cross-section for the phonon–vortex scattering can be determined employing the gravity analogs technique. According with Unruh [31], the effective action of a phonon propagating in a fluid is equivalent to the action of a scalar field in a curved background. This analogy can be used in two different ways; to study some aspects of general relativity, *e.g.* black hole evaporation, from the analogous hydrodynamical system; to study some properties of hydrodynamics using results of general relativity. In our case we want to determine the phonon–vortex cross-section in the CFL phase, using the analogous results obtained in general relativity [32]. In order to do this we separate the Nambu–Goldstone boson field as

$$\varphi(x) = \bar{\varphi}(x) + \phi(x)\,, \tag{17}$$

where $\bar{\varphi}(x)$ is a classical field describing the bulk motion of the superfluid and $\phi(x)$ is the fluctuations on the top of the superfluid. Thus we can write

$$S[\varphi] = S[\bar{\varphi}] + \frac{1}{2}\int d^4x\, \frac{\partial \mathcal{L}_{\rm eff}}{\partial(\partial_\mu\varphi)\partial(\partial_\nu\varphi)}\bigg|_{\bar{\varphi}} \partial_\mu\,\phi\partial_\nu\phi + \cdots \tag{18}$$

and introducing the acoustic metric

$$g_{\mu\nu} = \eta_{\mu\nu} + (c_{\rm s}^2 - 1)v_\mu v_\nu\,, \tag{19}$$

we have the action for the phonon field

$$S[\phi] = \frac{1}{2}\int d^4x\sqrt{-g}\, g^{\mu\nu}\partial_\mu\,\phi\partial_\nu\phi\,. \tag{20}$$

When the classical background has a vortex configuration it results in a non-trivial metric. Then one can obtain the vortex-phonon cross-section studying the propagation of the scalar field in this metric [33].

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