NEW VIEW OF THE QCD PHASE DIAGRAM*

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Quarkyonic matter is confining but can have densities much larger than Λ_{QCD}^3. Its existence is argued in the large N_c limit of QCD and implies that there are at least three phases of QCD with greatly different bulk properties. These are a Confined Phase of hadrons, a Deconfined Phase of quarks and gluons, and the Quarkyonic Phase. In the Quarkyonic Phase, the baryon density is accounted for by a quasi-free gas of quarks, and the antiquarks and gluons are confined into mesons, glueballs. Quarks near the Fermi surface also are treated as baryons. (In addition to these phases, there is a color superconducting phase that has vastly different transport properties than the above, but with bulk properties, such as pressure and energy density, that are not greatly different than that of Quarkyonic Matter.)

1. Introduction

The existence of Quarkyonic Matter [1]. was first argued in the limit of large number of colors, $N_c \to \infty$ with the number of quark flavors held fixed, N_f finite. The arguments were then generalized to the large N_c limit, with N_f/N_c fixed [2]. Computations based on he PNJL model, [3–5] were subsequently performed that argued that such a phase of matter may occur in systems with $N_c = 3$, and that the quarkyonic transition *might* be associated with chiral symmetry restoration [6, 7]. In a paper with Redlich and Sasaki, a model was proposed that could continuously vary N_c, and the behavior of the phase transition boundaries could be studied as a function of N_c. [7] In recent work with Hidaka, Pisarski and Kojo, we also argue that the quarkyoninc phase has a non-translational invariant breaking of chiral symmetry due to the formation of chiral density waves. [8] A picture of the revised phase diagram of QCD is shown in Fig. 1. The purpose of this talk is to provide a qualitative explanation of these developments.
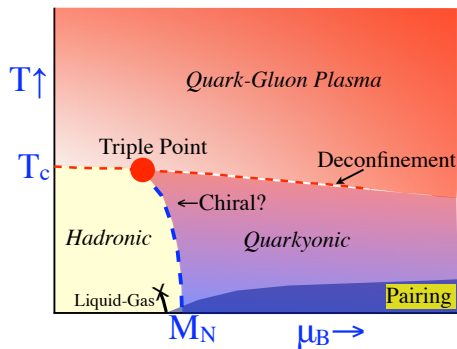

Fig. 1. The revised phase diagram of QCD.

The work on Quarkyonic matter provides perhaps the most recent study of new phases of matter at high baryon number density. There have been numerous speculations concerning high baryon number density matter, the most recent being work on color superconductivity [9, 10]. The properties of matter at finite baryon number density were in fact the first studies done in QCD concerning the properties of matter at very high energy density [11], and a history of the early developments in this area is reviewed in Ref. [12]. An issue that the work on Quarkyonic matter seeks to clarify is the relationship between confinement and high energy density matter. Our conclusion is somewhat surprising: Quarkyonic matter can have densities parametrically large compared to the QCD scale and still be confined.

2. The large N_c approximation

The large N_c approximation is the limit where the number of colors of QCD is taken to be very large, but the coupling strength $\lambda = g^2 N_c/4\pi$ is fixed. This theory is asymptotically free, and confining in vacuum. There is a Hagedorn spectrum of mesons, and mesons made of quarks interact with strength $1/N_c$ and glueballs with strength $1/N_c^2$. Baryons are made of N_c quarks, have a mass of order $N_c \Lambda_{\text{QCD}}$ and interact strongly with strength N_c.

At finite temperature and zero baryon number density, the low temperature phase is composed of light mass mesons. The number of degrees of freedom are of order one in the number of colors. At very low temperature, the Goldstone degrees of freedom are important, and there are $N_f^2 - 1$ degrees of freedom. Baryon degrees of freedom vanish in the confined phase since $e^{-M_B/T} \sim e^{-N_c}$ At very high temperatures, the degrees of freedom are $2(N_c^2 - 1) + 4N_c N_f$ arising from the gluons and quarks. Baryon degrees of freedom are unsuppressed in the Quark-Gluon Plasma.

Since the number of degrees of freedom is of order one in the hadron phase and is of order N_c^2 in the Quark-Gluon Plasma, there must be a jump in the energy density at a de-confinement temperature. Viewed from the perspective of the hadron phase, this is a limiting temperature where the energy density diverges. For finite but large N_c, at the Hagedron limiting temperature, the density of mesons becomes enormous, and at some point they must interact with one another. As this occurs, one makes a transition to a Quark-Gluon Plasma, that has a large but finite number of degrees of freedom. The rapid cross over for QCD at $N_c = 3$ is a remnant of this Hagedorn transition of large N_c QCD [13].

At finite baryon density, we see that $e^{(\mu_B - M_B)/T}$ will be of order e^{-N_c} for μ_B much less than M_b. In fact unless $(\mu_B - M_N)/M_N \sim 1/N_c$, the baryon number density is exponentially small. Therefore when μ_B is M_N, there is a baryon number generating transition. Of course, due to the very strong interactions of baryons in the large N_c limit, the threshold for baryon number generation is not the free nucleon mass, but is the mass of a nucleon in bound nuclear matter.

High density baryonic matter however remains confined. The contribution of fermion loops is suppressed by $\lambda \mu_Q^2 / N_c$, where $\mu_Q = \mu_B / N_c$ is a quark chemical potential. Such loops cannot Debye screen the confining potential until $\mu_Q \sim \sqrt{N_c} \Lambda_{\rm QCD}$ This means that the de-confinement phase transition is independent of baryon density. In spite of the fact that the baryonic matter is confined, at high density the typical bulk quantities such as pressure and energy density should be described by a weakly coupled gas of quarks. This is because for particles inside a Fermi sea, interactions are not infrared sensitive, and bulk quantities should be dominated by quark kinetic energies.

This matter that is simultaneously confined and yet has its energy density and pressure well approximated by a quasi-free gas of quarks, Rob Pisarski and I have named as Quarkyonic.

For $N_c = 3$, the phase transitions of the large N_c world can become cross overs. The Quarkyonic transition remains at $\mu_B = M_N$ for $T = 0$, since this is the physical threshold for producing baryon number. As T is increased, presumably the transition is at small μ_B. At high density, the confinement transition should become weaker since the effect of quarks will begin to reduce the effect of the confinement temperature at higher density. It also should move to lower values of T as μ_B increases. The width of the Quarkyonic transition is estimated from the baryon chemical potential which for Fermi momenta $k_{\rm F} \ll M_N$, $\mu_B = M_N + k_{\rm F}^2 / 2 M_N = M_N (1 + O(1/N_c^2))$, since the transition should be completed when $k_{\rm F} \sim \Lambda_{\rm QCD}$.

3. Physical picture of quarkyonic matter

The Hadron Phase is composed of mesons and glueballs. In the large N_c limit, there are no baryons. The number of Goldstone degrees of freedom are $N_f^2 - 1$. The Quark-Gluon Plasma is composed of quarks and gluons with $2(N_c^2 - 1) + 4N_f N_c$ degrees of freedom.

The Quarkyonic phase is composed of quarks, mesons and glueballs. The number of quark degrees of freedom are $2N_c N_f$ and in addition there are $N_f^2 - 1$ Goldstone boson degrees of freedom. The transition between these various phases should be thought of as a change in the number of degrees of freedom in a narrow range of temperature and baryon chemical potential. A phenomenological parameterization that embodies these constraints should be straightforward.

The quarkyonic phase can be thought of as a Fermi gas composed of quasi-free quarks. Near the Fermi surface the degrees of freedom are confined baryons. The thermal excitations are mesons and glueballs. Color superconductivity in the quarkyonic phase would not be allowed in the large N_c limit, but surely for finite N_c such phenomena are possible.

One can understand the Quarkyonic transition in large N_c with fixed N_c/N_f in a simple way: For large N_f there are exponentially large numbers of degenerate lowest mass baryon states, $N \sim e^{N_c F(N_c/N_f)}$. Near the Quarkyonic transitions, these states become important, since the probability that they contribution can be of order $e^{(N_c F(N_c/N_f) + \mu_B/T - M_N/T)}$. The accumulating density is however rapidly cutoff by the strong interactions of the baryons.

4. Chiral symmetry breaking

The nature of chiral symmetry breaking is not so simple to understand. In the high density phase, one expects that the effects of high density matter may lower or perhaps reduce to zero the chiral quark condensate [14–18]. Intuitively, this is because a high baryon density will exclude quarks unless they have energy near the Fermi surface. This means that if pairs form, they arise from particle hole pairs near the Fermi surface . It is indeed possible that pairing phenomena near the Fermi surface might generate chiral symmetry breaking through chiral density waves [19,20]. These pairs would break the translational invariance and a crystal would result [21]. (In previous considerations of chiral density waves, it was assumed that the potential between quarks was Coulombic. In Quarkyonic matter, the potential should be linear, and the possibility of condensation may be easier to realize.)

Together with Toru Kojo, Yoshimasa Hidaka and Rob Pisarski, [8], we have shown that such chiral density waves do occur. There is an $SU(2N_f)$ multiplet of massless Goldstone bosons associated with the breaking of chiral symmetry. The chiral order parameter is not translationally invariant, and there is a rotation between $\overline{\Psi}\Psi$ and a spin excitation $\overline{\Psi}\gamma^0\gamma^3\Psi$. The three direction is that normal to planes of equal values of the chiral order parameter. This breaking may be a one dimensional crystal structure or it might ultimately result in a three dimensional crystal. Thus in some sense, the three phases of QCD matter in the large N_c are similar to those of water. A liquid phase corresponding to confined hadronic matter, a gas phase corresponding to the Quark-Gluon Plasma and a solid phase corresponding to Quarkyonic Matter. The similarity has a further consequence: Like water, there is a triple point of phase coexistence between the three phases. This may have phenomenological consequences [22].

5. Validity of the large N_c limit

One can question whether or not the large N_c limit is applicable for $N_c = 3$. On the plus side of the equation is the fact that quarks seem to little influence the confining potential in vacuum out to a distance of about 2 fm, as seen from lattice Monte Carlo computations. The linear potential begins at a distance of about 0.2 fm. One may expect that there is some finite range of density before media quarks may short out the potential, so there is room for a Quarkyonic phase. There have also been model computations that argue for a Quarkyonic phase [6, 7]. These computations also argue that the chiral transition is approximately coincident with the Quarkyonic transition.

The negative side follows from some generic features of the large N_c limit that are not observed in baryons. The large N_c limit predicts that pion exchange has an intrinsic strength of order N_c at large distances. This requires that the ground state of large N_c baryonic matter is a Skyrme crystal [21]. On the other hand nuclear matter for $N_c = 3$ is a liquid, not a solid, and the biding energy is of order 15 MeV, not of order the nucleon mass. Moreover, lattice gauge theory computations argue that at long and intermediate distance, the nuclear force is not of strength of order N_c. How these facts become consistent with the large N_c approximation is a mystery. Perhaps the small binding energy of nuclear matter is an accident peculiar to $N_c = 3$. If not, then we do not understand something fundamental about the nature of QCD.

6. Conclusions

There may in fact be a rich substructure of phases of matter in the vicinity of $\mu_B = M_N$. This is because the density changes very rapidly there in a narrow range of chemical potential. Surely Quarkyonic Matter is a good generic acronym for the matter at high chemical potential, but the nature of this intermediate density matter may ultimately prove to be at least as interesting. At the minimum, the existence of Quarkyonic matter and of Color Superconductivity argue that the structure in the μ_B–T plane is richer than had originally been envisioned.

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